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Energy-optimal time allocation of a series of point-to-point motions: demonstration on a badminton robot

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Abstract—We consider the problem of energy-optimal time allocation of a series of point-to-point motions. When performing a series of point-to-point motions, the individual execution time of each motion is, in many cases, free to choose as long as the total execution time of the series of point-to-point motions is not exceeded. This paper presents an algorithm to compute the time allocated to each motion in a series of point-to-point motions in an energy-optimal way. The algorithm is demonstrated on a technology demonstration test facility at Flanders' Mechatronics Technology Centre.

I. INTRODUCTION

There is a growing interest in reducing the energy consumption of mechatronic systems such as industrial robots, pick-and-place machines, etc. [1], [2], [3], [4]. For linear time-invariant (LTI) systems for which the energy dissipation is given by a convex expression, energy-optimal point-to-point (PTP) motion trajectories can easily be computed by solving a convex optimization problem. The dissipated energy is minimized subject to linear equality constraints, which ensure that the system is at the desired endpoint at the specified time instants.

In case of a system performing a series of consecutive PTP motions with specified dwell time intervals, the energy-optimal system input can be obtained using the same methods as for the single PTP motion problem. In many practical situations, however, only the total execution time of the series of PTP motions is important, and hence the allocation of the total execution time over the different individual PTP motions is free such that the required energy to perform the series of PTP motions can be minimized even further.

This paper presents an algorithm to compute the energy-optimal time allocation of a series of PTP motions. The algorithm is applicable to systems for which the energy-optimal system input for a single PTP motion can be computed by solving a convex optimization problem. This is for instance the case for electrical motors for which the energy losses are typically computed as the sum of the internal motor resistance losses, and the losses due to viscous friction [2].

The developed energy-optimal time allocation algorithm consists of two parts. First, an energy map is computed for

each of the actuators. The energy map contains the required energy to perform a PTP motion as a function of the travel distance and the time in which this PTP motion needs to be performed. This part of the algorithm requires solving many convex optimization problems and is therefore the most computationally demanding part of the algorithm. Fortunately, these energy maps have to be computed only once since they depend on the system dynamics only. The energy maps are independent of the series of PTP motions for which the algorithm computes the energy-optimal time allocation. The second part of the algorithm uses the energy maps of each of the actuators to compute the actual energy-optimal time allocation using dynamic programming [5].

This paper also discusses the application of the energy-optimal time allocation algorithm on a badminton robot, available at Flanders' Mechatronics Technology Center (FMTC). The badminton robot serves as a test facility to demonstrate technological developments to FMTC's member companies. Details on the mechanical design, actuation, visual detection of the shuttle, shuttle trajectory estimation and interception, and control hardware and software of the badminton robot can be found in [6].

This paper is organised as follows. Section II describes the badminton robot and the way the energy-optimal time allocation algorithm is demonstrated. The energy-optimal time allocation algorithm is detailed in section III. Then, section IV discusses the results of the presented algorithm for a test case on the badminton robot. Finally, section V summarizes the conclusions.

II. THE BADMINTON ROBOT

Fig. II shows the 3 degree of freedom badminton robot:

- A linear motor is used to position the robot across the field.
- A rotary motor is used to adjust the position of the racket on the interception plane.
- Another rotary motor, called the hit motor, is used to perform the actual hit.

The position or angular position of each of the actuators is measured and fed back to a PID-position controller. Fig. 2

Fig. 1. Badminton robot with three degrees of freedom

Fig. 2. Control configuration of each of the actuators of the badminton robot

shows the control configuration of each of the actuators. In Fig. 2, k denotes the discrete time index, $r(k)$ denotes the reference signal (in metres or degrees), $u(k)$ denotes the controller output, which is proportional to the current reference of the actuator, and $y(k)$ represents the output, i.e. the (angular) position of the motor. In the remainder of this paper, the position of the linear motor is denoted by $x_{\text{lin}}(k)$, and the angular position of the rotary motor is denoted by $\theta(k)$. The motion trajectory of the hit motor is assumed to be fixed. For this reason, the energy dissipation of the hit motor is left out in the minimization of the total energy dissipation.

The energy dissipation of the linear motor E_{lin} and rotary motor E_{θ} are given by (1a) and (1b), where u_{lin} and u_{θ} denote the control signals, which are proportional to the current reference of the linear and rotary motor respectively [2].

$$E_{\text{lin}} = R_{\text{lin}} \|u_{\text{lin}}\|_2^2 + c_{\text{lin}} \|\dot{x}_{\text{lin}}\|_2^2 \quad (1a)$$

$$E_{\theta} = R_{\theta} \|u_{\theta}\|_2^2 + c_{\theta} \|\dot{\theta}\|_2^2 \quad (1b)$$

In (1a) and (1b), R_{lin} , R_{θ} , c_{lin} and c_{θ} are proportional to the motor resistances and viscous friction coefficients of the linear and rotary motor respectively.

During a normal play, a vision system detects the trajectory of the shuttle. Using this information, the arrival position and arrival time of the shuttle are estimated. The linear and rotary motor of the badminton robot are commanded to reach the arrival position before the shuttle does. Finally, the hit motor hits the shuttle when the shuttle arrives. To demonstrate the energy-optimal time allocation algorithm, the series of PTP motions must be known before executing the series of PTP motions. For this reason, the presented method is not applicable during a normal play where the desired endpoint and motion time of each PTP motion are only known after the opponent's hit.

In this demonstration, the aim is to serve a series of shuttles successively from different fixed locations within a given

Fig. 3. Illustration of the badminton racket's position x_r along the linear axis, the linear motor position x_{lin} and the rotary motor angle θ .

total time in an energy-optimal way. A number of shuttle drop mechanisms are mounted along the linear axis of the badminton robot. Only the position along the linear axis is fixed, the height at which the badminton robot hits the shuttle is not specified. In other words, the badminton racket has to reach the desired end position along the linear axis but the robot is free to divide the effort between the linear and the rotary motor. The badminton racket's position $x_r(k)$ along the linear axis is given by:

$$x_r(k) = x_{\text{lin}}(k) + l \cdot \sin \theta(k), \quad (2)$$

where $l = 0.65$ m is the length of the badminton racket. This is illustrated in Fig. 3.

In order to achieve the true energy-optimal solution, this extra degree of freedom should be taken into account in the optimization. This will result in an energy-optimal time allocation algorithm, specifically made for this demonstration, that not only optimizes the time allocation of the series of PTP motions, but also the angular position of the rotary motor during the dwells. The presented methodology is also applicable to other configurations of actuators such as XY-tables. This paper, however, only presents the dedicated algorithm for the badminton robot demonstration. This algorithm optimizes both the angular dwell position θ of the rotary motor and the allocated time, for each of the serves.

Energy-optimality is obtained by first running the energy-optimal time allocation algorithm and then optimizing the input signals of the different actuators. Currently, the algorithm does not yet account for system limitations such as input constraints, velocity constraints, etc. The time at which each badminton shuttle is dropped results from the energy-optimal time allocation algorithm.

III. ENERGY-OPTIMAL TIME ALLOCATION ALGORITHM

This section describes the energy-optimal time allocation algorithm. In the first part of this section, the calculation of the energy maps is discussed. The second part discusses the actual energy-optimal time allocation using the energy maps of each of the actuators.

A. Computing minimal energy maps

For each of the actuators, the minimally dissipated energy $E(d, N)$ to perform a single PTP motion as a function of the travel distance d and the execution time N needs to be stored in a map. Using the lifted system representation [7], the system

dynamics of each of the actuators of the badminton robot is written in the following form:

$$\begin{bmatrix} y \\ \dot{y} \\ u \end{bmatrix} = \begin{bmatrix} P_y \\ P_{\dot{y}} \\ P_u \end{bmatrix} r, \quad (3)$$

where P_y , $P_{\dot{y}}$ and P_u are lower-triangular Toeplitz matrices containing the impulse responses of the LTI systems with the reference signal $r(k)$ as input and position $y(k)$, velocity $\dot{y}(k)$, and control signal $u(k)$, respectively, as output. For the linear motor system these outputs are denoted by $x_{\text{lin}}(k)$, $\dot{x}_{\text{lin}}(k)$, and $u_{\text{lin}}(k)$. For the rotary motor system these outputs are denoted by $\theta(k)$, $\dot{\theta}(k)$, and $u_{\theta}(k)$.

At the end of each of the PTP motions, the badminton robot needs to stand still for a given number of samples denoted by N_{dwell} . For a given value of d and N , the energy-optimal reference signal $r \in \mathbb{R}^N$ to perform a single PTP motions is obtained by solving the following convex optimization problem:

$$\min_{r, u, \dot{y} \in \mathbb{R}^N} R \|u\|_2^2 + c \|\dot{y}\|_2^2, \quad (4a)$$

$$\text{subject to } u = P_u r, \quad (4b)$$

$$\dot{y} = P_{\dot{y}} r, \quad (4c)$$

$$\tilde{P}_y r = d, \quad (4d)$$

where $\tilde{P}_y = P_y(N - N_{\text{dwell}} + 1 \dots N, 1 \dots N)$, and $d \in \mathbb{R}^{N_{\text{dwell}}}$. In (4), the dissipated energy (4a) is minimized subject to a set of equality constraints, which represent the system dynamics (4b), (4c), and constraints (4d) which enforce the output to equal the desired value during the dwell of the PTP motion.

To obtain the energy maps for each of the actuators many convex optimization problems need to be solved. In order to reduce the computational cost of this step, we show that the optimal solution r^* of (4) is proportional to the travel distance d , and hence the energy dissipation, given by (1), is proportional to d^2 . As a result:

$$E(N, d) = d^2 E(N, 1). \quad (5)$$

This relation allows us to reduce the computational cost by computing the energy map $E_{\text{lin}}(N)$ of the linear motor for $d = 1$ m only, and the energy map $E_{\theta}(N)$ of the rotary motor for $d = 1^\circ$ only. Moreover, these energy maps have to be computed only once since they depend on the system dynamics only, the dependency on d is eliminated using (5).

To show that the optimal reference signal r^* of (4) is proportional to the travel distance d , the equality constraints (4b) and (4c) are substituted in the objective function (4a):

$$\min_{r \in \mathbb{R}^N} r^T (c P_{\dot{y}}^T P_{\dot{y}} + R P_u^T P_u) r, \quad (6a)$$

$$\text{subject to } \tilde{P}_y r = d, \quad (6b)$$

and the Karush-Kuhn-Tucker optimality conditions of (6) are derived [8]:

$$\begin{bmatrix} c P_{\dot{y}}^T P_{\dot{y}} + R P_u^T P_u & \tilde{P}_y^T \\ \tilde{P}_y & 0 \end{bmatrix} \begin{bmatrix} r^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} 0 \\ d \end{bmatrix}. \quad (7)$$

Solving this set of $N + N_{\text{dwell}}$ linear equations in $N + N_{\text{dwell}}$ variables gives the optimal primal variables r^* and dual variables λ^* of (6). The solution of (7) is clearly proportional to the travel distance d . Consequently, the energy dissipation, given by (1), is proportional to d^2 and (5) holds.

B. Energy-optimal time allocation using dynamic programming

Consider a series of Q PTP motions to be executed within N_{tot} samples starting from the initial position: $x_{\text{lin}} = 0$ m and $\theta = 0^\circ$. Let x_{ri} , θ_i , E_i , and N_i for $i = 1, \dots, Q$ respectively denote the desired badminton racket's position, the angular position during the dwell, the required energy, and the execution time of each of the Q PTP motions.

The second part of the energy-optimal time allocation algorithm starts by computing the minimal energy $E_{12}(N_{12}, \theta_2)$ to perform the first two PTP motions as a function of the execution time N_{12} of the first two PTP motions, and the angular position θ_2 during the dwell of the second PTP motion. The angular position θ is gridded such that the minimal energy $E_{12}(N_{12}, \theta_2)$ can be stored in a two-dimensional lookup table.

For each value of N_{12} and θ_2 , the minimal energy $E_{12} = E_1 + E_2$ is computed by solving the following minimization problem:

$$E_{12}(N_{12}, \theta_2) = \min_{N_1, \theta_1} E_1(N_1, \theta_1) + E_2(N_2, \theta_2, \theta_1) \quad (8a)$$

with

$$N_2 = N_{12} - N_1, \quad (8b)$$

$$E_1(N_1, \theta_1) = (x_{r1} - l \sin \theta_1)^2 E_{\text{lin}}(N_1) + \theta_1^2 E_{\theta}(N_1), \quad (8c)$$

$$E_2(N_2, \theta_2, \theta_1) = (x_{r2} - l \sin \theta_2 - x_{r1} + l \sin \theta_1)^2 E_{\text{lin}}(N_2) + (\theta_2 - \theta_1)^2 E_{\theta}(N_2). \quad (8d)$$

Note that the energy of each PTP motion, given by (8c) and (8d), contains the energy dissipation of both the linear and the rotary motor.

Problem (8) is solved by constructing the two-dimensional tensor $E_1(N_1, \theta_1) + E_2(N_2, \theta_2, \theta_1)$ and simply finding its smallest element. The minimizers $N_1^*(N_{12}, \theta_2)$, and $\theta_1^*(N_{12}, \theta_2)$ of these minimization problems are also stored in a two-dimensional lookup table.

In case of a series of only two PTP motions, the minimal energy to perform the series of PTP motions within N_{tot} samples and the corresponding optimal angular position θ_2^* can now easily be obtained by taking the minimum of $E_{12}(N_{12} = N_{\text{tot}}, \cdot)$. The energy-optimal time allocation N_1 , $N_2 = N_{12} - N_1$, and the optimal angular position θ_1^* are then retrieved from the lookup tables $\theta_1^*(N_{12}, \theta_2)$ and $N_1^*(N_{12}, \theta_2)$.

In case of a series of more than two PTP motions, the third PTP motion is added to the first two in a similar way resulting in the minimal energy $E_{123}(N_{123}, \theta_3)$ to perform the first three PTP motions. For each value of N_{123} and θ_3 , the minimal energy E_{123} is computed by solving the following

minimization problem:

$$E_{123}(N_{123}, \theta_3) = \min_{N_{12}, \theta_2} E_{12}(N_{12}, \theta_2) + E_3(N_3, \theta_3, \theta_2) \quad (9a)$$

with

$$N_3 = N_{123} - N_{12}, \quad (9b)$$

$$E_3(N_3, \theta_3, \theta_2) = (x_{r3} - l \sin \theta_3 - x_{r2} + l \sin \theta_2)^2 E_{\text{lin}}(N_3) + (\theta_3 - \theta_2)^2 E_{\theta}(N_3). \quad (9c)$$

Again, problem (9) is solved by constructing the matrix $E_{12}(N_{12}, \theta_2) + E_3(N_3, \theta_3, \theta_2)$ and simply finding its smallest element. The minimizers $N_{12}^*(N_{123}, \theta_3)$, and $\theta_2^*(N_{123}, \theta_3)$ of these minimization problems are again stored in two-dimensional lookup tables.

This procedure is repeated until the last PTP motion is added and we obtained the final three lookup tables: $E_{1..Q}(N_{1..Q}, \theta_Q)$, $N_{1..Q-1}^*(N_{1..Q}, \theta_Q)$, and $\theta_{Q-1}^*(N_{1..Q}, \theta_Q)$.

The total execution time of the series of Q PTP motions $N_{1..Q}$ is set equal to N_{tot} and the minimal energy is obtained by taking the minimum element of $E_{1..Q}(N_{1..Q} = N_{\text{tot}}, :)$. This also results in the corresponding energy-optimal angular position θ_Q^* for the last dwell. The energy-optimal angular position θ_{Q-1}^* and the energy-optimal time allocated to the first $Q-1$ PTP motions can now be read out from the lookup tables $\theta_{Q-1}^*(N_{1..Q}, \theta_Q)$ and $N_{1..Q-1}^*(N_{1..Q}, \theta_Q)$, hence $N_Q^* = N_{1..Q} - N_{1..Q-1}^*$ is also known. This procedure is repeated until the time allocated to each PTP motion and the optimal angular position of each PTP motion are read out from the lookup tables.

IV. SIMULATION RESULTS ON A BADMINTON ROBOT

The badminton robot is commanded to perform five serves in a given order within three seconds starting from the initial position: $x_{\text{lin}} = 0$ m and $\theta = 0^\circ$. The execution time for each of the PTP motions is free to minimize the required energy to perform the series of PTP motions. Five shuttle drop mechanisms are placed along the linear axis of the badminton robot at the following positions:

Fig. 4. Badminton robot in its initial position with the five shuttle drop mechanisms.

$$\begin{aligned} x_{r1} &= 1 \text{ m}, & x_{r2} &= -1 \text{ m}, \\ x_{r3} &= -0.7 \text{ m}, & x_{r4} &= -0.4 \text{ m}, \\ x_{r5} &= 0.4 \text{ m}. \end{aligned} \quad (10)$$

At each of these positions, the badminton robot needs to stand still for 0.1 s to perform the hit. Fig. 4 depicts the badminton robot in its initial position together with the locations of the five shuttle drop mechanisms.

First, the energy maps of the rotary and the linear motor are computed. The required energy to perform a PTP motion of an arbitrary distance is computed for execution times from 0.105 to 3 s. At a sampling rate of $f_s = 200$ Hz, this involves solving 580 convex optimization problems of the form (4) for each motor. The energy maps are computed in 77 s.

The second part of the algorithm optimizes both the angular position and the time allocation for each PTP motion by solving problem (8) and 3 problems of the form (9) until the final lookup tables $E_{1..5}(N_{1..5}, \theta_5)$, $N_{1..4}^*(N_{1..5}, \theta_5)$, and $\theta_4^*(N_{1..5}, \theta_5)$ are computed. The total execution time $N_{1..5}$ is set equal to $3 \text{ s} \cdot 200 \text{ Hz} = 600$ samples, and the optimal time allocation and optimal angular positions are recursively read out from the lookup tables. In case of an angular resolution of 2° , the computation time of the second part of the algorithm is 48 s. For other applications such as gantry robots, the time allocation is the only optimization variable of the energy-optimal time allocation algorithm. In this case, the second part of the energy-optimal time allocation algorithm can be computed in a few seconds or even less.

PTP motion	$d(\text{m})$	$N \cdot T_s(\text{s})$	$\theta(^{\circ})$	$E(\text{J})$
1	1	0.805	60	32.5
2	2	1.01	-56	91
3	0.3	0.255	-22	13.7
4	0.3	0.25	4	10
5	0.8	0.68	64	27.1

TABLE I
ENERGY-OPTIMAL MOTION TIMES, ANGULAR DWELL POSITIONS, AND DISSIPATED ENERGY FOR EACH OF THE PTP MOTIONS.

The energy-optimal time allocation algorithm finds the energy-optimal dwell moments and energy-optimal angular dwell positions for each of the PTP motions. Table I shows the travel distance, the execution time (including the dwell time of 0.1 s), the angular dwell position, and the dissipated energy for each of the PTP motions. This table shows that the relation between the travel distance and the optimally allocated time is not straightforward.

Fig. 5 shows the optimized pose of the badminton robot for each of the dwells. This clearly shows that allowing the rotary motor to cover a part of the distance results in a lower overall energy dissipation of the badminton robot. Fig. 6 and 7 show the optimal current reference and corresponding motion of the linear and rotary motor respectively.

The proposed time allocation scheme is compared with an equal time allocation scheme. For the considered example, the equal time allocation scheme allocates 0.6 s to each PTP motion such that the total execution time remains 3 s. The angular dwell positions are kept equal to the optimal values given in Table I. The minimal energy dissipation in case

Fig. 5. Initial position and energy-optimal poses of the badminton robot for each of the dwells

Fig. 6. Optimal current reference and corresponding motion of the linear motor.

of equal time allocation is 456 J whereas the energy-optimal time allocation results in a total energy dissipation of only 153 J. This comparison shows the potential energy savings of optimizing the time allocation of a series of PTP motions.

V. CONCLUSION

This paper presents an energy-optimal time allocation algorithm for a series of PTP motions. Optimizing the dwell moments of intermediate PTP motions can significantly reduce the energy consumption of mechatronic systems such as pick-and-place machines. The presented methodology is demonstrated on a technology demonstration test setup. Ongoing work focuses on extending the proposed method for constrained linear

Fig. 7. Optimal current reference and corresponding motion of the rotary motor.

systems. Experimental validation on the badminton robot is planned in short term.

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