On the applicability of static modes switching in gear contact applications

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Received: date / Accepted: date

Abstract Contact modeling is an active research area in the field of multi-body dynamics. Despite the important research effort of the last decades, two main challenging issues, namely accuracy and speed, are far from being jointly solved. One main issue remains the current lack of model order reduction schemes capable of efficiently treat systems where multiple, a priori unknown, input-output locations are present. This work discusses how a methodology named static modes switching can be extended for its use in gear contact simulation. The method proposes an on line strategy for the selection of residual attachment modes for accurate local deformation prediction. The applicability of the method is discussed by means of several numerical experiments. The result is an indication that the static modes switching underlying idea can be applied also in case of impulsive phenomena like gear impacts. Moreover, a numerical study on penalty factor values and number of eigenmodes and residual attachment modes shows an interesting relation between these three quantities, the dynamic behavior of the system and the solution accuracy. Finally, some simulations are performed to compare the adopted model order reduction strategy with a reference non-linear finite element simulation.

Keywords static modes switching · gear contact · impact · flexible multibody dynamics · model reduction · static augmentation

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1 INTRODUCTION

Contact modeling is an active research area in the field of multibody (MB) dynamics. Despite a large number of publications, e.g. [1, 2, 6, 7, 10, 11] describing in detail different contact methodologies for continuous contacts and impacts, two of the main challenging issues, namely accuracy and speed, are far from being jointly solved. In modern industrial applications, the use of lighter materials and more powerful machines translates directly into the need for more accurate numerical modeling techniques. This fact is especially true when contact and impact are involved as, e.g., for gear meshing in drivelines [13] where the structural compliance of the bodies need to be addressed. In this context, faster and more accurate simulations might be necessary to obtain the correct kinematics and reaction load distribution, but also stresses and strains, extending the model validity towards durability related quantities.

This work discusses how the methodology named Static Modes Switching (SMS), presented first in [5], can be adapted and used for efficient and accurate contact simulation in case of complex geometries and in the presence of impacts as for gear meshing simulation. The SMS methodology continuously adapts the mode set used to represent component flexibility during simulation by judiciously choosing only those residual attachment modes that are contributing actively to the body deformation. This technique is particularly useful when simulating mechanisms in which loading is possible in many degrees of freedom, but only few of them are simultaneously loaded at a given moment in time. With the intention to extend the SMS applicability to different branches of contact mechanics, this paper discusses how the problem of gear meshing can be efficiently solved. One contribution of this work lies in the fact that the SMS applicability is numerically tested using an explicit Runge-Kutta scheme that presents no numerical damping as opposed to previous studies [9] where numerical damping was necessary to remove spurious high frequency oscillations introduced by the method. This scheme allows the accurate simulation of gear meshing by means of a penalty formulation. The assumptions related to the modal uncoupling of the selected residual attachment modes are analyzed for the case of impacting gears, and proportional damping is used to tune the modal contributions of the residual attachment modes that should participate only quasi statically. The simulations carried out for this work are related to the impact of two teeth of the gears in Fig.1. Results indicate that SMS can be applied to complex multibody systems in which impacts are present. This conclusion is argued by analyzing the residual attachment modes contribution to the overall deformation. It is also shown that it is possible to predict which residual attachment modes contribute significantly to the overall system deformation purely based on geometrical information available during simulation.

The paper is structured as follows: First the SMS methodology is summarized together with some details related to the adopted contact strategy. In the second part a discussion on the local flexibility introduced by the use of the residual attachment modes and on the relations between penalty factor, eigen-

Fig. 1 View of the gear pair example adopted for the numerical simulations

modes, residual attachment modes and dynamics of the system is presented. The third part contains the investigation related to the SMS applicability in a penalty factor environment for gear meshing. Some conclusions follow.

2 STATIC MODES SWITCHING AND CONTACT STRATEGY

2.1 Static modes switching

The SMS procedure is summarized in the following. For more details related to the full procedure the interested reader can refer to [5] and [9]. The pre-processing steps related to SMS can be structured in three main steps that have the final goal of creating a reduction space for each flexible body. This space is composed by a certain number of dynamically responding modes (usually the first eigenmodes) and a set of statically responding modes (as a preferred choice residual attachment modes can be used) that have a one-to-one relation with each I/O point of the system. This means that each single statically responding mode can be uniquely related to a single nodal load in a defined direction. The one-to-one relation is necessary to exploit some physical characteristics of this mode or vector set that will be discussed in detail later. The three pre-processing steps are as follows:

- Selection of a set of linearly independent vectors that reduces the degrees of freedom (DOF) of a flexible body and is able to accurately represent the static behavior and the dynamic frequency response behavior of the system. Often, a good choice of this mode set is the inclusion of eigenmodes in the frequency range of interest and all the attachment modes that are

necessary to accurately define the static deformation for each of the possible I/O points in which the system will be loaded.

- After the model is reduced with the mentioned vector set, a mass orthonormalization procedure should be performed. In the particular case that the suggested set of vectors is chosen, the orthonormalization process does not change the shapes of the eigenmodes but modifies those of the attachment modes. The full vector space spanned by all the diagonalized attachment modes represents only the part of static contribution of the attachment modes that is residual to the mode shapes. Equation 1 shows the mass orthonormalization procedure

$$\left(-\hat{\Lambda}M_{ff} + K_{ff}\right)\hat{\Psi} = \mathbf{0}, \quad (1)$$

$$\mathbf{q}_f = \hat{\Psi}\hat{\mathbf{q}}_f, \quad (2)$$

$$\mathbf{M}_{ff}^{ortho} = \hat{\Psi}^T \mathbf{M}_{ff} \hat{\Psi}, \quad \mathbf{K}_{ff}^{ortho} = \hat{\Psi}^T \mathbf{K}_{ff} \hat{\Psi}. \quad (3)$$

Here $\mathbf{M}_{ff}$ and $\mathbf{K}_{ff}$ are the reduced mass and stiffness matrices of the flexible body, $\hat{\mathbf{q}}_f$ are the modal participation factors related to the mass-orthonormalized modes $\hat{\Psi}$, $\mathbf{q}_f$ is the modal deformation projected on the orthonormalized vector set while $\mathbf{M}_{ff}^{ortho}$ and $\mathbf{K}_{ff}^{ortho}$ are the diagonal, reduced mass and stiffness matrices.

- Based on the frequency range of interest only a few vectors are chosen as dynamically responding and the remaining to respond quasi statically. If the suggested set of vectors is adopted, the eigenmodes are chosen to respond dynamically while the full residual attachment vector space to respond statically. At this point it is possible to apply independently an unit load to each of the possible I/O points of the system and to compute vector shapes that represent the residual static deformation of the flexible body with the same degree of accuracy as the original model. It has to be noticed that the procedure is general and can be performed with any starting vector set. If the suggested one is used, the final space consists simply of eigenmodes and residual attachment modes, see Eq.4-5.

$$\mathbf{M}_{red} = \begin{bmatrix} \mathbf{M}_{dyn} & 0 \\ 0 & \mathbf{M}_{res} \end{bmatrix}, \quad \mathbf{K}_{red} = \begin{bmatrix} \mathbf{K}_{dyn} & 0 \\ 0 & \mathbf{K}_{res} \end{bmatrix}, \quad (4)$$

$$\mathbf{\Psi}_{red} = [\mathbf{\Psi}_{dyn} \ \mathbf{\Psi}_{res}], \quad \mathbf{q}_{red} = [\mathbf{q}_{dyn} \ \mathbf{q}_{res}]. \quad (5)$$

This set has the property of satisfying the dynamic and geometric boundary conditions related to the floating frame definition, constraints and external forces. A comprehensive investigation on different mode sets with the same characteristics can be found in [3] and [8]. Here, $\mathbf{M}_{dyn}$ and $\mathbf{K}_{dyn}$ are the diagonal mass and stiffness sub-matrices in the equations of motion and are obtained by pre and post multiplication with the mode shapes $\mathbf{\Psi}_{dyn}$ that respond dynamically and each of which is associated with the generalized coordinates $\mathbf{q}_{dyn}$. On the contrary $\mathbf{M}_{res}$ and $\mathbf{K}_{res}$ are the full mass and

Fig. 2 Static modes switching strategy

stiffness sub-matrices in the equations of motion and are obtained by pre and post multiplication with the residual attachment modes Ψ_{res} that respond quasi statically and each of which is associated with the generalized coordinates q_{res} .

A particularly desirable characteristic of the Ψ_{res} can be exploited during simulation. If the system is externally loaded in a few DOFs, only the Ψ_{res} corresponding to the loaded DOFs are excited and contribute to the system response. In practice, besides external loads, the Ψ_{res} can still be excited by inertia coupling and through the quadratic velocity terms of the equations of motion. To limit the importance of these terms, the use of the Buckens frame, together with the SMS technique is advised. It is an approximation, validated by numerical experiments presented in [5] and further confirmed in the following sections, to assume these weak couplings to be negligible with respect to the excitation given by external loads. Under this assumption, if a load acting on a certain DOF is removed, the modal participation factor (MPF) of the corresponding Ψ_{res} will damp out fast because of its intrinsic high frequency nature that makes it more sensitive to damping in the physical system. A single sentence can be used to summarize the principle behind SMS: If a DOF is no longer loaded, the corresponding Ψ_{res} can be excluded from the simulation and vice versa when a new DOF is loaded, the corresponding Ψ_{res} should be included. Practically this can be done by reassembling, during computation, the required matrices in the equations of motion see Fig.2. Numerically speaking, the on line inclusion/exclusion of a specific mode in the equations of motion introduces small amplitude impulses into the system. In this work an explicit Runge-Kutta scheme of order four is used. This scheme does not present any numerical damping and the SMS assumptions are tested

making use of proportional damping only. In this work the following vector set is always used: eigenmodes of the gears complying with the constraints are chosen to respond dynamically while residual attachment modes are chosen to respond quasi statically.

2.2 Contact strategy

The contact strategy is based on a penalty formulation, see e.g. [11] and [12], that makes use of a tunable master-slave approach by means of node to surface elements. The Finite Elements (FE) mesh is composed by brick elements with eight nodes and linear shape functions but extensions are easily implementable. The collision detection makes use of a two level approach. At first a coarse collision detection is performed to know which teeth pairs are in contact while in a second stage, the FE nodes of each flank that are possibly in contact are checked for penetration. The code is implemented in Matlab with the use of mex functions to allow the inclusion of the C coded collision detection strategy.

3 LOCAL FLEXIBILITY AND PENALTY PARAMETER

The results that follow are related to a simulation of contact between two gears meshing. The simulations are performed by giving an initial rotational velocity to one of the gears, while the other is at rest. The chosen value of 5rad/s is of the same order of magnitude of the relative velocity with which a gear pair might enter into contact during regular operation. Only one teeth pair is in contact at each time step and only the time of a single teeth impact, from collision to detachment, is simulated. The smaller gear has 18 teeth and it is reduced in two different ways by using 50 and 100 fixed-free eigenmodes and all the residual attachment modes related to the teeth flank that will be in contact during the simulation time. The total number of modes for this gear is 50 or 100 eigenmodes plus 702 residual attachment modes. The larger gear presents 37 teeth and is reduced in three different ways using 50, 100 and 200 fixed-free dynamic modes and all the residual attachment modes related to the teeth flank that will be in contact during the simulation time. The total number of modes for this gear is 50, 100 or 200 eigenmodes plus 675 residual attachment modes. Moreover, some simulations using an increasing number of eigenmodes only and no residual attachment modes are performed. All the simulations are tested against a non-linear FE analysis performed with the software package ABAQUS [4] and against a reference simulation including 200 and 400 eigenmodes for the two gears. This last simulation is considered to be a reliable approximation of the non-linear FE reference solution. Some details of the numerical simulations can be seen in Table 1. If the number of eigenmodes is increased gradually, it is observed that the impact time becomes gradually longer due to the higher system flexibility, see Fig.3. While increasing the number of dynamic modes from 100 to 2000 modes for this gear, the

Table 1 Time simulation details

simulation time [ms]	0.2 – 0.3
time step size [ms]	10^{-5}
penalty factor [N/m]	$10^8 - 2 \cdot 10^{10}$
master slave parameter	1 – 0.5
initial condition <i>ritzel</i> [rad/s]	5
initial condition <i>grossrad</i> [rad/s]	0

Fig. 3 Contact force for different modal reductions

contact time does not increase significantly anymore. A relevant preliminary result shows that scaling the penalty factor by 2.2 the contact time of the simulation including 2000 eigenmodes is reduced to a value that is comparable more to the unrealistic simulation performed using 20 eigenmodes than to the reference solution. This high sensitivity disappears when the residual attachment modes are used, as it can be seen in Fig. 4. It can be seen here that the penalty factor c_p can be raised up by a factor 50, nicely converging towards the reference solution. The simple explanation for this lies in the inclusion of the residual attachment modes that lower the contact stiffness and for this reason, the influence of the very high penalty factor becomes less relevant since it is outweighed by the physical compliance. This is a particularly desirable characteristics of the residual attachment modes. Given this observation and the ones in the next paragraphs, the following can be stated:

- When setting up a simulation of this type, at first a value of c_p that is high enough compared to the system stiffness should be chosen. The second step is then to select how many eigenmodes are necessary to represent the system dynamics properly. Interestingly Fig. 5 shows that if all the residual attachment modes are included in the simulation to accurately model the

Fig. 4 Influence of penalty factor on the contact force

Fig. 5 Influence of eigenmodes on contact force

teeth local deformation, then the solution of the contact problem at force level is relatively insensitive to the amount of eigenmodes included.

- Noticing this, c_p can be gradually increased as in Fig. 4 until a further variation does not produce sensible changes in the force evaluation or system instability appears in the form of a numerically ill-conditioned problem.
- Once the convergence at force level is reached, the kinematic of the system needs to be inspected. If the penalty factor does not sensibly modify these quantities, then the correct dynamics is likely to be included in the system, else more eigenmodes should be introduced. It can be seen in Fig. 6 (top graph) that in this case the rotational velocity of the large gear remains

Fig. 6 Influence of c_p on the rotational velocity of the large gear (top graph) and small gear (bottom graph)

almost unchanged if the penalty factor is increased. This means that when the large gear is reduced with 200 eigenmodes, the relevant dynamics is present in the system. On the contrary (bottom graph), if the small gear is reduced using only 50 eigenmodes, a variation of the penalty factor results in a noticeable variation of the rotational speed. This fact calls for the need of more eigenmodes to be introduced in the flexible representation of the small gear. Specifically the curve representing a reduction using 100 eigenmodes for the small gear, the solution is as expected closer to the reference solution.

- Once the number of eigenmodes is increased, a new sensitivity analysis with respect to c_p should be performed. If all the above mentioned quantities nicely converge it is a good indication that the penalty factor is high enough to have a good solution accuracy and small enough to preserve well-conditioning of the system. Moreover the included eigenmodes are able to correctly represent the dynamics of the system up to the frequency of interest and the residual attachment modes take care of the local stiffness contact.

In Fig. 7 it is shown that by using the proposed approach the results are very close to the reference non-linear FE simulation.

4 SMS APPLICABILITY

The main drawback of the outlined reduction scheme is the very large amount of residual attachment modes that should be included in order to obtain accurate results. The SMS method proposes a way to select, during simulation, only

Fig. 7 Contact force solution comparison

those few residual attachment modes that participate significantly to the system solution, namely those ones related to the few nodes in contact at a certain moment in time. Two main physical effects would need to be observed in order to validate the possibility to use the static modes switching strategy. First, as anticipated, the modal contributions related to those modes associated with I/O points but that are not lying in the contact region, should not significantly contribute to the system deformation. Figure 8 shows the modal participation factors of some residual attachment modes of the two gears. The top graph shows four residual attachment modes corresponding to nodes inside the contact region or in its close vicinity, for the small gear. Particularly, only three of these nodes are in the contact region. It can be seen that the curve related to the residual attachment mode related to the node outside of the contact patch, is some orders of magnitude lower than the others. Very interestingly also, the contribution of the residual attachment modes corresponding to nodes in the contact patch damps out very fast after the contact time ends. Similar considerations can be done for the larger gear (bottom graph). Secondly, due to the fact that after a reference simulation, the contact patch related to the first impact was known, a simulation using only the residual attachment modes related to the nodes in the contact patch could be performed. The amounts of residual attachment included dropped from 702 to 81 for the small gear and from 675 to 54 for the large gear, making the simulation faster while the level of accuracy remained practically unchanged. Figure 9 shows the impact force for the simulation including all the residual attachment modes and the one including only the contributing modes. It is quite evident that the differences between the two are very small. Figure 10 shows the rotational speed for the simulation including all the residual attachment modes and the one including only the contributing modes. The top graph refers to the small gear that is

Fig. 8 Modal participation factors related to the residual attachment modes of the two gears

Fig. 9 Difference in contact forces between simulations with all the residual attachment modes and only the active ones

reduced with only 50 eigenmodes. In this case a slight variation appears between the two solutions. This is due to the fact that not enough eigenmodes are included in the simulation. To underline this even further it can be seen that these differences do not appear in the graph that refers to the larger gear that is reduced with enough eigenmodes. This fact completes the investigation related to the static modes switching idea and its applicability for gear simulations. In the present work, the switching of the residual attachment modes during time simulation is not investigated and it is subject of current research.

Fig. 10 Difference in rotational velocity between simulations with all the residual attachment modes and only the active ones for the two gears

Preliminary results show that the expected low amplitude discontinuities introduced by the methods can be successfully eliminated by exploiting both the physical damping of the structure and the master-slave contact strategy that allows the introduction of contact penetration averaging.

5 CONCLUSIONS

This work discusses the applicability of static modes switching for gear contact. The numerical validation of the underlying hypotheses of this method has been shown by analyzing the modal contribution of the residual attachment modes related to nodes in the contact region. It has also been shown that the contribution of the residual attachment modes related to nodes outside of the contact region is negligible with respect to the overall and local deformation of the two gears. This fact allows reducing the number of residual attachment modes necessary to properly simulate the local contact stiffness between two teeth in contact. It has also been shown that the local stiffness introduced by the residual attachment modes and the kinematic quantities can be related to the penalty factor and indirectly to the number of eigenmodes to be included. When all the necessary residual attachment modes are included in the simulation, the contact stiffness reduces considerably as compared to simulations where only eigenmodes are used and this allows an increase of the penalty factor. The accuracy of the proposed reduction strategy has been shown by means of comparison between a non-linear FE simulation and a model reduced with a large number of eigenmodes at force level. Future work will concentrate on the implementation of an automatic switching strategy, purely based on geomet-

rical information. The numerical difficulties related with such an automatic procedure will be also addressed.

6 ACKNOWLEDGEMENTS

The European Commission is gratefully acknowledged for the support within the Marie Curie ITN "VECOM" FP7 - 213543. The IWT Flanders within the MODRIO project is also gratefully acknowledged for their support. This work benefits from the Belgian Programme on Interuniversity Attraction Poles, initiated by the Belgian Federal Science Policy Office (DYSCO).

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