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Numerical Magnitude Representations and Individual Differences in Children's Arithmetic
Strategy Use

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Abstract

Against the background of neuroimaging studies on how the brain processes numbers, there is now converging evidence that numerical magnitude representations are crucial for successful mathematics achievement. One major drawback of this research is that it mainly investigated mathematics performance as measured through general standardized achievement tests. We extended this research by investigating the association between these numerical magnitude representations and children's strategy use during single-digit arithmetic. Our findings reveal that children's symbolic but not nonsymbolic numerical magnitude processing skills are associated with individual differences in arithmetic. Children with better access to magnitude representations from symbolic digits, retrieve more facts from their memory and are faster in executing fact retrieval as well as procedural strategies. These associations remain even when intellectual ability, digit naming and general mathematics achievement were additionally controlled for. This all indicates that particularly the access to numerical meaning from Arabic symbols is key for children's arithmetic strategy development, which suggests that educators and remedial teachers should focus on connecting Arabic symbols to the quantities they represent.

Keywords: numerical magnitude; arithmetic strategy use; distance effect; access deficit; dyscalculia

Numerical Magnitude Representations and Individual Differences in Children's Arithmetic Strategy Use

One of the ways in which cognitive neuroscience has impacted on educational research – against the background of studies on how the brain processes number – is by drawing the attention to the importance of numerical magnitude representations for successful mathematical development (Butterworth, Varma, & Laurillard, 2011; De Smedt et al., 2010; Price & Ansari, 2012). Originally, neuroimaging research has inspired the study of children with dyscalculia – a specific learning disorder in mathematics – but more recently, studies in typically developing children have attributed a crucial role to numerical magnitude representations in predicting individual differences in mathematics achievement (e.g., Bugden & Ansari, 2011; De Smedt, Verschaffel, & Ghesquière, 2009; Halberda, Mazocco, & Feigenson, 2008; Holloway & Ansari, 2009). One major drawback of this research is that it mainly investigated mathematics performance with broad general standardized achievement tests, which typically assess a wide variety of mathematical skills (e.g., arithmetic, problem solving, geometry, etc.) without time constraint, and which only yield a total score that reflects performance averaged across various mathematical domains. It remains, however, to be determined how numerical magnitude representations are associated with specific mathematical skills, such as children's use of arithmetic strategies. The focus on arithmetic strategy use may help us to pinpoint associations between mathematics achievement and numerical magnitude representations in a more precise way, and such knowledge will be beneficial to devise appropriate educational interventions.

Extending the existing body of data, the present study aims to examine how numerical magnitude representations contribute to one specific and crucial aspect of mathematics development, namely children's use of strategies when solving elementary arithmetic. This focus is guided by the observation that arithmetic constitutes a major building block for

subsequent mathematics achievement (Kilpatrick, Swafford, & Findell, 2001) and that deficits in arithmetic fact retrieval constitute the hallmark of children with mathematical learning disabilities or dyscalculia (e.g., Geary, 2010; Jordan, Hanich, & Kaplan, 2003).

Various studies have examined the strategies that children (but also adults) use when solving single-digit additions and subtractions. These problems are typically solved either by directly retrieving the answer from long-term memory or by using a procedural strategy, such as counting or decomposing the problem into smaller facts (Siegler, 1996). There is large-subject variability in the use of these strategies (Dowker, 2005; Imbo & Vandierendonck, 2007) and these individual differences might be explained by variations in the ability to represent numerical magnitudes. For example, it might be that more precise representations of numerical magnitude help children to develop more advanced procedural strategies, which in turn results in a faster reliance on arithmetic fact retrieval. To the best of our knowledge, however, there are currently no studies that have investigated the specific associations between arithmetic strategy use and numerical magnitude representations, even though there is consistent evidence that the ability to represent numerical magnitudes is related to individual differences in general mathematics achievement (e.g., Butterworth et al., 2011; Piazza, 2010, for a review).

The ability to represent numerical magnitudes is typically investigated by means of Arabic digit and dot comparison tasks, in which children have to identify the larger of two numerosities. Performance on these tasks is typically characterized by a distance effect: children are slower and less accurate in deciding which of two numerosities is larger, when the numerical distance is small (e.g., 3 vs. 4), than when the distance is large (e.g., 3 vs. 8) (Moyer & Landauer, 1967). This distance effect is assumed to arise from overlapping internal representations of numerical magnitudes: numbers closer to each other have larger representational overlap than numbers further apart (Noël, Rousselle, & Mussolin, 2005). The

size of this distance effect provides an indication of the distinctness or preciseness of numerical magnitude representations. Moreover, it has been uniquely related to individual differences in mathematics achievement, indicating that more precise representations of numerical magnitude are associated with higher mathematics achievement (Bugden & Ansari, 2011; Holloway & Ansari, 2009). Even more compelling are data by De Smedt et al. (2009), who revealed by means of longitudinal data that the size of the distance effect at the start of first grade predicted subsequent mathematics achievement one year later in second grade.

A crucial remaining question is, however, whether the representation of numerical magnitudes per se, or its access via symbolic digits is important for mathematical achievement (Rousselle & Noël, 2007; De Smedt & Gilmore, 2011) and, consequently, arithmetic strategy use. To disentangle between both possibilities, performance on numerical magnitude comparison tasks with (Arabic digits) and without (dots) symbolic processing requirement should be compared. If both nonsymbolic and symbolic tasks predict arithmetic strategy use, this favors the idea that numerical magnitude representations in itself are crucial for arithmetic strategy development. If symbolic but not nonsymbolic tasks predict individual differences in arithmetic strategy use, this suggests that the access to numerical meaning from symbolic digits is key. This issue has been approached by studies that examined the associations between numerical magnitude representations and children's general mathematics achievement, but the findings of this research remain inconclusive. For example, Holloway and Ansari (2009) showed that the symbolic, but not the nonsymbolic, distance effect was uniquely related to individual differences in mathematics achievement. Similarly, De Smedt and Gilmore (2011) and Rousselle and Noël (2007) revealed that children with mathematical difficulties are impaired on symbolic but not nonsymbolic comparison tasks, indicating that they have specific difficulties in the access to numerical magnitude from symbolic digits. On the other hand, it has been shown that nonsymbolic magnitude

representations predict individual differences in mathematics achievement in preschoolers (Libertus, Feigenson, & Halberda, 2011; Mazzocco, Feigenson, & Halberda, 2011) and adolescents (Halberda et al., 2008) and some studies reported that children with dyscalculia have impairments on both symbolic and nonsymbolic tasks (Mussolin, Mejias, & Noël, 2010), which suggests that magnitude representations per se might be important for successful mathematics development.

One explanation for these inconsistent findings might be that each of these studies has examined mathematical performance with broad measures, such as general standardized achievement tests. It might be that the association between mathematics achievement and numerical magnitude representations depends on the specific mathematical skill under investigation, and this might vary across different standardized achievement tests. Therefore, a more refined measure of mathematical performance, such as the assessment of children's arithmetic strategies may help to pinpoint associations between numerical magnitude representations and mathematical development in a more precise way.

Against this background, the present study set out to explore the role of numerical magnitude representations in arithmetic strategy use. The study comprised children who were in the middle of third grade. All completed numerical magnitude comparison tasks, presented in symbolic (digits) and nonsymbolic (dots) formats. They also solved a single-digit addition and subtraction task, in which they were asked to report the strategy they used to solve a particular problem on a trial-by-trial basis. Against the background of the studies reviewed above, we verified whether representation of numerical magnitudes per se or the access to numerical magnitudes from symbolic digits was associated with arithmetic strategy use. To evaluate alternative explanations for associations between numerical magnitude representations and arithmetic strategy use, general intellectual ability and digit naming skills were assessed as control measures. We also evaluated children's general mathematics ability

to replicate previous work.

Method

Participants

Participants were 49 typically developing children (30 boys, 19 girls) with a mean age of 8 years 10 months ($SD = 4$ months), in the middle of third grade. Children were recruited from two elementary schools in Flanders (Belgium) and came from middle- to upper middle-class families. Their native language was Dutch. The children had no history of developmental disorders and none had repeated a grade.

Materials

Computerized Tasks. All computerized tasks were designed with the E-prime 1.0 software (Schneider, Eschmann, & Zuccolotto, 2002). Children sat in front of a Dell Latitude C800 computer and watched white stimuli (Arial font, 72-point size) appearing on a black background. They were asked to perform both accurately and fast. A trial started with a 200ms fixation in the center of the screen. After 1000ms, stimuli appeared and remained visible until the child responded, except for the nonsymbolic magnitude comparison task, where the stimuli disappeared after 840ms, to avoid counting of number of dots. Each trial was initiated by the experimenter with a control key.

Numerical magnitude comparison. In these tasks, children had to compare two simultaneously presented numerical magnitudes, one displayed on the left side of the computer screen, and one displayed on the right. Children had to indicate the larger of those two numerical magnitudes by pressing a key on the side of the larger one. The left response key was d; the right response key was k. Reaction times and answers were registered by the computer. Both symbolic and nonsymbolic magnitude comparison tasks were administered, consisting of Arabic digits and dot arrays, respectively. Stimuli comprised all combinations of numerosities from 1 to 9, yielding 72 trials for each task. The position of the largest

numerosity was counterbalanced. The nonsymbolic stimuli were generated with the MATLAB script provided by Piazza and colleagues (2004) and were controlled for non-numerical parameters, such as dot size, total occupied area, and density. On one half of the trials, dot size, array size, and density were positively correlated with number, and on the other half of the trials, dot size, array size, and density were negatively correlated. This prevented that decisions were dependent on non-numerical cues or perceptual features. To familiarize children with the key assignments, three practice trials were included per task.

Arithmetic strategy use. We administered a single-digit addition and subtraction task. Stimuli were selected from the so-called standard set of single-digit arithmetic problems (Lefevre, Sadesky, & Bisanz, 1996), which excludes tie problems (e.g., $6 + 6$) and problems containing 0 or 1 as operand or answer. Only one of each pair of commutative problems was selected, resulting in a set of 28 problems per operation. The position of the largest operand was counterbalanced. Children were instructed to perform both accurately and fast. Responses were verbal. A voice key registered the child's reaction time, after which the experimenter recorded the child's answer. Children could use whatever strategy they wanted to. On a trial-by-trial basis, the experimenter asked the children to verbally report the strategy the child used to solve the arithmetic problem. Similar to other studies in arithmetic (e.g., Imbo & Vandierendonck, 2007; Torbeyns, Verschaffel, & Ghesquière, 2004), strategies were classified into retrieval (i.e., if the child immediately knew the answer and there was no evidence of overt calculations), procedure (i.e., if the child indicated that he or she used counting or decomposed the problem into smaller sub-problems to arrive at the solution), or other (i.e., if the child did not know how he or she solved the problem). This classification method, is a valid and reliable way of assessing children's arithmetic strategy use (Siegler & Stern, 1998). Two practice trials were presented to familiarize children with task administration.

Digit naming. In this task, each of the numbers 1 to 9 were successively presented twice on the computer screen. The child was asked to name each digit. Reaction time was registered by a voice key, after which the answer was entered on the keyboard by the experimenter. There were two practice trials to make the child familiar with task administration.

Standardized tests.

Intellectual ability. Raven's Standard Progressive Matrices (Raven, Court, & Raven, 1992) was administered as a measure of intellectual ability. For each child, a standardized score ($M = 100$, $SD = 15$) was calculated.

General mathematics achievement. Children's general mathematics achievement was assessed using a curriculum-based standardized achievement test for mathematics (Dudal, 2000). This test measured multidigit calculation, word problem solving and geometry.

Procedure

Children completed individually the computerized tasks in a quiet room at their own school. Standardized tests were group-based.

Results

Descriptive Analyses

Numerical magnitude comparison. Descriptive statistics on the numerical magnitude comparison tasks are presented in Table 1. The reaction time analyses considered correct responses only; trials deviating more than 3SDs from a participant's mean reaction time were also excluded (0.9% symbolic trials; 1.9% nonsymbolic trials).

To provide more detailed information about children's representation of numerical magnitudes, we determined for each child the size of the distance effect. This was done by calculating a linear regression in which numerical distance predicted reaction time, for each comparison task separately. The slope of this regression reflects the effect of distance on

children's reaction time when comparing magnitudes, with a steeper slope indicating a larger distance effect (De Smedt et al., 2009). We calculated these regressions for reaction times only because we obtained ceiling levels and limited variation for accuracy at large distances. This expected slope should be negative because the distance effect predicts a negative relationship between distance and reaction time. As such, the distance effect in the current study provides an indication of how fluent magnitude information is available from symbolic and nonsymbolic magnitudes. As expected, the mean slopes of the symbolic ($M = -26.35\text{ms}$, $SD = 14.57\text{ms}$) and the nonsymbolic ($M = -37.41\text{ms}$, $SD = 15.82\text{ms}$) comparison tasks were negative. One-sample t -tests further indicated that both slopes differed significantly from 0 (symbolic $t(48) = -12.65$, $p < .01$; nonsymbolic $t(48) = -16.55$, $p < .01$). There was a significant difference between symbolic and nonsymbolic slope ($t(48) = 4.14$, $p < .01$), indicating that the distance effect was larger for the nonsymbolic than for the symbolic task.

Arithmetic strategy use. Descriptive statistics of the arithmetic task are displayed in Table 1. The mean accuracy on this task was 95.44% ($SD = 4.24$). Reaction times were calculated for the correct responses only. Trials with incorrect voice-key registration were further excluded from the reaction time analyses (5% addition trials; 5% subtraction trials). Trials deviating more than 3SDs from a participant reaction time on each task were further excluded (3% addition trials; 3% subtraction trials). The mean reaction time on the arithmetic task was 2659.92ms ($SD = 937.91$). Additions were solved more accurately ($F(1,48) = 19.15$, $p < .01$) and faster ($F(1,48) = 45.56$, $p < .01$) than subtractions .

Children's arithmetic strategy use was examined by investigating the strategy distribution and strategy efficiency. Strategy distribution was determined by calculating the frequency with which a strategy was used to solve arithmetical problems. Because the frequency of trials belonging to the 'other' category was very low (0.5%), these trials were excluded from further analyses.

The mean frequencies of procedure use for addition and subtraction were $M = 16.08$ ($SD = 1.71$) and $M = 16.86$ ($SD = 2.17$), respectively. The mean frequency for fact retrieval was $M = 11.65$ ($SD = 1.74$) for addition and $M = 10.71$ ($SD = 2.32$) for subtraction. A repeated measures analysis on the frequencies of fact retrieval with operation (addition vs. subtraction) as within-subject factor, yielded a significant effect of operation ($F(1,48) = 8.01$, $p < .01$), showing that fact retrieval was more frequently used during addition than during subtraction.

Strategy efficiency was determined by calculating the mean accuracy and reaction time with which retrieval and procedural strategies were executed. The mean accuracy for retrieval was 99.48% ($SD = 2.08$) for addition and 98.44% ($SD = 4.12$) for subtraction, whereas the mean accuracy for procedural strategies on addition and subtraction was 95.87% ($SD = 4.35$) and 91.09% ($SD = 10.43$), respectively. The mean reaction time for fact retrieval was 1705.34ms ($SD = 581.20$ ms) for addition and 2156.79ms ($SD = 739.72$ ms) for subtraction. The mean reaction times to execute procedural strategies on addition and subtraction were 2878.77ms ($SD = 1216.31$ ms) and 3538.74ms ($SD = 1324.25$ ms), respectively. A 2×2 repeated measures ANOVA with strategy (retrieval vs. procedure) and operation (addition vs. subtraction) as within-subject factors was calculated on the accuracy and reaction times. With regard to accuracy, there was a main effect of strategy ($F(1, 48) = 31.62$, $p < .01$), indicating that retrieval strategies were executed more accurately than procedural strategies. There was a main effect of operation ($F(1, 48) = 15.73$, $p < .01$) showing that additions were solved more accurately than subtractions. A significant strategy $\times$ operation interaction ($F(1, 48) = 6.42$, $p < .05$) emerged, suggesting that differences between addition and subtraction were larger for procedural strategies than for fact retrieval. Post-hoc t -tests revealed that the operation differences were significant for procedural strategies ($p < .01$) but not for fact retrieval ($p = .14$). Turning to reaction times, there was a main effect

of strategy ($F(1, 48) = 122.23, p < .01$): Retrieval strategies were executed faster than procedural strategies. There was a main effect of operation ($F(1, 48) = 52.94, p < .01$) indicating that additions were solved faster than subtractions. There was no strategy $\times$ operation interaction ($F(1, 48) = 2.17, p = .15$).

Control tasks. All children were 100% accurate on the digit naming task, and their mean reaction time was 543.60ms ($SD = 71.03$ ms). Children's intellectual ability ($M = 108.37, SD = 11.70$) and general mathematics achievement ($M = 102.87, SD = 13.91$) were within the normal range.

Correlational Analyses

Pearson correlation coefficients were calculated to examine the associations between the different variables under study (Table 2). For the purpose of correlational analyses, the accuracy, speed and strategy data were initially averaged across operations.

Children's symbolic magnitude processing skills were significantly associated with arithmetic and general mathematics achievement, indicating that children with a smaller distance effect showed better mathematical performance. On the other hand, nonsymbolic magnitude processing skills were not correlated with the mathematical tasks. We observed a significant association between symbolic magnitude processing skills and arithmetic fact retrieval: Children with smaller symbolic distance effects retrieved more facts from their memory (Figure 1A). No such association was observed for nonsymbolic magnitude processing (Figure 1B). The size of the symbolic distance effect was also significantly associated with the speed of retrieving arithmetic facts and the speed of executing procedures.

The same pattern of associations was observed for addition and subtraction separately. The symbolic distance effect was significantly correlated with the frequency of retrieval in addition ($r = .31, p < .05$) and subtraction ($r = .43, p < .01$). Likewise, there were significant associations for both operations between the symbolic distance effect and the speed of

executing retrieval (addition: $r = -.44, p < .01$; subtraction: $r = -.57, p < .01$) and procedural strategies (addition: $r = -.57, p < .01$; subtraction: $r = -.53, p < .01$).

Additional Analyses

Hierarchical regression analyses were calculated to assess the amount of unique variance in arithmetic strategy use that was explained by symbolic magnitude representations. Table 3 displays the results of the hierarchical regression analysis of fact retrieval frequency. In step 1, Raven's matrices was entered into the model to control for the potential effect of intellectual ability. In step 2, we included children's general mathematics achievement to carefully examine the additional role of children's symbolic distance effect when explaining individual differences in fact retrieval frequency. Finally, the symbolic distance effect was entered into the model. Our findings revealed that the symbolic distance effect remained to predict a significant amount of unique variance in children's fact retrieval frequency, even when the effects of intellectual ability and general mathematics achievement were controlled for.

A similar hierarchical regression analysis was run on arithmetic strategy efficiency (Table 4), but we included digit naming in addition to intellectual ability in Step 1, to control for possible effects of processing speed. The findings in Table 4 indicate that the symbolic distance effect remained a significant predictor of children's arithmetic strategy efficiency, even when the effects of intellectual ability, digit naming and general mathematics achievement were taken into account.

Discussion

There is a converging body of data – often backed by neuroimaging studies on how the brain processes numbers – suggesting that numerical magnitude representations are crucial for successful mathematical development (Butterworth et al., 2011; Price & Ansari, 2012). One major drawback of this research is that it mainly investigated mathematics performance

as measured through general standardized achievement tests. From these tests it is unclear how children's numerical magnitude representations contribute to specific mathematical skills, information which is crucial in order to devise appropriate educational interventions. The present study investigated how symbolic and nonsymbolic numerical magnitude representations contribute to individual differences in arithmetic strategy use. Our findings reveal that children's symbolic but not nonsymbolic numerical magnitude processing skills are associated with individual differences in arithmetic. Turning to strategy use, children with better access to magnitude representations from symbolic digits retrieve more facts from their memory and are faster in executing fact retrieval as well as procedural strategies. These associations even remain when intellectual ability, digit naming and general mathematics achievement are additionally controlled for.

Consistent with previous studies (Holloway & Ansari, 2009; Rousselle & Noël, 2007; Sasanguie, De Smedt, Defever, & Reynvoet, 2011), the current findings reveal that symbolic but not nonsymbolic magnitude processing skills correlate with individual differences in general mathematics achievement. The present findings go beyond the previous ones, by showing that this pattern of associations also applies to specific mathematical skills, such as arithmetic strategy use. More specifically, smaller symbolic distance effects coincide with an increasing reliance on arithmetic fact retrieval and with a faster execution of arithmetic strategies. Such smaller distance effects might reflect more exact mappings between Arabic symbols and the magnitudes they represent, leading to a greater fluency or speed in accessing magnitude representations from symbolic digits. Access to numerical meaning is beneficial for children's arithmetic development, because without good understanding that numbers represent quantities, arithmetic becomes an exercise in rote memory (Griffin, 2002; Robinson, Menchetti, & Torgesen, 2002).

How may this access to magnitude representations from symbolic digits be

particularly important for arithmetic fact learning? Firstly, it could be that arithmetic facts that contain meaningful numbers are easier to store and to recall from memory (Robinson et al., 2002). Secondly, it is also possible that a good understanding of numerical meaning helps children to develop more advanced counting strategies, which in turn might lead to a faster reliance on arithmetic facts. For instance, at the earliest stages of arithmetic development, children count all the numbers in a problem and they gradually move on to more advanced counting procedures by counting on from the larger number in the problem (Geary, Bow-Thomas, & Yao, 1992). This advanced counting strategy requires a decision on the larger number for which access to numerical meaning of the presented Arabic symbols is needed. Against this background, it could be that more precise representations of numerical magnitude lead to a faster acquisition of this counting-on-larger strategy and this in turn contributes to a faster development of problem-answer associations in memory. Future longitudinal research is needed to test these hypotheses more carefully.

The present findings contrast with other studies who observed significant associations between nonsymbolic magnitude representations and math performance (Halberda et al., 2008; Libertus et al., 2011; Mazzocco et al., 2011). The comparison tasks used in these studies comprised larger numerical magnitudes (e.g., 5 - 17 in Halberda et al., 2008; 4 - 15 in Libertus et al., 2011; 1 - 14 in Mazzocco et al., 2011) than in the present study (1 - 9). It is also important to note that these studies used Weber fractions to index the numerical distance effect, whereas we used reaction time slopes. These differences in measurements might explain the difference between those previous studies and the current findings. Furthermore, studies differ in terms of age of the participants under study (e.g., 14-year olds in Halberda et al., 2008; 4-year olds in Libertus et al., 2011; 8-year olds in present study). Without doubt, the association between numerical magnitude processing and arithmetic skills changes over developmental time (see Noël and Rousselle, 2011, for an extensive discussion). Future

studies that measure numerical magnitude processing and arithmetic at different time points are therefore needed to unravel this issue.

Our data showed that associations between symbolic magnitude representations and arithmetic strategy use occur both in addition and subtraction. This is somewhat inconsistent with adult models of arithmetic, that have suggested that subtraction relies to a larger extent on numerical magnitude processing than addition (e.g., Dehaene, Piazza, Pinel, & Cohen, 2003). It might be that this operation specificity only emerges over developmental time and that this might only be observed in adults. Longitudinal data are needed to examine this possibility.

The current study did not observe an association between digit naming and arithmetic fact retrieval. This might be surprising, given that both digit naming and the symbolic distance effect might measure the access to numerical magnitudes from Arabic symbols. However, we found no association between digit naming and the symbolic distance effect in the current study, which suggests that both measures do not access magnitude representations to the same extent. One option could be that digit naming is carried out non-semantically. Alternatively, both measures might access numerical magnitude but this access may be much more explicit in the context of the symbolic distance effect. This distance effect arises from an explicit comparison of the magnitudes of the presented symbolic digits and this is not the case in number naming, where the access to numerical magnitude is not crucial to perform the task.

It is important to point out that the current data are cross-sectional in nature and that they do not allow us to establish causal connections. From these data it cannot be inferred whether shifts in strategy development to more advanced procedural strategies and fact retrieval are driven by more precise magnitude representations of symbolic digits. This again calls for longitudinal research, which should delineate the developmental trajectories of children's numerical magnitude representations and their associations with arithmetic strategy

use. This research should preferably start early in children's mathematical development as these symbolic magnitude representations might be crucial in the early transition to more advanced counting strategies.

Even more compelling would be to conduct carefully controlled intervention research that examines the effect of training children's access to numerical representations on subsequent arithmetic strategy use. It has been shown that interventions that use number lines improve children's errors on an arithmetic task, indicating an increase in close misses after training (Booth & Siegler, 2008). Based on the current data, it would be interesting to investigate if such interventions also lead to an increase in more advanced counting strategies or to an increase of fact retrieval use when solving arithmetic problems. As a next step, it could be useful to develop screening instruments to identify children with difficulties in accessing numerical meaning from symbols and to set up targeted interventions for remediating those difficulties.

In conclusion, the current study shows that children with smaller symbolic distance effects retrieve more facts from their memory when solving elementary arithmetic problems. This all indicates that particularly the access to numerical meaning from Arabic symbols is key for children's arithmetic strategy development. It suggests that educators and remedial teachers should focus on connecting Arabic symbols to the quantities they represent.

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Table 1

Descriptive statistics for numerical magnitude comparison and arithmetic

	Accuracy (% correct)	Reaction time (ms)
	<i>M (SD)</i>	<i>M (SD)</i>
Numerical magnitude comparison		
Symbolic	94.81 (3.65)	744.41 (121.00)
Nonsymbolic	92.49 (5.83)	745.01 (127.58)
Arithmetic		
Addition	97.38 (2.61)	2364.46 (904.71)
Subtraction	93.51 (6.94)	2955.38 (1062.37)

Table 2

Correlations between different variables (n = 49).

	1	2	3	4	5	6	7	8
1. Symbolic DE								
2. Nonsymbolic DE	.24							
3. Arithmetic Accuracy	.41**	-.18						
4. Frequency retrieval	.45**	-.16	.28					
5. Retrieval RT	-.46**	-.04	-.33*	-.53**				
6. Procedural RT	-.58**	-.12	-.42**	-.45**	.79**			
7. Digit naming	.13	.03	-.12	-.17	.41**	.33*		
8. General mathematics achievement	.41**	.03	.58**	.31*	-.39**	-.48**	-.08	
9. Intellectual ability	.14	-.01	.31*	.03	-.16	-.17	-.14	.51**

Note. DE = Distance effect. * $p < .05$; ** $p < .01$

Table 3

Hierarchical regression analysis predicting the frequency of fact retrieval ($n = 49$).

Step	Predictor	β	t	ΔR^2
1	Intellectual ability	.03	.22	.00
2	General mathematic achievement	.40	2.48*	.12
3	Symbolic distance effect	.38	2.68**	.12

Note. * $p < .05$; ** $p < .01$. The regression model is significant $F(3,48) = 4.72$,

$p < .01$, $R^2 = .24$.

Table 4

Hierarchical regression analysis predicting arithmetic strategy efficiency (n = 49).

Step	Predictor	Retrieval speed			Procedural speed		
		β	t	ΔR^2	β	t	ΔR^2
1	Intellectual ability	-.10	-.75	.01	-.13	-.92	.02
	Digit naming	.39	2.91**	.15	.31	2.22*	.09
2	General mathematic achievement	-.42	-2.87**	.12	-.53	-3.69**	.20
3	Symbolic distance effect	-.44	-3.56**	.16	-.53	-4.75**	.23

Note. * $p < .05$; ** $p < .01$. Both regression models are significant. Retrieval speed

$F(4,48) = 9.34, p < .01, : R^2 = .46$. Procedural speed: $F(4,48) = 13.72, p < .01, R^2 = .56$.

Figure 1. Scatter plots showing the association between children's mean frequency of retrieval and symbolic (panel A) as well as nonsymbolic (panel B) distance effects. The solid line represents the linear regression for this relationship.