Hybrid Finite Element – Wave Based techniques for interior vibro-acoustics. Application to a large-sized vehicle model

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Abstract

Nowadays, in automotive industry, a large part of the design process takes place in a virtual environment in order to shorten the time-to-market while keeping high-quality standards. Numerous methodologies based on numerical techniques have been developed in the past decades, aimed at assessing functional performance attributes. In this paper ,the focus is on three-dimensional acoustic analysis for interior truck cabin applications in the low and low-medium frequency range, which is increasingly analyzed by means of deterministic methods. Among those there is the Finite Element Method (FEM), of which the range is practically limited to the low-frequency range because of its intrinsic features such as interpolation and pollution. Recently, a new class of deterministic techniques has been developed, under the name of Wave Based Methods (WBMs), which are based on the indirect Trefftz approach. Hybrid formulations have been developed in order to combine their main features with a wide range of other numerical techniques.

In this paper such a hybrid FE-WBM is applied to an interior truck application in order to illustrate its potential towards industry-relevant applications in the low- and mid-frequency range.

1 Introduction

Nowadays, aspects like legislation, safety and market highly influence industrial design processes, especially in the automotive sector. Functional performance attributes such as Noise, Vibration & Harshness (NVH), crashworthiness and comfort become of growing relevance in the design process. An ever increasing number of variants must be delivered in order to satisfy requirements from both legislation and customization. Therefore, a deeper and more dedicated design process must guarantee high standards on a larger number of products, while minimizing the costs and time-to-market. For these reasons, in the last decades, researchers have developed several methodologies and techniques to calculate the aforementioned attributes within virtual environments. Computer Aided Engineering (CAE) techniques, based on these numerical methods, are increasingly used during the design process for the assessment and sign-off of components and systems. As a result, dedicated efforts on experimental analysis of physical prototypes are drastically reduced.

Nowadays, the most commonly used CAE approaches for steady-state vibro-acoustic problems are the Finite Element Method (FEM) and the Boundary Element Method (BEM).

The FEM [1] is based on the discretization of the domain into a relatively large number of small domains (i.e. elements). Within these elements, the acoustic pressure field is approximated by means of the nodal variables, linearly combined with so-called shape functions, which are often polynomials. However, since the shape functions are not the solution of the governing differential equation, a fine discretization is required to limit the pollution error [2]. This results in large models, yielding practically unsolvable problems at higher frequencies. The BEM [3] is based on the boundary integral formulation of the physical problem, thus only the boundaries are discretized using surface elements. Within the surface elements, the acoustic variables are expressed by using simple polynomial shape functions. The solution is obtained in two steps: first the boundary conditions are enforced on boundaries and the system is solved with respect to the nodal unknowns. Finally, the pressure field is recovered by application of the boundary integration formulation.

Apart from the FEM and BEM family, there exists another class of methods descending from the so-called Trefftz method [4-6], whose main difference from the element based methods consists in the selection of the weighting functions and field variable expansion. Instead of approximating the variable expansion with simple shape functions, exact solutions for the governing equation are chosen. Amongst those methods applied to steady-state acoustic problems one has the Variational Theory of Complex Rays (VTCR) [7,-9] and the Wave-Based Method (WBM) [10].

In the WBM, the function set used for the variable expansion is exact solution of the Helmholtz equation, but may still violate the boundary conditions. The problem is solved by enforcing the boundary conditions in an integral sense, through a weighted residual formulation using a Galerkin scheme. As a result, the model has a moderate size respect to the FEM, yielding enhanced convergence behavior. Nevertheless, the applicability of the WBM is rather limited due to its intrinsic geometrical limitations: it has been proven that a sufficient condition for convergence is the convexity of the domain. Recent research activities [11-13] proposed a class of hybrid techniques that combine the high convergence of the WBM with the flexibility of the FE in discretizing rather complex geometries, which would otherwise require too many WBM subdomains. The hybrid FE-WBM was also applied in a context of numerical-experimental identification of lightweight structures, by combining the potential with a novel type of test setup [14]. With another research focus on the inclusion of complex and localized damping models in the WBM [15,16], the identification approach can be extended towards the identification and optimization of fully trimmed lightweight components.

In this paper, the hybrid FE-M is applied to an industry-relevant (vibro-)acoustic problems. The main modeling characteristics are discussed and investigated, in order to provide the reader with insights on applicability fields, main features and, finally, performance.

2 Numerical methods and techniques

The basic principles and the mathematical formulation behind the Wave Based Methods are presented in this section. Furthermore, the hybrid formulation, which combines the high convergence rate typical of WBMs with the versatility of FEM in discretizing acoustic domains, is presented.

2.1 Wave-Based Methods

Figure 1 illustrates a steady-state uncoupled acoustic problem, whose solution is described by the pressure field $p(\vec{r})$ within the volume V, filled with air. The acoustic excitation is a monopole q, located at the position $\vec{r}_q$.

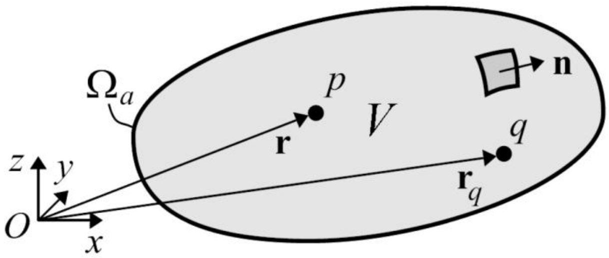

Figure 1: Steady-state acoustic problem definition. Uncoupled acoustic domain

In Eq. 1, $k_a = \dfrac{\omega}{c}$ is the wavenumber, ω is the circular frequency and c is the speed of the pressure wave in the medium. Whereas in the right hand side q is the strength of the source and $\delta(\vec{r}, \vec{r}_q)$ is the Dirac Delta function.

The boundary of the acoustic domain, Ω_a, is partitioned into 3 non-overlapping boundaries such that $\Omega_a = \Omega_p \bigcup \Omega_v \bigcup \Omega_z$. On each of the parts, pressure, velocity and impedance boundary conditions are applied, according to Eqs. 2.

- Dirichlet BCs:
  $R_p(\vec{r}) = p(\vec{r}) - \overline{p}(\vec{r}) = 0$
- Neumann BCs:
  $$R_v(\vec{r}) = \frac{j}{\rho_a \cdot \omega} \cdot \frac{\partial p(\vec{r})}{\partial \hat{n}} - \overline{v}_n(\vec{r}) = 0 \tag{2}$$
- Robin BCs:
  $R_z(\vec{r}) = \dfrac{j}{\rho_a \cdot \omega} \cdot \dfrac{\partial p(\vec{r})}{\partial \hat{n}} - \dfrac{p(\vec{r})}{\overline{Z}_n(\vec{r})} = 0$

An important characteristic of the WBM [10] is the field variable expansion (Eq. 3), which is expressed as linear combination of a set of so-called Wave Functions (Eqs. 5), which exactly satisfy the homogeneous Helmholtz differential equation.

The field variable $p(\vec{r})$ can be then be expressed as follows:

$$p(\vec{r}) \approx \hat{p}_{wb}(\vec{r}) = \sum_{a=1}^{n_a} \Phi_a(\vec{r}) \cdot p_{wb,a} + \hat{p}_q(\vec{r}) = \{\Phi\} \cdot \{p_{wb}\} + \hat{p}_q(\vec{r}) \tag{3}$$

The particular solution for Eq. 1 is described in Eq. 4

$$\hat{p}_q(\vec{r}) = \frac{j\rho\omega \cdot q}{4\pi} \cdot \frac{e^{-jk_a d(\vec{r}, \vec{r}_q)}}{d(\vec{r}, \vec{r}_q)} \tag{4}$$

In which $d(\vec{r}, \vec{r}_q)$ is the distance between the points $\vec{r}$ and $\vec{r}_q$.

In [9] the author proposes the definition of three sets of wave functions, namely r-set, s-set and t-set

$$\begin{cases} \Phi_r(\vec{r}) = \cos(k_{xr}x)\cdot\cos(k_{yr}y)\cdot e^{-jk_{zr}z} \\ \Phi_s(\vec{r}) = \cos(k_{xs}x)\cdot e^{-jk_{ys}y}\cdot\cos(k_{zs}z) \\ \Phi_t(\vec{r}) = e^{-jk_{xt}x}\cdot\cos(k_{yt}y)\cdot\cos(k_{zt}z) \end{cases} \tag{5}$$

This wave function set exactly satisfies the Helmholtz equation, provided that the condition $k_{x,i}^2 + k_{y,i}^2 + k_{z,i}^2 = k_a^2$ holds. Desmet [10] shows that a convergent set of wave functions is obtained if a limited set of $k_{x,i}$ values is selected, which satisfies the following condition:

$$\begin{cases} (k_{xr},k_{yr},k_{zr}) = \dfrac{a_1\pi}{L_x}, \dfrac{a_2\pi}{L_y}, \pm\sqrt{k_a^2 - \left(\dfrac{a_1\pi}{L_x}\right)^2 - \left(\dfrac{a_2\pi}{L_y}\right)^2} \\ (k_{xs},k_{ys},k_{zs}) = \dfrac{a_3\pi}{L_x}, \pm\sqrt{k_a^2 - \left(\dfrac{a_3\pi}{L_x}\right)^2 - \left(\dfrac{a_4\pi}{L_z}\right)^2}, \dfrac{a_4\pi}{L_z} \\ (k_{xt},k_{yt},k_{zt}) = \pm\sqrt{k_a^2 - \left(\dfrac{a_5\pi}{L_y}\right)^2 - \left(\dfrac{a_6\pi}{L_z}\right)^2}, \dfrac{a_5\pi}{L_y}, \dfrac{a_6\pi}{L_z} \end{cases} \tag{6}$$

In Eq. 6, L_x, L_y and L_z are the dimensions of the smallest bounding box that fully encloses the acoustic domain. The parameters $a_i = \{0,1,...a_{MAX,i}\}$ are integer numbers describing the wavelength associated to the functions. For the truncation of the series expansion (Eq. 3), the following frequency-dependent criterion is proposed

$$\frac{n_{a1}}{L_x} \approx \frac{n_{a2}}{L_y} \approx \frac{n_{a3}}{L_z} \approx \frac{n_{a4}}{L_x} \approx \frac{n_{a5}}{L_y} \approx \frac{n_{a6}}{L_z} \geq N\cdot\frac{k}{\pi} \tag{7}$$

This way, functions with wavenumber up to $N\cdot k_a$ are considered in the field variable expansion, with $k_a = {2\pi\cdot f}/{c}$ the physical wavenumber of the problem.

The wave function set in Eq. 5, may violate the boundary condition of the problem, which are enforced through a weighted residual formulation, as described in Eq. 8

$$-\int_{\Omega p} \frac{j}{\rho_a\omega}\cdot\frac{\partial\tilde{p}(\vec{r})}{\partial n}\cdot R_p(\vec{r})\cdot d\Omega + \int_{\Omega v}\tilde{p}(\vec{r})\cdot R_v(\vec{r})\cdot d\Omega + \int_{\Omega z}\tilde{p}(\vec{r})\cdot R_z(\vec{r})\cdot d\Omega = 0 \tag{8}$$

The application of these conditions, according to the Galerkin scheme, yields the following matrix equation, solved with respect to the wave functions coefficients.

$$[A_{ww}]\cdot\{p_{wb}\} = \{f_{wb}\} \tag{9}$$

The system matrix $[A_{ww}]$ is fully populated, frequency dependent and complex valued. The main advantage consists in the enhanced convergence rate and the size, which is sensibly smaller than FE matrices. The advantages of both methods, WB and FE, are combined together into the so-called Hybrid FE-WB formulation.

2.2 Hybrid Finite Element-Wave Based (FE-WB) methodologies

The FEM is based on the partitioning of the whole domain into a series of non-overlapping, relatively small subdomains, called elements. The field variables within these elements are approximated using a linear combination of shape functions (polynomials) and the nodal values.

$$p(\vec{r}) \approx \hat{p}_{fe}(\vec{r}\,') = \sum_{n=1}^{n_k} N_n(\vec{r}\,') \cdot p_{fe,n} = \{N\} \cdot \{p_{fe}\} \tag{10}$$

The shape functions do not satisfy the governing differential equation and may violate the boundary conditions. The error is minimized through a weighted residual formulation, based on the Galerkin method. As a result, the matrix system is generated (as in Eq. 11), which is solved for the nodal values $\{p_{fe}\}$.

$$\left([K] + j\omega \cdot [C] - \omega^2 \cdot [M]\right) \cdot \{p_{fe}\} = [Z] \cdot \{p_{fe}\} = \{f_{fe}\} \tag{11}$$

The matrix system can be split into frequency independent matrices that are relatively large, sparsely populated and real valued. The discretization scheme together with the polynomial shape functions yields two sources of error:

- *Pollution error:* the FE discretization yields the calculation of the solution at a frequency that is different from the actual frequency of the problem, due to the approximation of the wavelength.
- *Interpolation error:* the choice of simple shape functions yields an approximation regarding the actual solution. Interpolation errors are known to be the major error source at low frequencies. To reduce this effect, a rule of thumb states that a certain number of elements per wavelength is needed [17].

Given a certain FE discretization, characterized by its element size, both interpolation and pollution errors increase with frequency. Mesh refinement can be applied to improve FE solution quality, with a significant increase in computational efforts.

In automotive applications, the acoustic domain can be split into two kinds of region: large portions with a relatively simple shape and smaller domains, irregular and typically close to the acoustic boundaries. For the latter type, an FE discretization is optimal, whereas the rest is easily tackled with a number of WB domains. Following this reasoning, Pluymers [11] proposed a Direct Coupling Approach to combine FE domains and WB domains. The continuity conditions for the pressure and normal velocity are directly enforced on the WB and FE field variable approximations along the coupling interface, Ω_H in Figure 2.

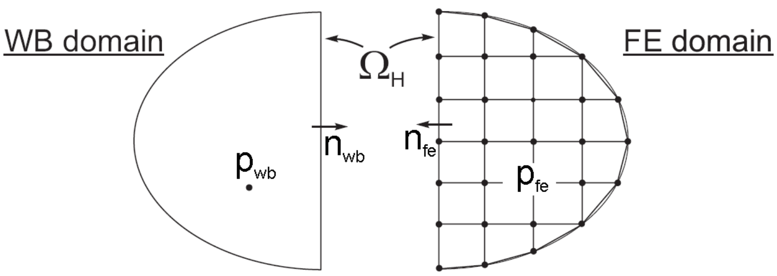

Figure 2: Direct FE-WB coupling approach

In order to take into account the mutual influence, the residual formulation for uncoupled problems must be extended with coupling terms. In the direct coupling approach, the pressure continuity is accounted in the WB residual, Eq. 8 must be enriched with the following term

$$\int_{\Omega_H} \frac{j}{\rho_a \omega} \cdot \frac{\partial \tilde{p}(\vec{r})}{\partial n} \cdot R_{h,p}(\vec{r}) \cdot d\Omega \tag{12}$$

In Eq. 12, the term $R_{h,p}(\vec{r})$ represent the pressure residual at the hybrid interface, according to Eq. 13

$$R_{h,p}(\vec{r}) = p_{wb}(\vec{r}) - p_{fe}(\vec{r}) = 0 \tag{13}$$

The continuity of the velocity field at the hybrid interface is enforced through the FE formulation.

As a result, the matrix equation for the hybrid acoustic problem is as follows

$$\begin{bmatrix} A_{ww} + C_{ww} & C_{wf} \\ C_{fw} & Z \end{bmatrix} \cdot \begin{Bmatrix} p_{wb} \\ p_{fe} \end{Bmatrix} = \begin{Bmatrix} f_{wb} + f_{fw} \\ f_{wb} + f_{wf} \end{Bmatrix} \tag{14}$$

In Eq. 6, it is possible to recognize terms regarding uncoupled acoustic problems. C_{ww} is the acoustic back-coupling matrix, while C_{wf} and C_{fw} account for the mutual interaction, occurring at the hybrid interface. The additional terms on the right hand side are due to the presence of acoustic loads in the WB domain. The global matrix equation for the coupled problem requires a large amount of computational efforts, since it is considerably large due to the FE submodel and has zones with fully populated and complex values: hence, no readily available solver can be straightforwardly applied.

In order to fully exploit the matrix properties of the different techniques, Van Hal [12,18] and Pluymers [11] proposed a three-step solution procedure, described as follows:

1) First the FE degrees of freedom (dofs) are eliminated by expressing them as function of the WB dofs (manipulation of the bottom equation, and inverting the sparse matrix Z). The FE dofs are plugged into the first equation, such that a dense matrix equation is obtained.
2) Solution of dense matrix equation for the wave contributions p_{wb}
3) Recovering the p_{fe} by simple matrix operation.

The field variable can then be calculated for the whole cavity by applying Eqs. 3 and 10, for the WB domain and the FE, respectively.

2.3 Hybrid MRFE-WB formulations

Recently, Van Genechten [13,19,20] has proposed an innovative variant of the hybrid WB formulation for vibro-acoustic problems, namely the hybrid Modally Reduced FE-WB technique.

Reduction techniques allow describing a FE model by means of so-called generalized coordinates, instead of physical coordinates (i.e. pressure, displacement…). As a result, a large sized problem can be studied by means of a reduced set of so-called generalized coordinates. One of the most commonly used substructuring techniques was proposed by Craig and Bampton [21] and is based on modal reduction techniques. The dynamic behavior is described by means of the modal coordinates, calculated for rigid wall boundary conditions. As the normal modes violate the velocity continuity at the hybrid interface, additional enrichment vectors are needed.

For the Craig-Bampton reduction the following vectors are used:

- *Fixed interface modes*. These modal vectors are computed by enforcing zero pressure condition along the interface (i.e. hybrid interface). According to a rule of thumb, the normal modes basis must be calculated up a frequency which is 2-3 times the maximum frequency of interest.
- *Constraint modes*. This set of static vectors is calculated by enforcing unit pressure to one of the interface FE dofs, while applying zero pressure on the rest of the interface dofs; the constraint modes set is obtained by repeating the procedure for every interface dof. Therefore, as many modes as the number of interface dofs are needed.

Finally, the FE nodal values can be expressed as in Eq. {15}:

$$\{pfe\} = [V_F \quad V_C] \cdot \begin{Bmatrix} \Psi_F \\ \Psi_C \end{Bmatrix} \tag{15}$$

In Eq. 6 the matrices V_F and V_C represent the fixed interface normal modes and the constraint modes, whereas the vectors Ψ_F and Ψ_C are the modal coordinates according to the Craig-Bampton approach.

The projection of the FE physical coordinates onto a modal basis, yields a new form for the residual formulation in Eqs. 8 and 12. Hence, the system of equations for the coupled FE-WB acoustic problem in Eq. 13 becomes

$$\begin{bmatrix} A_{ww} + \tilde{C}_{ww,C} & 0 & \tilde{C}_{wf,C} \\ 0 & \tilde{Z}_{FF} & \tilde{Z}_{FC} \\ \tilde{C}_{fw,C} & \tilde{Z}_{CF} & \tilde{Z}_{CC} \end{bmatrix} \cdot \begin{Bmatrix} p_{wb} \\ \Psi_F \\ \Psi_C \end{Bmatrix} = \begin{Bmatrix} f_{wb} + \tilde{f}_{fw,c} \\ \tilde{f}_{fe,F} + \tilde{f}_{wf,F} \\ \tilde{f}_{fe,C} + \tilde{f}_{wf,C} \end{Bmatrix} \tag{16}$$

The coupled problem is now solved for the wave contributions p_{wb} and the modal participation factors Ψ_F and Ψ_C. As for the direct approach, the three steps procedure is used to solve the problem. Furthermore, Van Genechten [19,22] proposes to perform the calculation of the static modes assuming that pressure at the boundaries is given by an analytical, known distribution of loads. The interface modes must be coupled to the wave functions to enforce the continuity conditions; thus a logical choice is to use the wave function set to describe the pressure distribution at the interface. As a result, a global numerical integration to describe the continuity at the hybrid interface can be applied, with a significant saving of computational costs (recall that a local integration scheme requires that at least one integration point must be used for each interface element). Although the accuracy is comparable with the direct hybrid FE-WB approach, the performance is highly improved. Aspects such as accuracy and performance regarding hybrid techniques are discussed further in this paper.

3 Industrial Application

Section 3 reports on the application of hybrid FE-WB techniques on industry-relevant acoustic problems: more specifically, the acoustic pressure field is calculated for an enclosed cavity, being part of a commercial truck’s interior cavity. Thus, the driving elements in these analyses are both the geometrical complexity and the large size, allowing to fully exploit the hybrid FE-WB formulation.

Finally, aspects such as accuracy, performance and modeling are reported and discussed, in order to show the potential of the hybrid FE-WB techniques to tackle industry-relevant applications.

3.1 Problem description

The acoustic domain, shown in Figure 6, consists of an acoustic cavity filled with air at room temperature ($\rho_a = 1.225 \ {kg}/{m^3}$, $c = 340 \ {m}/{s}$). The geometrical complexity refers the lower part of the domain, being in direct contact with the floor panel of a truck cabin. The cavity encloses a volume of 8.27 m^3 and a surface of 26.05 m^2. The excitation consists of an acoustic point source located in the central area of the domain. Several approaches are applied to solve this problem with and without the presence of absorbing material. Finally, the applied techniques are compared to each other in terms of performance.

In order to reach high efficiency, hybrid FE-WB methods are used to tackle the problem. FE discretization is applied on the lower area, because of its geometrical complexity. The upper part is modeled with one WB domain, coupled to the FE domain through a single hybrid interface (i.e. rectangular surface).

In FE-WB approaches, a numerical parameter which describes the hybridization is defined as ratio between domains volume, according to Eq. 17.

$$hr = \frac{VOL\_WB}{VOL\_TOT} \cdot 100 \tag{17}$$

The FE model for the cavity in Figure 6 is discretized with 2.523.833 finite elements (linear tetrahedral) and 433.046 nodes. According to the rule of thumb, stating that a minimum of 6 elements per wavelength is needed, the validity of the FE mesh is up to 846 Hz (and 80% of the elements are valid up to 1.316 Hz).

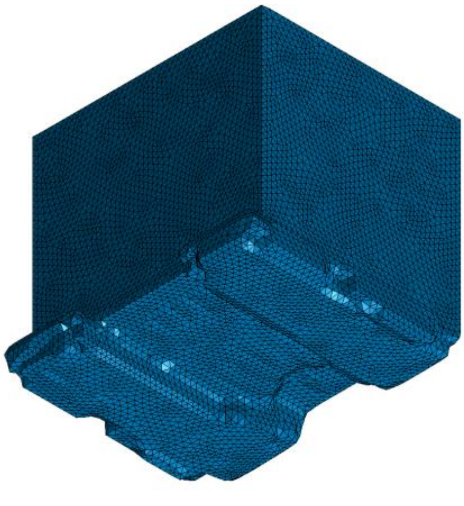

Figure 6: Industry-relevant acoustic problem. Truck's cab.

3.2 Hybrid Direct FE-WB. Robin boundary conditions

The first industrial problem under investigation consists of an acoustic cavity, with the presence of a point source with fictitious unit amplitude; in the area in contact with the cab floor, a normal impedance boundary condition is applied, with value $\bar{z} = \rho_a \cdot c \cdot (1.81 - 5.55j) \ {Pa \cdot s}/{m}$. The hybrid FE-WB, using the direct coupling approach is used to tackle the problem (Figure 7). The upper part (green) is modeled by means of a unique WB domain, directly connected to the FE counterpart through a rectangular shaped hybrid interface. The impedance boundary condition is applied on a surface of the FE part (i.e. 5.56 m^2) and it is modeled with 14.385 triangular absorbing elements (CAABSF/PAABSF in MSC/Nastran [23]), that are built upon the faces of the tetra elements.

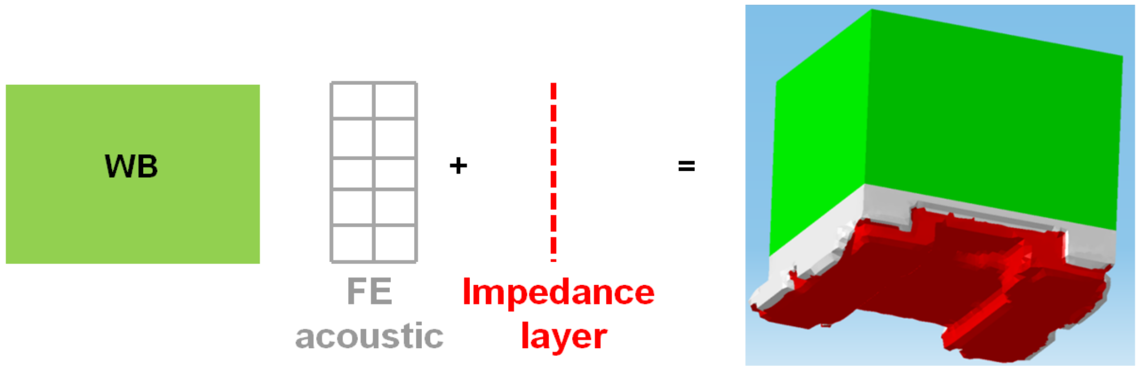

Figure 7: Hybrid FE-WB formulations. Normal impedance boundary conditions (red portion = impedance layer)

The WB domain consists of a brick-like box with dimensions 2.20 x 1.89 x 1.54 m^3, yielding a hybridization level of 78%, while the remaining FE domain consists of 596.977 tetra elements and 107.100 nodes.

The hybrid FE-WB approach is compared to the pure FE. In order to have a clear comparison, the hybrid model is built by removing elements from the original FE model; as a result, the FE discretization for the lower part is identical.

The acoustic response is virtually measured and averaged out over 9 field points and summarized in Figure 8, which reports the FRFs until 1 kHz and with a frequency resolution of 2.5 Hz.

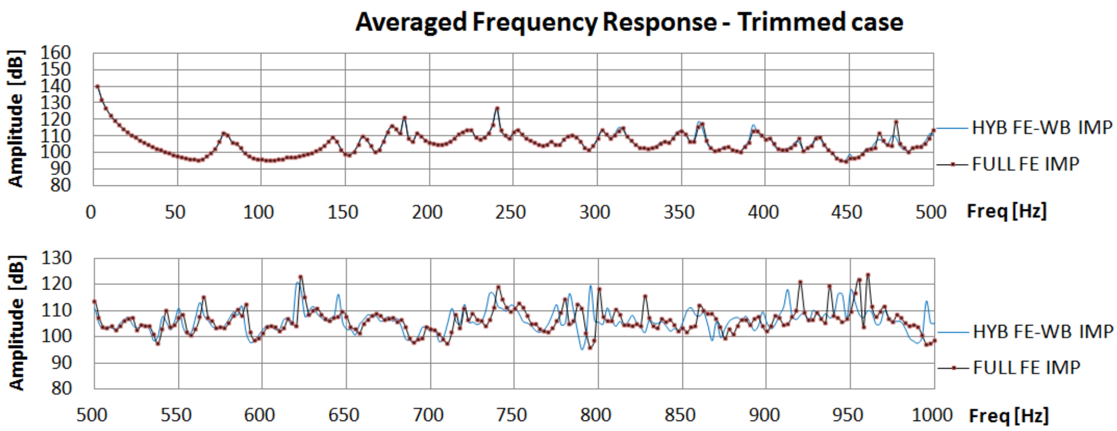

Figure 8: Example acoustic response averaged over 9 field points, under a volume velocity source as calculated with FE and with the hybrid FE-WB approach - with impedance boundary conditions.

As expected, the 2 approaches yield very similar results. The models –FE and hybrid- show identically the same behavior up to 600 Hz, where some pollution effects shift the FE response towards higher frequencies. Note that also the hybrid models suffer from pollution, but as these errors are, next to frequency, also driven by the overall problem dimensions, a geometrically smaller sized FE part in the hybrid models results in lower pollution errors. The difference in frequency between the 2 approaches can be estimated to be about 10 Hz at 900 Hz.

The hybrid FE-WB techniques show higher accuracy than classical FEM having the same mesh discretization. Alternatively, a given mesh can be used at higher frequency if deployed in hybrid techniques. This industry relevant application has proven the potential of the hybrid WBMs in the low and mid-frequency range.

3.3 Hybrid Modally Reduced FE-WB. Neumann boundary conditions

The second industrial case investigated, concerns the cab cavity previously presented, subject to an acoustic point source and rigid wall boundary conditions. The problem is tackled with the same hybrid technique and its modally reduced counterpart. Figure 9 gives an overview on the WBM modeling scheme. In the MRFE-WB the FE part is reduced according to the Craig-Bampton approach and subsequently coupled to the WB domain, as shown in section 2.

As in the previous case, the upper part of the cavity is modeled with a single WB domain and for the lower part a (modally reduced) FE discretization is used. To apply the MRFE-WB techniques, an uncoupled modal analysis is performed up to 1.000 Hz. This yields a set of 287 fixed interface normal modes, which are added to the static interface modes, calculated by applying the wave functions as loads, as described in section 2.3. Finally, a modally reduced representation for the FE domain is achieved: 287 normal modes and 4.737 static vectors (one for each interface dof) replace the original formulation (i.e. 107.100 dofs) (see also Table 1).

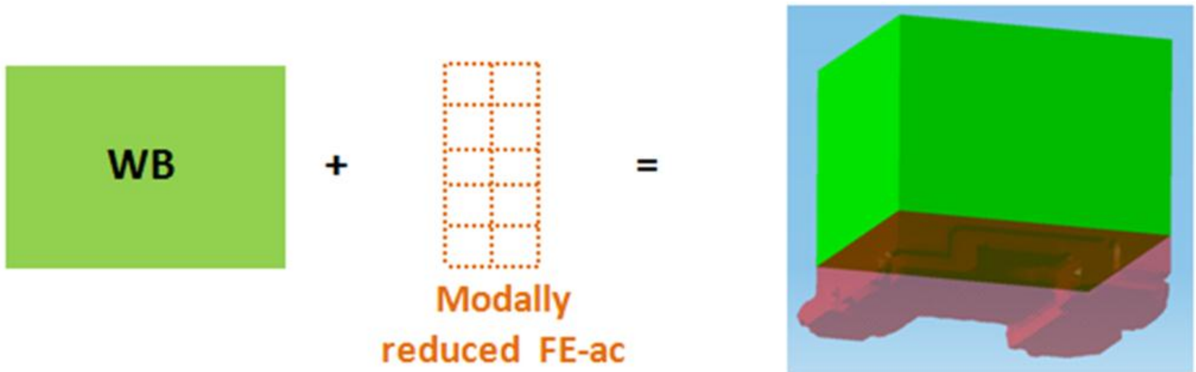

Figure 9: Hybrid MRFE-WB formulations. Rigid walls boundary conditions are applied

The mode computation is the most demanding step, since a huge number of vectors must be calculated. The modal basis, used for the Craig-Bampton reduction step, requires 3.7 GB of disk space.

The acoustic response of the problem is calculated and averaged over 9 field points, located in the upper part of the acoustic cavity. The FRFs are summarized in Figure 10, whereas Figure 11 illustrates the resulting pressure field at a given frequency.

	Full FE	**Hybrid FE-WB**	**Hybrid MRFE-WB**
WB Hybridization	0 %	78%	78%
# FE nodes	433.046	107.100	0
# Finite Elements	2.523.833	596.977	0
# WB dofs	0	(Fig. 12a)	(Fig. 12a)
# modal dofs (Craig-Bampton)	-	-	287
# interface dofs (Craig-Bampton)	-	-	4737

Table 1: Numerical models

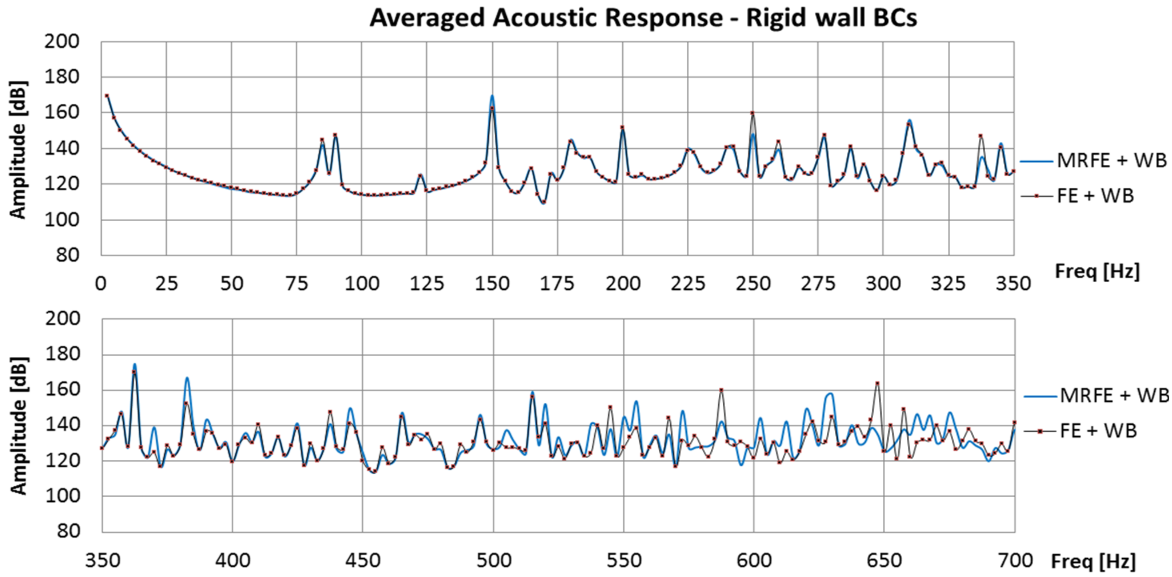

Figure 10: Acoustic response averaged over 9 field points, under a volume velocity source as calculated with the hybrid FE-WB approach: standard FE-WB and MRFE-WB. Rigid walled boundary conditions.

A comparison between the two hybrid techniques is provided up to 700 Hz, with a frequency resolution of 2.5 Hz. The comparison between the hybrid techniques, illustrates that there is a good match up to 550 Hz, which is in agreement with the rule of thumb, stating that the modal basis has to be calculated up to a frequency which is 2 times higher than the maximum frequency of interest (normal modes up to 1.000 Hz have been calculated).

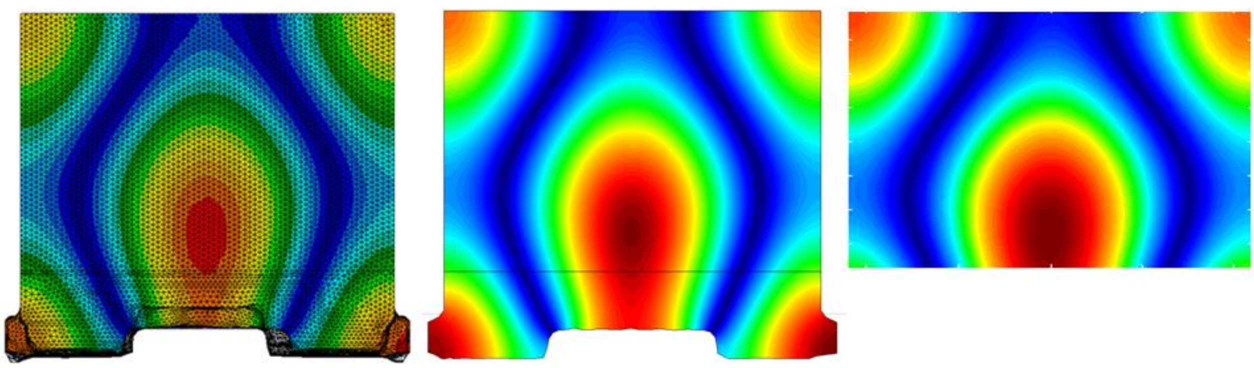

Figure 11: Example pressure maps at 150Hz for the undamped case (cross section y=0). a) left: Full FE; b) middle: WB + Physical coordinates FE; c) right: WB + MR FE.

The applied methodologies have also been compared in terms of pressure contour plots, as shown in Figure 11. The pressure field, resulting from the full FE calculation, is shown in Figure 11a. Figures 11b and 11c illustrate the results of hybrid FE-WB method, direct FE and modally reduced FE, respectively. For the latter case, only the pressure field in the WB domain is visualized (the FE pressure field is represented by its modal coordinates). Although different post processors have been used, the three methods highly match with each other.

Finally, an analysis of the performance is reported as well. All calculations are performed on a Windows-based 2.66GHz Intel Xeon system with 12 CPU’s with hyperthreading and 60 GB RAM. Figure12b summarizes the calculation time, by using different approaches and, for the FEM, different number of processors. The number of wave functions used in the variable expansion, are reported in Figure 12A. The number of wave functions for the field variable expansion increases with frequency, according to the adaptive truncation criterion in Eq. 7.

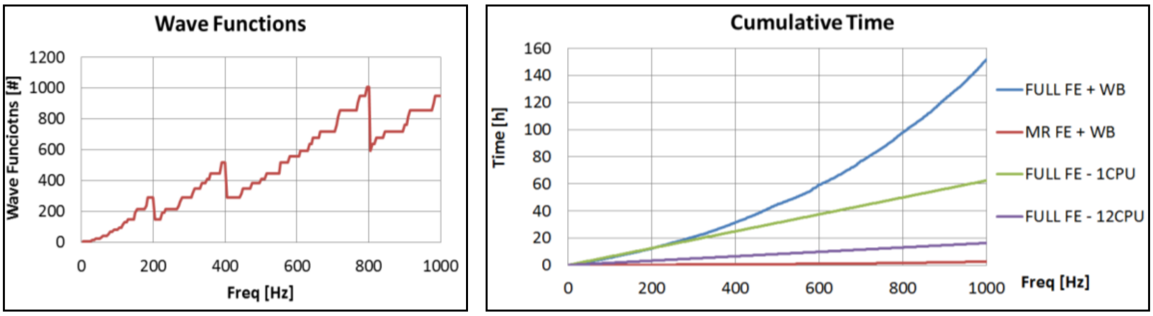

Figure 12: Hybrid MRFE-WB approach. a) Total elapsed time. b) Number of wave functions for the MRFE-WB approach as function of frequency

The elapsed times are reported in Figure 12b, for the frequency range up to 1.000 Hz and with a resolution of 2.5 Hz. The hybrid FE-WB based methods are reported and compared with direct FE. The hybrid FE-WB calculation is run on a C++ platform developed for research purposes; however, the largest part of the computational efforts is spent on solving the sparse system. For the FE methods two configurations are used, namely single core and multicore (12 CPUs), both with MSC/Nastran. The best performance is obtained with the MRFE-WB approach (2.65 hours for the full range), running on a Matlab research code. Although the code is not optimized, some phases of the MRFE-WB solution process make use of multiple CPUs. Within the hybrid methods, the main gain comes from enhancements concerning the FE counterpart. The modal reduction allows describing the uncoupled FE problem with 5.024 dofs (originally the FE domain had 107.100 dofs).

4 Conclusion

This paper reports on the application of innovative numerical techniques for 3D steady-state vibro-acoustics. A new class of deterministic methods, the so-called hybrid FE-WB methods, is discussed and applied to the analysis of an industry-relevant acoustic problem, being a large sized domain with high geometrical complexity, linked to a truck cabin.

A comparison of several techniques is presented and the currently most suitable technique for undamped problems is the hybrid MRFE-WB, since it significantly reduces the burden for the involved FE calculations, while accuracy and high convergence are guaranteed by the WB domain. Accuracy and performance have been assessed and compared with the well-established FE schemes.

Future research will focus on reduction schemes to also further enhance the hybrid models with global and especially local damping definitions.

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