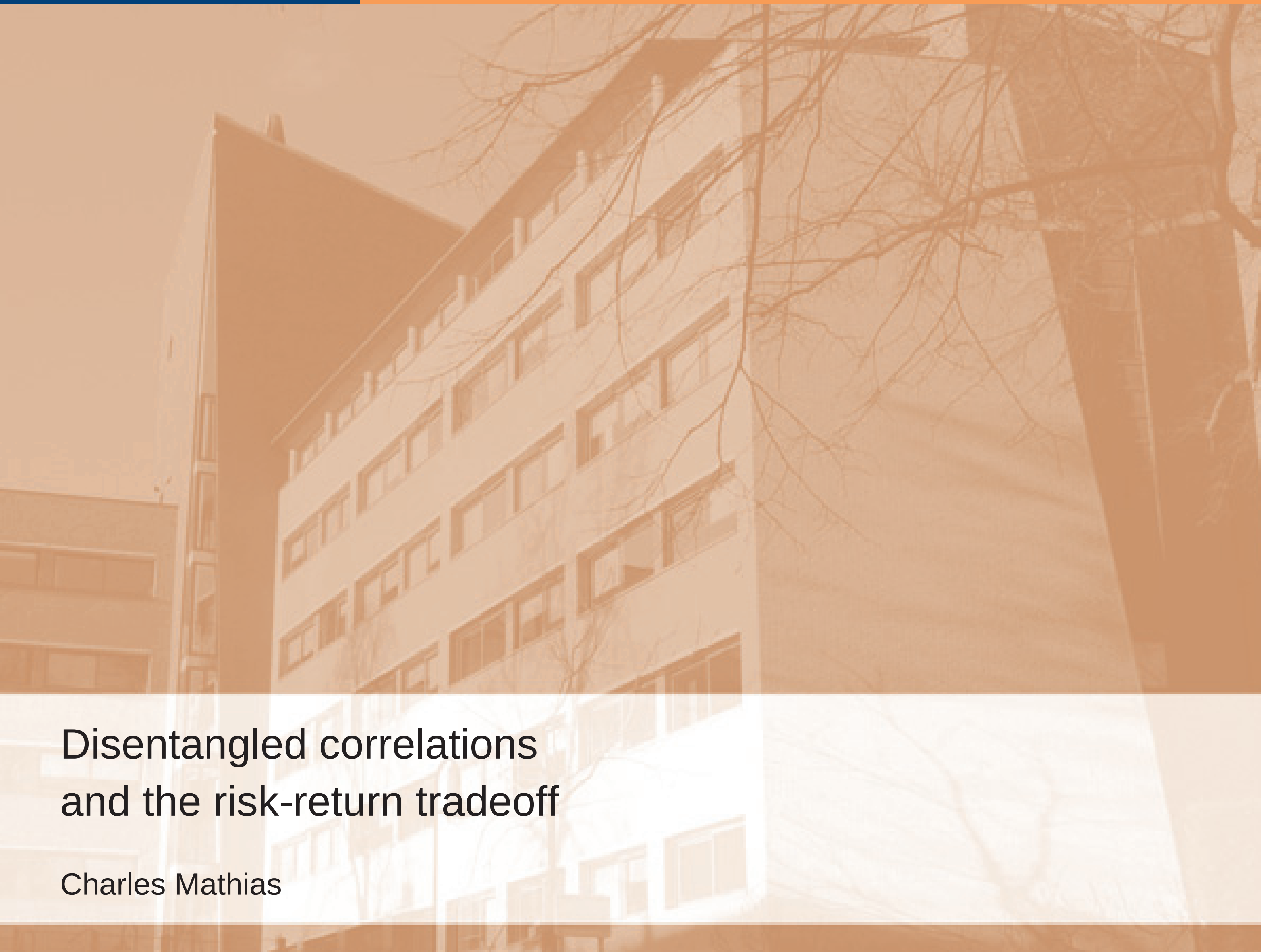
KATHOLIEKE UNIVERSITEIT LEUVEN

Faculty of Business and Economics



Disentangled correlations and the risk-return tradeoff

Charles Mathias

DEPARTMENT OF DECISION SCIENCES AND INFORMATION MANAGEMENT (KBI)

KBI 1214

Disentangled Correlations and the Risk-Return Tradeoff*

Charles Mathias†‡

March 2012

Abstract

This paper evaluates the impact of co-movement in equity return correlations on the equity risk-return trade-off. By applying a principal components analysis on conditional correlations, conditional covariances between the return of a security and the market return are decomposed in a sum of three terms: pure volatility dynamics, the interaction of volatility and market-wide (common) correlation dynamics, and the interaction of volatility and idiosyncratic correlation dynamics. The importance of each of these covariance terms on the risk-return trade-off is analysed, in different cross-sections. For portfolios sorted on industry, size and momentum, the risk-return trade-off is originated by the interaction of volatility and common correlation dynamics, whereas in the book-to-market cross-section, the trade-off comes from the interaction of volatility and idiosyncratic correlation dynamics. This suggests that investors conditionally price book-to-market differently than industry, size and momentum.

*I am grateful to Andrew Ang, Kris Boudt, Christophe Croux, Geert Dhaene, Michael Froemmel, Roxana Halbleib, Hugues Pirotte and David Veredas, as well as the participants to the ENTER 2011 Jamboree at Tilburg University and the KU Leuven doctoral seminar for useful comments. I also thank Kevin Sheppard for the UCSD-GARCH Matlab Toolbox, and Kenneth French and Robert Shiller for their online databases.

†Faculteit Economie and Bedrijfskunde, KULeuven, Email: `charles.mathias@kuleuven.be`. Address: Naamsestraat 69, B-3000 Leuven, Belgium. Phone: +3216326968.

‡ECARES, Solvay Brussels School of Economics and Management, Université libre de Bruxelles and ECORE. Email: `charles.mathias@ulb.ac.be`. Address: Avenue F. D. Roosevelt 50 (CP 139), B-1050 Brussels, Belgium. Phone: +3226504133.

Introduction

The trade-off between risk and return is one of the most important relations of financial theory. Since the capital asset pricing model of [32], the risk of a security can be considered as the covariance of its return with the market return. In that setting, the correlation with the market return is an important building block, since it measures the asset's diversification benefit in the market portfolio. Correlations are however not constant through time and even exhibit comovement. The present paper assesses whether this dynamic structure of conditional correlations is informative about how expected risk is reflected in returns.

Empirical evidence of the time-varying and comoving nature of correlations is very large. [27, 28], [21] and [18] observe that correlations are time-varying. In an international study, [8] find significant changes in correlations over time in Europe. All these studies also exhibit that correlations tend to go up in bear markets, which can be considered as a sort of correlations co-movement. Existence of this asymmetric nature of conditional correlations is confirmed by formal tests, as proposed in [4] and [22]. Co-movement in correlations is also intimately linked to the growing body of literature on financial contagion, as indicated by [8]. Even though the concept of contagion remains fairly elusive and there exists no formal definition of it, measuring its existence generally goes through a study of common changes in correlations, as suggested by [19]. From this perspective, [25] establish the existence of co-movement in excess correlations, which is the amount of correlations beyond what one may expect given fundamentals.

Examining the effect of time-varying correlations on asset returns is a rather recent research topic. On one hand, some authors have considered the asset pricing implications of the stochastic nature of correlations. [26] study the risk premium generated by stochastic correlations from an ICAPM perspective and show that investors hedge against stocks that perform badly when the aggregate level of correlations goes up. By looking at the option market, [14] show that uncertainty about correlations is priced because ex ante option-implied correlations are higher than their ex post realizations. [9] develop a theoretical model with portfolio implications in case of stochastic correlations.

On the other hand, two recent contributions have focused on the role of time-varying conditional correlations in the traditional risk-return trade-off. First, [6, 7] estimate the risk-return trade-off by modelling covariances with the Dynamic Conditional Correlations (DCC) GARCH model of [15]. The authors obtain a positive risk premium for various return cross-sections. They suggest that the explicit choice of modelling conditional covariances using a DCC-GARCH is a major ingredient for obtaining this positive trade-off. Second, [30] theoretically demonstrate that, because of the [31] critique, both the average level of asset volatilities and of asset correlations should be priced in the market return. In a way, the authors suggest that the price of correlations may have other components than the traditionally documented price of diversification benefits. They suggest that this additional price should be generated by the aggregate level of correlations, and hence that investors may be more risk averse to common than to idiosyncratic changes in correlations.

The goal of the present paper is to provide deeper empirical evidence of the effect of the comoving structure of conditional correlations on the risk-return trade-off. For four different return cross-sections—portfolios sorted on industry, size, book-to-market and momentum—conditional correlations of portfolio returns with the market return are disentangled in the sum of a static, a common dynamic and an idiosyncratic dynamic component. Conditional covariances can therefore be decomposed likewise, in a first term capturing pure volatility dynamics, a second accounting for the interaction of volatility and common correlation dynamics, and a third term reflecting the interaction of volatility and idiosyncratic correlation dynamics. In a standard asset pricing regression, the portfolio returns will be regressed on the three covariance components, to identify which aspects of time-varying conditional correlations are at the basis of the observed risk-return trade-off.

Given the bounded nature of correlations, it is impossible to come up with a correlation decomposition such that components are orthogonal and bounded, and that their sum is also bounded. At least one constraint has to be relaxed. Therefore, I consider four different procedures for decomposing conditional correlations, each of them working under different regularity constraints. As a basis for these different decompositions, I use the DCC-GARCH estimated conditional correlations, and the static correlation component is always defined as their time series average. In the first decomposition methodology, I apply a standard Principal Components Analysis on the DCC-GARCH correlations. In the second, a PCA is applied to Fisher transformed conditional correlations, and both common and idiosyncratic components are transformed back onto the unit circle. In the third and fourth methodology, only one of both components—common or idiosyncratic—of transformed correlations are transformed back onto the unit circle; the other one is defined as the residual with respect to DCC-GARCH correlations. For the four decomposition methodologies, the corresponding covariance decomposition is obtained by multiplying each of the correlation components by the product of GARCH conditional volatilities of the portfolio and market returns.

Conditional covariances (decomposed and non decomposed) are used as explanatory variables against the observed monthly returns, in a procedure very similar to [6]. For every panel—industry, size, book-to-market and momentum—the system of regressions is estimated in a Seemingly Unrelated Regression (SUR) framework. Intercepts are allowed to vary across individuals, but the slopes of covariance components on returns are assumed to be constant across individuals and time. Parameters are estimated using a stationary bootstrap two-step-least squares approach, which allows to account for autoregressive heteroskedasticity of returns and for the fact that explanatory variables are point estimates themselves.

When regressing returns on non decomposed covariances, results of [6, 7] are partially retrieved. I find a significant trade-off for industry, size and book-to-market portfolios and do not reject the null of zero intercepts for size and book-to-market. When looking at decomposed covariances, I see that for industry, size and momentum sorted portfolios, the highest trade-off coefficient is associated with the interaction of volatility and common correlation dynamics, with a significant result for the size and momentum cross-sections. In the book-to-market cross-section, on the other hand, the largest trade-off comes from the interaction between volatility and idiosyncratic correlation dynamics, although non significant in the basic setting.

Robustness of these results is assessed under various ICAPM control specifications. First, I consider the inter-temporal pricing of six macroeconomic variables: default spread, term spread, relative short term interest rate, inflation rate, output gap and aggregate dividend yields. Default spread, term spread, relative interest rate and output gap are not priced inter-temporally, aggregate dividend yields and inflation rates are priced in industry sorted portfolios. None of the controls however affect the results. Second, I control for inter-temporal pricing of the financial factors for size and book-to-market of [17] and the factor for momentum of [12]. The size factor is not priced in any cross-section, the book-to-market factor is in all and the momentum factor is priced for momentum portfolios. The significance of the parameters of interest increases when controlling for these financial factors: common correlation dynamics generate a significant positive trade-off for industry, size and momentum sorted portfolios, and idiosyncratic correlation dynamics now significantly account for the trade-off in the book-to-market panel. Third, I also look at the pricing of aggregate volatility, as suggested by [3]; volatility is not priced and does not affect the results. Finally, I follow [26] and verify whether the aggregate level of correlations is priced inter-temporally; I find that they do not have a significant impact on returns and therefore they do not affect results.

The result obtained in the book-to-market panel is compelling, since it is a cross-sectional counterargument for [30] or any conjecture on the effects of contagion (which would suggest a dominant role to common correlation dynamics). To interpret this peculiar finding, I explore what the idiosyncratic and common correlation dynamics precisely capture in the covariances between the returns on the portfolios sorted on size, book-to-market and momentum, and the market return. For all panels, I find that middle deciles are characterized by important correlation commonness, whereas the highest and lowest deciles are dominated by correlation idiosyncrasy. This suggests that correlation dynamics are differently rooted in the book-to-market premium than they are in the premia related to industry, size or momentum. One could indeed state that investors price time-varying expected diversification benefits only *within* the equity portfolio from a book-to-market perspective, whereas from an industry, size or momentum point of view, these dynamics in expected diversification benefits are only priced *between* the equity portfolio and other asset classes.

The paper is structured as follows. Section 1 explains the role of correlations in the literature on risk-return trade-off. Section 2 describes the dataset used. Section 3 explains the disentangling conditional correlations. The main results are shown and interpreted in Section 4. Robustness for these results is provided in Section 5. Section 6 concludes.

1 Theoretical background

The conditional version of the capital asset pricing model[1] (henceforth CAPM) states that the expected return of a security conditional on all available information should be proportional to the conditional covariance of the stock's return with market return and with any random

[1] [32], [23]

variable affecting the stochastic investment opportunity set,

$$\mathrm{E}_t\left[R_{i,t+1}\right] = \gamma_m \sigma_{im|t} + \gamma_x \sigma_{ix|t}, \tag{1}$$

where $\mathrm{E}_t[\cdot]$ denotes the expectation operator based on all available information at t, $R_{i,t+1}$ is the excess return of security i at time $t+1$, $\sigma_{im|t}$ and $\sigma_{ix|t}$ are the expected covariances for period $t+1$ conditional on information available at t, between security i and respectively the market return and a stochastic variable affecting the investment opportunity set x_t (examples of such variables are interest rate levels, aggregate volatility, the average spread between investment grade and high yield corporate bonds, etc.). Merton (1980) suggests that γ_m can be interpreted as the risk aversion coefficient of an agent with constant relative risk aversion (CRRA). By considering the market return on the left hand side of the equation, one obtains the trade-off between expected market return and expected market variance,

$$\mathrm{E}_t\left[R_{m,t+1}\right] = \gamma_m \sigma^2_{m|t} + \gamma_x \sigma_{mx|t}. \tag{2}$$

The existence of the positive trade-off ($\gamma_m > 0$) expressed in (1) and (2) has been tested by several authors with mixed success. Exhaustively reviewing all these contributions goes beyond the scope of the present paper, but interesting overviews can be found, among others, in [20] or [6]. In the present paper, only [6, 7] and [30] will be explored in more detail, since the role of time-varying correlations in the positive risk-return trade-off relation plays a key role in these papers.

[6] estimate (1) with daily data between 1963 and 2008 and model conditional covariances using the DCC-GARCH method of [15]. They estimate conditional covariances $\sigma_{im|t} = \rho_{im|t}\sigma_{i|t}\sigma_{m|t}$, where $\sigma_{i|t}$ and $\sigma_{m|t}$ are GARCH(1,1) standard deviations of the return on stock i and on the market, and where $\rho_{im|t}$ is a GARCH(1,1)-type estimate of the conditional correlation between both returns, parametrized independently from the conditional volatilities. The authors then plug the series of $\sigma_{im|t}$ as explanatory variables in a standard system of asset pricing regressions,

$$R_{i,t+1} = \alpha_i + \gamma_m\, \sigma_{im|t} + \sum_x \gamma_x\, \sigma_{ix|t} + e_{i,t+1}, \tag{3}$$

which is estimated using using a two-step-least-squares in a SUR framework. The authors consider several cross-sections (Dow Jones 30 constituents, industry, size, book-to-market, momentum, investment-to-assets and return-on-assets). They are all characterized by a positive trade-off, and the null hypothesis of intercepts being jointly equal te zero is rejected only for momentum, investment-to-assets and return-on-assets cross-sections.

[7] go a step further, by developing a multivariate DCC-GARCH-in-mean for estimating conditional covariances and trade-off in one step. They use monthly data from 1927 until 2009. The authors identify a positive trade-off of market return on market variance, and suggest that this comes from the joint action of (i) looking at the whole cross-section and (ii) using DCC-GARCH covariances. The present paper will investigate more closely this issue of the role of time-variation in conditional correlations for obtaining a positive risk-return trade-off.

The other recent contribution highlighting the particular importance of correlations in evaluating the risk-return trade-off is [30]. Their starting point is the critique of Roll (1977), which states the limitations of empirical tests on (1) and (2) due to the fact that the true return on aggregate wealth is unobservable and incorrectly replaced by an observable part of it (the return on the stock market). The authors demonstrate that, when stock returns have total market and stock market beta's, both the average level of return correlations and of return volatilities should be positively priced across asset returns. This suggests that the price of correlations in the cross-section of asset returns may be more than the price of diversification benefits only. In this way, [30] point out that, if time-varying conditional correlations exhibit co-movement, the common correlation factor should bear a higher price than the idiosyncratic.

The present paper will combine the conclusions of [6, 7], which highlight the importance of parametrizing separately correlations and volatilities, and the main result of [30], which states that a higher price may be attributed to common movements compared to idiosyncratic. More specifically, I will decompose conditional correlations in the following way: $\rho_{im|t} = \kappa_i + \chi_{i|t} + \xi_{i|t}$, where κ_i accounts for the unconditional level of correlations, $\chi_{i|t}$ is the common dynamic correlation (accounting for dynamics shared by all individuals) and $\xi_{i|t}$ the idiosyncratic dynamic correlation component. This correlation decomposition yields a covariance decomposition $\sigma_{im|t} = \kappa_i \sigma_{i|t} \sigma_{m|t} + \chi_{i|t} \sigma_{i|t} \sigma_{m|t} + \xi_{i|t} \sigma_{i|t} \sigma_{m|t}$, which can be used to estimate the impact of common *versus* idiosyncratic correlation dynamics in asset pricing. Indeed, each of these covariance terms could generate a different risk premium in the return cross-section.

One may wonder why it is important to pass by a correlation decomposition to achieve a decomposition in covariances. First of all, the idea of focusing specifically on correlations is coherent with [6, 7], according to which it is better to separate correlation parametrization from volatility parametrization. Second, theory suggests that it is comovement in *correlations*, not in covariances, to which a larger asset pricing premium may be associated. This is indeed the conclusion of [30]. Moreover, although no formal model exists for it, the literature on contagion also intuitively suggests that investors may require a higher premium when the aggregate level of correlations is higher: markets are more sensitive to contagion dynamics, since a negative market shock will have a more uniform impact on investor portfolios, potentially requiring more investors to simultaneously liquidate their positions. Third, the specific nature of volatility time series, being always larger than zero, makes a multiplicative decomposition more appropriate to volatility than an additive one. Therefore, it seems wise to apply the additive decomposition only to the non-volatility part of covariances, *i.e.* the correlations.

2 Dataset

Monthly stock portfolio returns are downloaded from Kenneth French's from July 1963 until December 2010, accounting for $T = 570$ observation dates. Four different panels of returns are considered: 30 portfolios sorted on industry (hereafter IND), 10 on market equity (SIZE), 10 on book-to-market (BTM) and 10 on past 12 months performance (MOM). The number of individuals in each panel (10 or 30) is denoted by n. The choice of these return panels has two reasons: industry portfolios are included because they constitute an intuitive way of looking at the stock market cross-section; size, book-to-market and momentum portfolios are considered given their widely documented deviation with respect to the CAPM. The market return is the market capital weighted return on all NYSE, AMEX and NASDAQ stocks (from CRSP), and the one-month Treasury bill rate is used as risk-less rate. The particular choice of the starting date is motivated by the fact that it allows performing all regressions and robustness checks on the same sample. The use of monthly frequency, rather than daily, is to avoid problems of multicollinearity in the regression, which will be explained in more detail in Section 3.

Six macroeconomic variables are considered for robustness, the same as in [7]: default spread, term spread, relative short term interest rate, inflation rate, output gap and aggregate dividend yield. Monthly yields of the Federal Fund effective rate, three-month Treasury bill and the ten-year Treasury bond are downloaded from the H.15 database of the Federal Reserve Board, as well as yields on AAA and BAA rated corporate bonds. Default spread (def) is defined as the difference in yields between BAA and AAA rated corporate bonds. Term spread ($term$) is the difference between ten-year and three-month Treasury yields. Short term relative interest rate ($rrel$) is defined as the difference between the three-month Treasury bill yield its 12 month moving average. Monthly aggregate dividend yields and the consumer price index are downloaded from Robert Shiller's website. The aggregate dividend yield (div) is the ratio of monthly dividends on S&P 500 index to the current level of the index. The inflation rate (inf) is the monthly growth of the consumer price index. Finally, the Industrial Production Index is downloaded from the G.17 database of the Federal Reserve Board. Output gap (out) is defined as the growth rate on the latter index. Innovations on the macroeconomic variables are computed by taking first differences and DCC-GARCH conditional covariances of portfolio returns with these innovations will be used to assess their inter-temporal pricing.

Following [16], [17], [24] and [12], the financial factors of size (smb), book-to-market (hml) and momentum (umd) are also introduced. All are downloaded from Kenneth French's website. I will assess whether DCC-GARCH estimated time-varying covariances of portfolio returns with each of these factors help in the pricing across time and assets.

Finally, I control for investors hedging against changes market volatility and the aggregate level of correlations. [10], [11, 13] suggest that investors hedge against time-varying market volatility, since it affects the investment opportunity set. [3] and [1] confirm that covariances with innovations in volatility are priced in the return cross-section. Therefore, I also control for time-varying covariances with innovations in the log of monthly market volatility, estimated as the first differences in the standard deviation of daily returns of the S&P 500 for each month, and noted with $\tilde{\sigma}_t$. Following [14] and [26], I also control for the investor's hedging against innovations in the aggregate level of correlations. I look at time-varying covariances between asset returns and innovations in monthly aggregate correlations, which are computed by taking the first differences in the monthly averages of correlations between daily returns on 30 industry sorted portfolios and the market return, and are noted by $\tilde{\rho}_t$.

3 Disentangling conditional correlations

Although an additive decomposition is *more* appropriate to correlations than to volatilities, capturing correlation co-movement in an additive setting is not a straightforward problem. Indeed, one should at the same time take into account the boundedness of correlations, and the orthogonality of common versus idiosyncratic dynamics. More specifically, one wants to model variations in expected correlations as the sum of two terms—a common and and idiosyncratic—such that neither the one, nor the other, nor their sum, leads to conditional correlations lying outside the unit circle. Developing such a model is conceptually contradictory: one wants both common and idiosyncratic terms to be moving independently from one another, but having a bounded sum, meaning that their domains should be non-independent. This issue can be viewed as an optimization problem with a negative number of degrees of freedom, since there are more constraints than unknowns. Intuitively, one therefore has to relax at least one of the constraints. One possibility is to impose both terms to be orthogonal and bounded, but to allow their sum lying outside the unit circle. As an alternative, the reverse would be to impose the sum common and idiosyncratic terms lying within the unit circle, but relaxing their orthogonality and hence the boundedness of at least one of the two terms (the common, the idiosyncratic, or both).

Proposing a one-step model going from observed returns to decomposed conditional correlations is a difficult task and goes beyond the scope of the present paper. Therefore, the general procedure for reaching a decomposition of conditional correlations will go in two steps. First, the series of non disentangled conditional correlations will be estimated, using a DCC-GARCH of [15]. Second these series will serve as the basic input for a decomposition in common and idiosyncratic conditional correlation terms, considering the different paths of decomposing highlighted above. Once this conditional correlation decomposition is achieved, the resulting decomposition of conditional covariances an easily be computed.

3.1 DCC-GARCH conditional correlations

The Dynamic Conditional Correlations Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) model of [15] is applied to estimate the conditional second order moments of each bivariate series $\mathbf{R}_{i,t+1} \equiv (R_{i,t+1}, R_{m,t+1})'$, where i denotes a particular portfolio and m is the market. In the DCC-GARCH(1,1), conditional correlations and volatilities are parametrized according to

$$\mathbf{R}_{i,t+1} = \boldsymbol{\mu}_i + \boldsymbol{\varepsilon}_{i,t+1},$$

where $\boldsymbol{\mu}_i = (\mu_i, \mu_m)'$, $\boldsymbol{\varepsilon}_{i,t+1} = (\varepsilon_{i,t+1}, \varepsilon_{m,t+1})'$, and

$$\mathrm{E}_t[\boldsymbol{\varepsilon}_{i,t+1}\boldsymbol{\varepsilon}'_{i,t+1}] = D_{i,t} R_{i,t} D_{i,t}.$$

The diagonal matrix $D_{i,t} = \mathrm{diag}\left(\sigma_{i|t}, \sigma_{m|t}\right)$ contains the univariate GARCH(1,1) volatilities, $\sigma^2_{i|t} = \omega_i + \alpha_i(R_{i,t} - \mu_i)^2 + \beta_i \sigma^2_{i|t-1}$, and each element $\rho^{dcc}_{jk|t}$ (with $j,k \in \{i,m\}$) of the correlations matrix R_t is parametrized by

$$\rho^{dcc}_{jk|t} = \frac{q_{jk|t}}{\sqrt{q_{jj|t} q_{kk|t}}}, \tag{4}$$

with $q_{jk|t} = \bar{\rho}_{jk} + \alpha(\varepsilon_{j,t}\varepsilon_{k,t} - \bar{\rho}_{jk}) + \beta(q_{jk|t-1} - \bar{\rho}_{jk})$. The reason why correlations are written as a ratio of GARCH elements is to make sure they always respect $-1 < \rho^{dcc}_{jk|t} < 1$. The estimation of conditional second order moments for the n return couples per panel is done using the UCSD-GARCH Matlab toolbox of Kevin Sheppard. This toolbox applies Maximum Likelihood approach assuming innovations follow a Generalized Error Distribution.

Conditional correlations of portfolio returns with the market return for the different panels are plotted in Figure 1. Looking at these charts, several observations can be made. First, one sees that co-movement is an important feature in conditional correlation time series, especially for portfolios sorted on size, book-to-market and momentum, where the correlation most often stays above 0.8. Cross-sectional diversity in conditional correlations seems highest along the industry dimension and lowest in the size panel. A closer study of the latter panel shows that the nearly parallel lines exhibit monotonicity in terms of their average, where smaller size is associated with the lower average correlations. Book-to-market and momentum sorted portfolios also exhibit strong co-movement, but one can graphically detect that idiosyncratic dynamics are higher than in the size panel. Moreover, there is not the same phenomenon of monotonically increasing/decreasing average correlations as a function of deciles.

An important feature revealed by Figure 1 is that correlations exhibit a sharp decrease during the Tech bubble. This is very clear in the book-to-market and momentum cross-sections, and also from a sector perspective. Closer analysis[2] of the data shows that this downward spike is most important in sectors not affected by the bubble, like healthcare, construction, utilities, transport, wholesale and retail, while the sector exhibiting the least change in correlation during that period is Telecom. The book-to-market and momentum panels confirm this observation, since the dip appears to be happening in the middle deciles, and not among the very high or very low book-to-market or momentum portfolios. A plausible economic explanation of the phenomenon may be that, as a result of sky-rocketing valuations for Tech companies, their weight in investor's portfolios increased likewise, and comovement of other sectors with this "market" mechanically decreased. In a way, this correlation dip would then be an interesting verification of the bubble, since it suggests that the market is not what it should be.

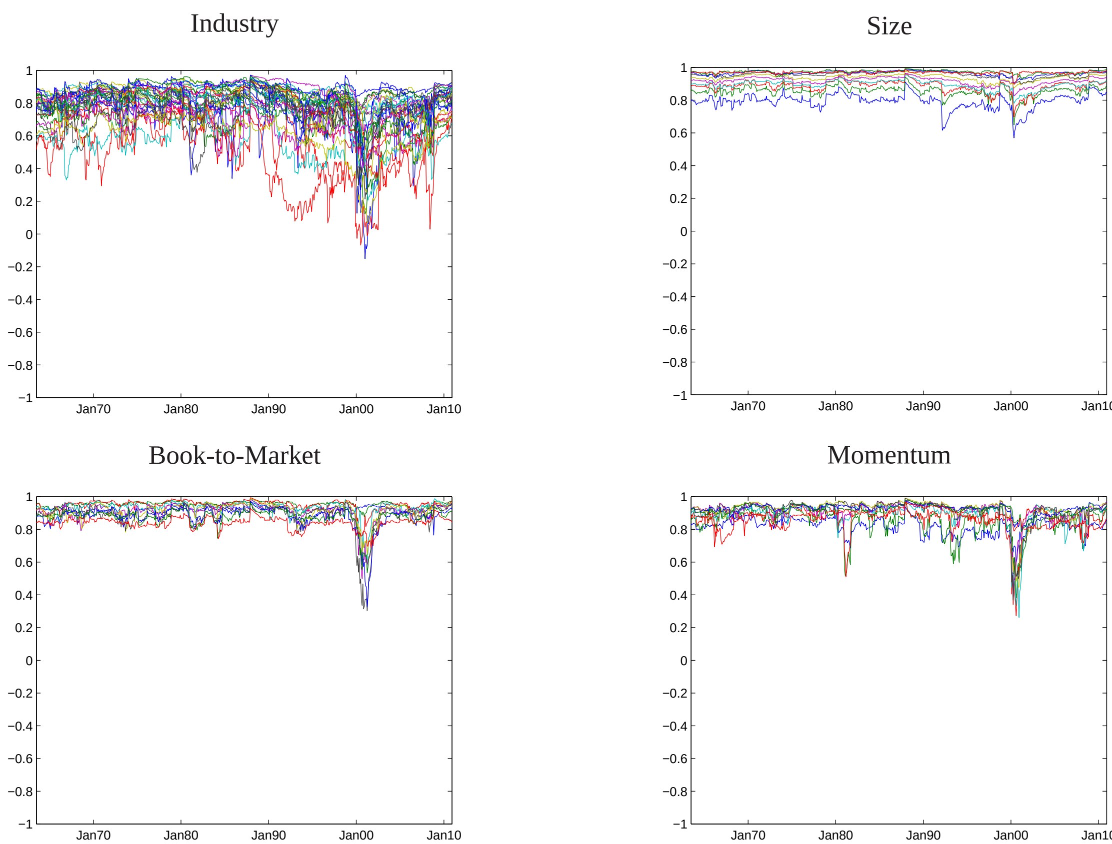
3.2 Decomposition of conditional correlations

The goal of this section is, for every panel (IND, SIZE, BTM and MOM), to write conditional correlations as a sum of static, common dynamic and idiosyncratic dynamic terms,

$$\rho_{im|t} = \kappa_i + \chi_{i|t} + \xi_{i|t}, \tag{5}$$

[2] Charts available upon request.

Figure 1: Conditional correlations estimated using DCC-GARCH(1,1)



The conditional correlations are estimated with DCC-GARCH using the UCSD-GARCH Matlab toolbox of Kevin Sheppard.

where the static component κ_i captures the unconditional expectation of correlations and is computed by taking the time-series average of these DCC-GARCH conditional correlations,

$$\kappa_i = \frac{1}{T}\sum_{t=1}^{T} \rho_{im|t}^{dcc}.$$

Descriptive statistics of κ_i's per panel are shown in Table 1. We see that, on average, correlations are high and lie close to each other. As it is also reflected in Figure 1, Table 1 confirms that cross-sectional variation is highest when looking at the industry sorted portfolios where the minimum static correlation term is of 0.52 and the maximum of 0.89, whereas along the size, book-to-market and momentum dimensions, correlations tend to stay on average between 0.8 and 1.

The common component $\chi_{i|t}$ will be a mean-zero process driven by panel-wide dynamics and the idiosyncratic component $\xi_{i|t}$ a mean-zero process driven by individual-specific dynamics. The sum of the latter two terms will be called the dynamic component, $\delta_{i|t} = \chi_{i|t} + \xi_{i|t}$. The DCC-GARCH conditional correlations $\rho_{im|t}^{dcc}$ will serve as an input for the several decomposition approaches; the latter will be distinguished in superscript of the components, $\chi_{i|t}^{(j)}$ and $\xi_{i|t}^{(j)}$, with $j = 1, 2, 3, 4$.

Table 1: Descriptive statistics of κ_i

	$\bar{\kappa}$	$\kappa_{1/2}$	IQR_κ	$\min_\kappa$	$\max_\kappa$
IND	0.77	0.78	0.11	0.52	0.89
SIZE	0.91	0.93	0.08	0.79	0.97
BTM	0.90	0.91	0.06	0.84	0.95
MOM	0.89	0.90	0.05	0.82	0.93

Descriptive statistics of static components of DCC-GARCH estimated conditional correlations. From left to right, the arithmetic mean, the median, the interquartile range, the minimum and the maximum are shown.

3.2.1 PCA on conditional correlations

The first methodology is to apply a standard Principal Component Analysis on the demeaned conditional correlations, $\delta_{i|t}^{(1)} \equiv \rho_{im|t}^{dcc} - \kappa_i$. The information criterion of [2], which generalizes [5] for selecting the number of factors in approximate static factor models, suggests that the industry panel (with $n = 30$) has a common space of dimension $q = 4$, while the SIZE, BTM and MOM panels (with $n = 10$) have $q = 1$ common factor. We then easily get to the representation

$$\delta_{i|t}^{(1)} = \chi_{i|t}^{(1)} + \xi_{i|t}^{(1)}, \tag{6}$$

where $\chi^{(1)}_{i|t}$ is the common and $\xi^{(1)}_{i|t}$ the idiosyncratic component. The variance decomposition is represented in Table 2. The table confirms that co-movement is lowest in the industry panel (where R^2 is of only 82%, even though there are 4 factors considered) and that co-movement is highest in the size panel (with an R^2 of 85%). The main problem with this methodology is however that one does not impose any constraints on the domain of common and idiosyncratic components. By construction, $\kappa_i + \chi^{(1)}_{i|t} + \xi^{(1)}_{i|t}$ will always lie within the unit circle, and $\chi^{(1)}_{i|t}$ and $\xi^{(1)}_{i|t}$ will be orthogonal, but the price to pay is that $\kappa_i + \chi^{(1)}_{i|t}$ and $\kappa_i + \xi^{(1)}_{i|t}$ may not always lie between -1 and 1. This can be observed in Figures 2 and 3. Figure 2 shows that the sum of the static and common dynamic components sometimes exceeds the upper bound for the IND, BTM and MOM panels, be it only rarely and to a very limited extent. In Figure 3, one remarks that the sum of static and dynamic idiosyncratic components sometimes exceeds the upper bound for SIZE, BTM and MOM panels. This happens only once for the size sorted portfolios. For book-to-market portfolios the upper bound is crossed several times around the burst of the Tech bubble. For momentum sorted portfolios, finally, the upper bound is crossed at three different periods: in the early 80s, at the burst of the Tech bubble and during the Lehman crisis.

Table 2: Variance decompositions

	IND	SIZE	BTM	MOM
n	30	10	10	10
q	4	1	1	1
s^2_χ / s^2_δ	0.82	0.85	0.83	0.72

Percentage of the total variance explained by common components in the decomposition (6). n is the number of individuals in the panel, q the number of factors used, as suggested by the criterion.

3.2.2 Transformed PCA on transformed conditional correlations

An alternative is to look for $\chi^{(2)}_{i|t}$ and $\xi^{(2)}_{i|t}$ such that $-1 < \kappa_i + \chi^{(2)}_{i|t} < 1$ and $-1 < \kappa_i + \xi^{(2)}_{i|t} < 1$, but allowing for $\kappa_i + \chi^{(2)}_{i|t} + \xi^{(2)}_{i|t}$ to lie outside the unit circle. I do this by performing a PCA on transformed correlations, and transforming back the obtained components. More specifically, the DCC-GARCH conditional correlations $\rho^{dcc}_{im|t}$ are first projected on the real line by the Fisher transformation,

$$z_{i|t} \equiv \operatorname{arctanh}\left(\rho^{dcc}_{im|t}\right) = \frac{1}{2}\ln\left(\frac{1+\rho^{dcc}_{im|t}}{1-\rho^{dcc}_{im|t}}\right).$$

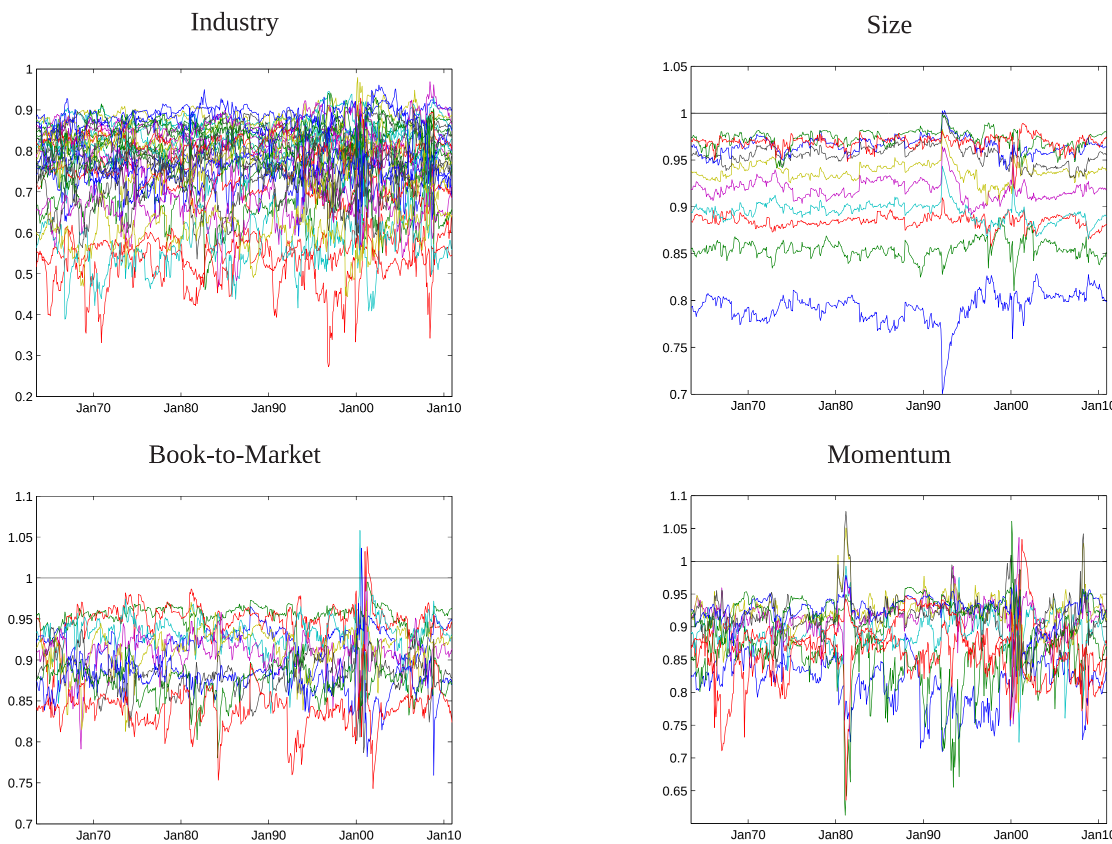
A PCA on the transformed dynamic component $d_{i|t} \equiv z_{i|t} - k_i$, where $k_i \equiv 1/T \sum_{t=1}^{T} z_{i|t}$, allows then to write $d_{i|t} = c_{i|t} + x_{i|t}$[3], with $c_{i|t}$ the common and $x_{i|t}$ the idiosyncratic com-

[3]The dimensions of common spaces are the same as above.

Figure 2: PCA on DCC-GARCH conditional correlations: $\kappa_i + \chi^1_{i|t}$

I represent the sum of the static and common dynamic component under the first decomposition procedure, i.e. PCA on DCC-GARCH conditional correlations. No constraints are set on the boundedness of common nor idiosyncratic components, implying that they may sometimes fall outside the unit circle. For common components, this is only rarely observed, and only for industry, book-to-market and momentum panels.

Figure 3: PCA on DCC-GARCH conditional correlations: $\kappa_i + \xi^1_{i|t}$



I represent the sum of the static and common dynamic component under the first decomposition procedure, i.e. PCA on DCC-GARCH conditional correlations. No constraints are set on the boundedness of common nor idiosyncratic components, implying that they may sometimes fall outside the unit circle. For idiosyncratic components, this is never observed for industry, rarely for size but several times for book-to-market and momentum.

ponent. Common and idiosyncratic components are then projected back to the unit circle. In order to keep the appropriate level of concavity in the transformation, I define

$$\chi_{i|t}^{(2)} \equiv \tanh\left(k_i + c_{i|t}\right) - \frac{1}{T}\sum_{t=1}^{T}\tanh\left(k_i + c_{i|t}\right),$$

and

$$\xi_{i|t}^{(2)} \equiv \tanh\left(k_i + x_{i|t}\right) - \frac{1}{T}\sum_{t=1}^{T}\tanh\left(k_i + x_{i|t}\right).$$

We now have that the DCC-GARCH estimated $\rho_{im|t}^{dcc} \neq \kappa_i + \chi_{i|t}^{(2)} + \xi_{i|t}^{(2)}$. Define the dynamic component as $\delta_{i|t}^{(2)} \equiv \chi_{i|t}^{(2)} + \xi_{i|t}^{(2)}$. By construction and by Jensen's inequality, we have that $-1 < \kappa_i + \chi_{i|t}^{(2)} < 1$ and $-1 < \kappa_i + \xi_{i|t}^{(2)} < 1$, but one cannot make sure that $-1 < \kappa_i + \chi_{i|t}^{(2)} + \xi_{i|t}^{(2)} < 1$. Figure 4 illustrates that this problem is not encountered for IND, SIZE and MOM, but that a small number of data points of the BTM panel have an estimated conditional correlation larger than one, during the 2008 financial crisis.

3.2.3 Partially transformed PCA on transformed conditional correlations

A last alternative is to re-transform only one of the two components $c_{i|t}$ and $x_{i|t}$ obtained above and define the other as the difference between DCC-GARCH estimated conditional correlations and the other parameters. Indeed, define

$$\chi_{i|t}^{(3)} \equiv \chi_{i|t}^{(2)} \equiv \tanh\left(k_i + c_{i|t}\right) - \frac{1}{T}\sum_{t=1}^{T}\tanh\left(k_i + c_{i|t}\right),$$

and

$$\xi_{i|t}^{(3)} \equiv \rho_{im|t}^{dcc} - \kappa_i - \chi_{i|t}^{(3)}.$$

This fully takes away the orthogonality condition, $\xi_{i|t}^{(3)}$ is now computed as a residual to the other terms. Moreover, although $\chi_{i|t}^{(3)}$ satisfy the correct boundary condition of $-1 < \kappa_i + \chi_{i|t}^{(3)} < 1$, $\xi_{i|t}^{(3)}$ will not. In a way, the cost of having the sum of all correlation terms lying inside the unit circle is that one of the components is not correctly convexified. Figure 5 shows that for IND, BTM and MOM, the idiosyncratic correlation component falls outside the required bounds during the burst of the 2001 Tech bubble.

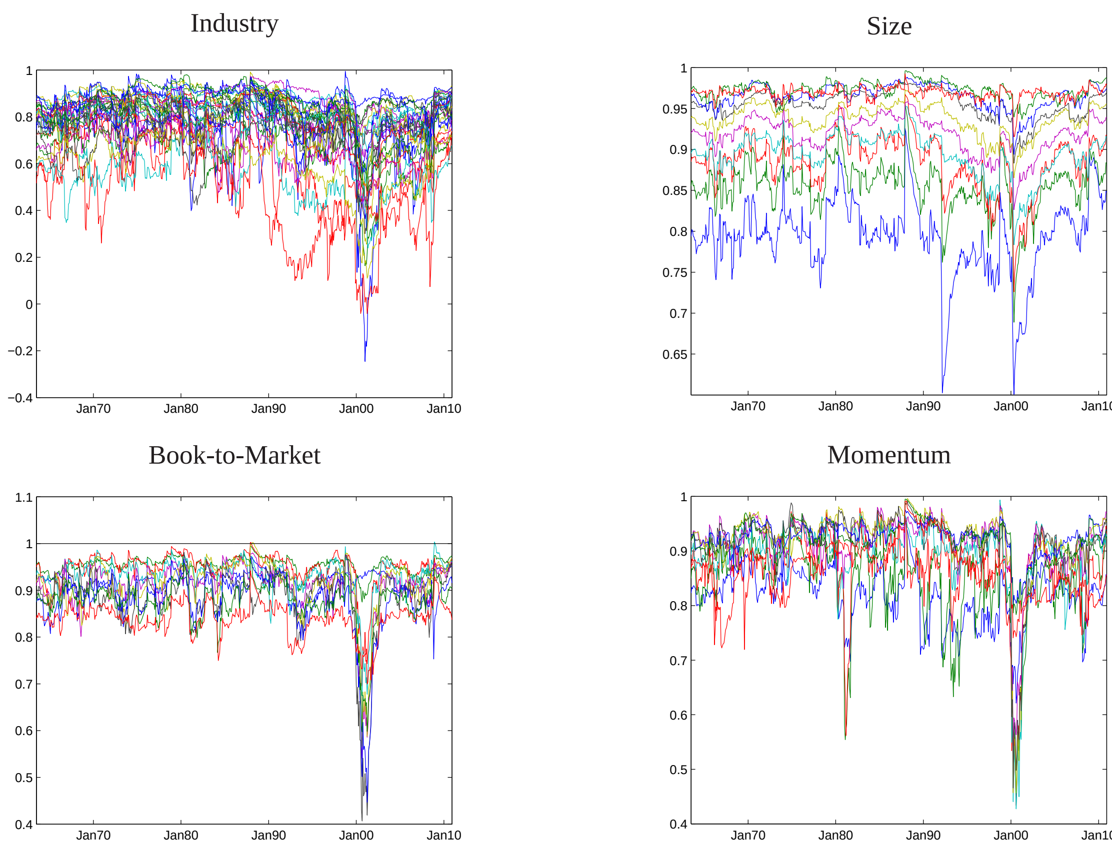
Likewise, one could apply the same procedure to idiosyncratic components:

$$\xi_{i|t}^{(4)} \equiv \xi_{i|t}^{(2)} \equiv \tanh\left(k_i + x_{i|t}\right) - \frac{1}{T}\sum_{t=1}^{T}\tanh\left(k_i + x_{i|t}\right),$$

and

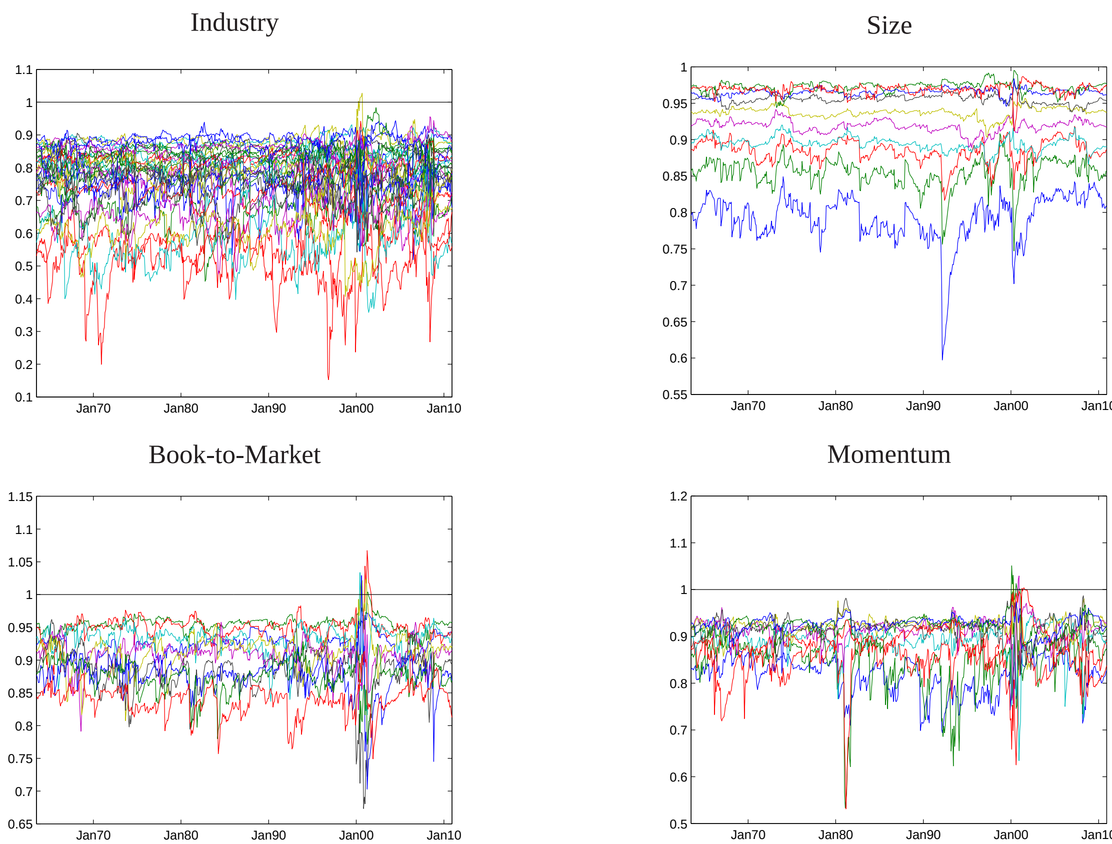
$$\chi_{i|t}^{(4)} \equiv \rho_{im|t}^{dcc} - \kappa_i - \xi_{i|t}^{(4)}.$$

Figure 4: Transformed PCA on transformed DCC-GARCH conditional correlations: $\kappa_i + \chi^2_{i|t} + \xi^2_{i|t}$



Under the second decomposition procedure, common and idiosyncratic components will always lie within the unit circle, but their sum may not. We nevertheless see that this only happens a very limited number of times for the book-to-market cross-section, and never for industry, size and momentum.

Figure 5: Partially transformed PCA on transformed DCC-GARCH conditional correlations: $\kappa_i + \xi_{i|t}^{(3)}$



Under the third decomposition procedure, common components and the total conditional correlations are bounded by construction, but the idiosyncratic components, computed as residuals, may sometimes lie outside their bounds. As we see, this never happens for size, happens for one industry portfolio and for several book-to-market and momentum sorted portfolios around the burst of the Tech bubble early 2001.

The same comments apply as for the previous; estimated common components are shown in Figure 6 and exceed their upper bound in a limited number of data points for IND, BTM and MOM. Curiously enough, the violations of correlation constraints do not happen during crisis periods, but at the beginning and the end of the 90s.

Figure 6: Partially transformed PCA on transformed DCC-GARCH conditional correlations: $\kappa_i + \chi_{i|t}^{(4)}$

Under the fourth decomposition procedure, idiosyncratic components and the total conditional correlations are bounded by construction, but the common components, computed as residuals, may sometimes lie outside their bounds. As we see, this never happens for size, happens for one industry and book-to-market portfolio, and still very rarely for momentum sorted portfolios.

3.3 From decomposed correlations to decomposed covariances

Since conditional correlations and volatilities are parametrized separately, the conditional covariance between the return on a portfolio and the market can easily be written as

$$\sigma_{im|t} = \rho_{im|t}\sigma_{i|t}\sigma_{m|t}. \tag{7}$$

Given the correlation decomposition (5), one can therefore write

$$\sigma_{im|t} = \kappa_i \sigma_{i|t} \sigma_{m|t} + \chi_{i|t} \sigma_{i|t} \sigma_{m|t} + \xi_{i|t} \sigma_{i|t} \sigma_{m|t}. \tag{8}$$

Since κ_i accounts for the *unconditional* aspects of correlations, one could say that the first term at the right hand side of the equation in (8) corresponds to the covariance under the assumption of *constant* conditional correlations (CCC). We can thus say that

$$\sigma_{im|t}^{ccc} = \kappa_i \sigma_{i|t} \sigma_{m|t}.$$

By writing conditional covariances according to (8), one states that they are the sum of a term driven by pure volatility dynamics, one driven by the interaction of volatility and common correlation dynamics, and one driven by the interaction of volatility and idiosyncratic correlation dynamics.

The next section will examine how important each of these covariance terms is for asset pricing in different cross-sections. Since all three regressors contain the conditional volatility product $\sigma_{i|t}\sigma_{m|t}$, one should care about multicollinearity issues. Indeed, although correlation components are by construction orthogonal, multiplying these orthogonal terms by the same time series will make the covariates correlated. A descriptive analysis suggests that a good time window is to take monthly data from 1963 until 2010. Given the high comovement in daily volatility data, considering a daily frequency during the same period generates correlations between covariance terms of over .95. The same comment holds when looking at monthly data starting in 1929: volatility comovement has been so big during the 1929 crisis that problems of multicollinearity may also arise. Table 3 reports, for each panel, the maximum and minimum correlations between each pair of components in the monthly 1963-2010 window, as used in this study. As one can see, correlations are between -0.51 and 0.68, which is reasonable from a regression point of view; results in the next section will confirm that there is no problem of multicollinearity.[4]

Table 3: Correlations across time of covariance components

	$\min r_{\kappa\chi}$	$\max r_{\kappa\chi}$	$\min r_{\kappa\xi}$	$\max r_{\kappa\xi}$	$\min r_{\chi\xi}$	$\max r_{\chi\xi}$
IND	-0.39	0.64	-0.54	0.50	-0.32	0.28
SIZE	0.10	0.40	-0.51	0.34	-0.24	0.25
BTM	-0.05	0.36	-0.42	0.68	-0.20	0.34
MOM	-0.13	0.28	-0.35	0.57	-0.33	0.21

Minima and maxima of correlations between the conditional covariance components for various panels. Correlations should not be too high, to avoid issues of muliticolinear in the next step where each of the terms will be used as regressors.

[4]Results for the other windows are available upon request.

4 Main results

As a matter of introduction, I first look at the basic relation of [6],

$$R_{i,t+1} = \alpha_i + \gamma_m \sigma^{dcc}_{im|t} + e_{i,t+1}. \tag{9}$$

The system of regressions in (9) is estimated using a stationary block bootstrap TSLS. Bootstrap is appropriate to get reliable confidence intervals on the estimated parameters, given that the explanatory variables are point estimates themselves. Block bootstrap is used to cope with autoregressive heteroskedasticity of estimation errors. Blocks are overlapping and are randomly drawn (with replacement) from the original dataset to form new panels of equal length. To conserve stationarity of the overlapping blocks, their lengths are random and follow a geometric distribution, as suggested by [29]. This procedure of bootstrapping is also called stationary bootstrap. At every bootstrap iteration, the Newey-West corrected (unconditional) variance-covariance matrix of the first step OLS residuals is used as weighting matrix for the second step GLS.[5] This methodology lies in the same spirit as [6], who also estimate their asset pricing equation using this TSLS procedure, but do not use bootstrap for taking into account additional uncertainty. [7], on the other hand, use a one-step method by estimating a multivariate DCC-GARCH-in-mean model. Developing a one-step model for disentangled correlations would be preferable to the two-step procedure considered in the present paper, but, as explained above, it is not straightforward to come up with such a model that disentangles correlations, and it is therefore beyond the goal of the present paper.

The first line of Table 4 shows the estimated parameters and their p-values of the regression in equation (9). The trade-off for the industry cross-section is found to be significantly positive, but the the null hypothesis of zero intercepts is rejected. Returns on size sorted portfolios behave coherently with the conditional CAPM, since the trade-off is significantly positive and it cannot be rejected that intercepts are different from zero. The same holds for the book-to-market cross-section, even though the level of significance of the slope is slightly larger (6%). The momentum portfolios have positive but non significant risk-return trade-off and significant intercepts. These results are quite in line with [6]. The only difference is that the latter find non significant intercepts for the industry cross-section and a significant slope for momentum portfolios. Plausible reasons for these differences in outcomes can be the difference in used samples or the use of bootstrap in the present paper, increasing the standard errors of estimated parameters.

I will now take the closer look at which aspects of time-varying covariances—pure volatility, the interaction of volatility and commonness in correlations, or the interaction of volatility and idiosyncrasy in correlations—are responsible for the generated premia.

[5] I use the `covnw.m` Matlab function of Kevin Sheppard to compute this unconditional covariance matrix. Consistent with the univariate Newey-West estimator, it computes the unconditional covariance matrix between residuals as a weighted sum of covariance matrices between contemporaneous and lagged residuals: $\widehat{\mathrm{E}\left[\mathbf{e}_t\mathbf{e}_t'\right]} = \sum_{l=0}^{L} w_l \left(\frac{1}{T-l} \sum_{t=l+1}^{T} \mathbf{e}_t \mathbf{e}_{t-l}' \right)$, where $w_l = 1 - \frac{l}{L+1}$ and L is set to 9 months.

First, I look at

$$R_{i,t+1} = \alpha_i + \gamma_\kappa \left(\kappa_i \sigma_{i|t} \sigma_{m|t}\right) + e_{i,t+1}, \tag{10}$$

which corresponds to (9) but assuming constant instead of dynamic conditional correlations GARCH covariances. Estimation results are shown in line 2 of Table 4. The slope coefficients are less significant for IND and SIZE. They do not loose their significance for BTM and become significant for MOM. This result is a first illustration of the importance of considering the dynamic structure of conditional correlations in the risk-return trade-off. Strictly speaking, there is indeed no reason why conditional covariance changes that are driven by pure volatility dynamics would have a more significant price than overall covariance changes. The data however suggest that the dynamic aspect of correlations plays a major role in understanding the significantly positive risk-return trade-off.

I now consider the importance of co-movement in correlations in the risk-return trade-off, by estimating

$$R_{i,t+1} = \alpha_i + \gamma_\chi \left(\chi_{i|t} \sigma_{i|t} \sigma_{m|t}\right) + e_{i,t+1}. \tag{11}$$

The three alternatives for $\chi_{i|t}$ are considered: the common term of DCC-GARCH conditional correlations ($\chi_{i|t}^{(1)}$), the transformed common component of transformed DCC-GARCH conditional correlations ($\chi_{i|t}^{(2)}$), or the residual between conditional correlations and the transformed idiosyncratic component of DCC-GARCH conditional correlations ($\chi_{i|t}^{(4)}$). Estimation results of the three cases are shown respectively in lines 3, 4 and 5 of Table 4. For IND, SIZE and MOM, lines 3 and 5 indicate a significantly positive trade-off. Line 4, on the other hand, where commonality is computed as a transformation of common components of transformed correlations, exhibits a positive but non significant slope. For BTM, none of the common correlation components are associated with a significantly positive risk premium.

Likewise, I also look at idiosyncratic correlation changes,

$$R_{i,t+1} = \alpha_i + \gamma_\xi \left(\xi_{i|t} \sigma_{i|t} \sigma_{m|t}\right) + e_{i,t+1}, \tag{12}$$

again considering the three ways of modelling idiosyncrasy in conditional correlations ($\xi_{i|t}^{(1)}$, $\xi_{i|t}^{(2)}$ and $\xi_{i|t}^{(3)}$). Columns 6 to 8 of Table 4 illustrate that idiosyncrasy does not generate a significantly positive trade-off for any of the cross-sections.

Consistency of these results is now assessed by performing the estimation with the three covariance components jointly,

$$\begin{aligned} R_{i,t+1} &= \alpha_i + \gamma_\kappa \left(\kappa_i \sigma_{i|t} \sigma_{m|t}\right) + \gamma_\chi \left(\chi_{i|t} \sigma_{i|t} \sigma_{m|t}\right) \\ &\quad + \gamma_\xi \left(\xi_{i|t} \sigma_{i|t} \sigma_{m|t}\right) + e_{i,t+1}. \end{aligned} \tag{13}$$

These results are shown in the last four lines, where the four different decomposition methods are used. For IND, none of the terms is still significant at the 5% level, but smallest p-values are associated with the common components. For SIZE, only common correlation terms generate a significant trade-off in three cases of the four. For MOM, common correlation changes always generate a positive trade-off. For BTM, no clear pattern can be observed.

Table 4: Estimation of asset pricing regressions

	γ_m	γ_κ	γ_χ	γ_ξ	W_α
Industry cross-section					
(1)	2.10**				1722.04***
(2)		1.49			586.13***
(3)			6.41**		111.13***
(4)			0.89		95.98***
(5)			5.83**		105.25***
(6)				2.33	108.57***
(7)				0.90	96.58***
(8)				3.94	108.45***
(9)		0.83	4.63*	1.63	98.73***
(10)		0.84	3.89	3.10	100.44***
(11)		0.89	3.95*	3.86	95.98***
(12)		0.90	3.99	3.20	96.58***
Size cross-section					
(1)	1.87**				16.40*
(2)		5.18*			18.31**
(3)			18.42**		17.92*
(4)			3.95*		9.75
(5)			26.06***		19.19**
(6)				4.32	18.89**
(7)				3.99	10.33
(8)				7.24	20.25**
(9)		1.57*	13.70*	3.81	8.90
(10)		1.59*	27.01**	2.26	9.29
(11)		1.67*	27.73**	2.48	9.75
(12)		1.45*	22.90**	3.11	10.33
Market-to-Book cross-section					
(1)	1.72*				9.28
(2)		1.59*			7.67
(3)			4.02		27.35***
(4)			3.86		14.74
(5)			-2.20		27.38***
(6)				11.88*	25.86***
(7)				3.20	17.33*
(8)				8.71	28.05***
(9)		1.18	3.28	8.52	16.07*
(10)		1.19	2.09	10.58	16.54*
(11)		1.35	1.55	6.43	14.74
(12)		1.21	-0.51	11.03	17.33*

Estimation results for regressions (9) to (13).

Table 4: (continued)

	γ_m	γ_κ	γ_χ	γ_ξ	W_α
Momentum cross-section					
(1)	1.48				90.51***
(2)		9.66**			84.37***
(3)			19.86**		63.71***
(4)			1.67*		51.63***
(5)			14.91**		62.93***
(6)				-2.62	59.32***
(7)				1.45*	51.90***
(8)				-2.47	57.92***
(9)		1.18	18.46**	-4.13	56.66***
(10)		1.42	23.57**	-5.75*	58.67***
(11)		1.50	23.02**	-6.75*	51.63***
(12)		1.31	14.49**	-5.73	51.90***

Estimation results for regressions (9) to (13). (1) [6]. (2) constant conditional correlations. (3)-(5) interaction of volatility and common correlation dynamics: PCA on conditional correlations (3), transformed PCA on transformed correlations (4) and residuals of the partially transformed PCA on transformed correlations (5). (6)-(8) interaction of volatility and idiosyncratic correlation dynamics: PCA on conditional correlations (6), transformed PCA on transformed correlations (7), and residuals of the partially transformed PCA on transformed correlations (8). (9)-(12) full decomposition—each line uses a different correlation decomposition, in the same order as presented in Section 3. Asterisks represent the significance of parameters: at 10% (*), 5% (**) and 1% (***).

Even though the regression results suggest that some covariance components generate a significant trade-off while others do not, they do not ascertain that regression coefficients are significantly different from each other, in a statistical sense. Therefore, I perform a Wald test on the three main parameters γ_κ, γ_χ and γ_ξ, jointly testing the hypotheses $\gamma_\kappa = \gamma_\chi$ and $\gamma_\kappa = \gamma_\xi$. As Table 5 shows, the Wald test statistic is too small to suggest different coefficient values in IND, SIZE and BTM, but is significantly different from zero in MOM. Unless for momentum sorted portfolios, results do thus not entirely go against CAPM.

Table 5: Wald test on equality of coefficients

	IND	SIZE	BTM	MOM
(1)	1.18	1.54	0.97	6.47**
(2)	1.04	3.12	1.13	6.37**
(3)	1.25	3.21	0.36	7.85**
(4)	0.98	3.42	1.26	8.25**

Wald test statistic. Columns indicate the considered panel. Lines indicate the method used for decomposing conditional correlations: (1) PCA, (2) transformed PCA on transformed data, (3) partially transformed PCA, only on common components, (4) partially transformed PCA, only on idiosyncratic components. Test statistics are computed using the bootstrap variance-covariance matrix of the coefficients. Asterisks indicate significance of the statistic: 10% (*), 5% (**) and 1% (***)

5 Robustness

Robustness tests are performed on the results obtained in the previous Section from an ICAPM perspective under various specifications. The general regression is

$$\begin{aligned} R_{i,t+1} &= \alpha_i + \gamma_\kappa \left(\kappa_i \sigma_{i|t} \sigma_{m|t}\right) \\ &\quad + \gamma_\chi \left(\chi_{i|t} \sigma_{i|t} \sigma_{m|t}\right) + \gamma_\xi \left(\xi_{i|t} \sigma_{i|t} \sigma_{m|t}\right) \\ &\quad + \sum_x \gamma_x \sigma_{ix|t} + e_{i,t+1} \end{aligned} \tag{14}$$

where x denote innovations in any variable potentially priced intertemporally, and $\sigma_{ix|t}$ is the DCC-GARCH conditional covariance between returns on portfolio i and these innovations. I use the same maximum likelihood procedure as above to estimate the DCC-GARCH. In what follows, I will first control for inter-temporal pricing of six macroeconomic variables: 3 variables on fixed income (default spread, term spread, short term interest rate), 2 variables on the general state of the economy (inflation rate and output gap) and aggregate dividend yield. Second, I will check for robustness with inter-temporal pricing of the usual financial factors (size, book-to-market and momentum). Third, I allow for the pricing of changes in aggregate volatility. Finally, I also consider the aggregate level of correlations.

5.1 Macroeconomic variables

As in [6, 7], I introduce time-varying correlations with innovations in macroeconomic variables, to account for potential inter-temporal pricing of these variables. First, I first look at default spread, term spread and relative short term interest rate, i.e. $x = ddef,\ dterm,\ drrel$. Table 6 shows that, as in BE10b, none of these variables are priced inter-temporally. Hence, their introduction does not affect previous results. Consistent with the results obtained above, one still obtains that none of the covariance terms seem to generate a trade-off for IND and BTM, and that for SIZE and MOM the positive trade-off is generated by the interaction of common correlation changes with volatility changes. The null hypothesis of zero intercepts is not rejected for SIZE and BTM and only in MOM the test statistic suggests a significant difference between all parameters.

Next, I look introduce innovations in dividend yield and output gap: $x = dinf,\ dout$. Results are shown in Table 7. Inflation rate is priced inter-temporally for the industry cross-section. SIZE and MOM remain the only two cross-sections that are characterized by a positive risk-return trade-off, generated by the interaction of volatility and common correlation dynamics. Intercepts are not significantly different from zero for SIZE and BTM and significantly different for MOM.

Finally, I consider aggregate dividend yields: $x = ddiv$. As Table 8 indicates, dividend yields are very significantly priced inter-temporally in IND, but not in any other panel. Looking at the other parameters of interest, one observes decreased significances. In SIZE, there is no significant tradeoff at the 5% level and in MOM the p-values are also lower. Results on the Wald test statistics are identical as before.

5.2 Financial factors

In the present subsection, I check for robustness when controlling for inter-temporal pricing of financial factors for size (smb) and book-to-market (hml) of [17] and the factor for momentum (umd) of [12], i.e. I consider (14) with $x = smb,\ hml,\ umd$. Estimates are shown in Table 9. Non of the panels significantly price smb, but all do so for hml; umd has a significant impact on IND and MOM. Consistent with [6, 7], the introduction of time-varying conditional covariances with these financial variables increases the significance of the parameters of interest. Indeed, one now observes a significantly positive risk-return trade-off for all cross-sections. For IND, SIZE and MOM, this trade-off is driven by the interaction of volatility changes and common correlation changes. For BTM, on the contrary, the trade-off is generated by covariance dynamics coming from an interaction of volatility changes and idiosyncratic correlation changes. The decomposition procedure only marginally affects results for IND and MOM, point towards a higher significance of using a transformation for SIZE, and suggest for BTM that significance decreases when defining idiosyncratic correlation components as the residual of the common. The same conclusions as before hold concerning the difference of parameters.

5.3 Aggregate volatility

I now turn to the inter-temporal pricing of aggregate volatility, measured by introducing DCC-GARCH conditional covariances between portfolio returns and first differences in the realized daily volatility of the S&P 500 index for each month. Table 10 indicates that innovations in aggregate volatility are never priced and that parameter values are not affected compared to Section 4.

5.4 Aggregate correlations

To finish, I look at innovations in the aggregate level of correlations, $\tilde{\rho}$. Regression results are shown in Table 11 and indicate that innovations in aggregate correlations are not significantly priced, although p-values are smaller than 10% for IND.

6 Concluding discussion

The results obtained in the book-to-market panel are rather surprising, given that they go against [30] or against any intuitive conjecture on the effect of expected contagion risk on returns. It is therefore interesting to assess how correlation commonness and idiosyncrasy are reflected in the conditional covariances of returns on size, book-to-market and momentum portfolios with the market return. I look at a partial variance decomposition on conditional covariances: of the total variation in conditional covariances arising from the interaction of volatility and correlation dynamics, which fraction is to be attributed to common and which to idiosyncratic correlation dynamics? Table 12 represents these results, considering each of the four correlation decompositions. Except for the first decomposition on the size panel, for which the importance of correlation commonness in covariances seems to increase with size, all other variance decompositions follow a U-shaped pattern: the highest degree of idiosyncrasy is observed for the smallest and largest deciles, whereas the covariances of middle deciles are very much influenced by correlation commonness and behave hence very similarly from the perspective of correlation dynamics. In other words, the idiosyncratic terms capture the correlation dynamics of the first and last deciles, whereas the common terms capture the correlation dynamics of the middle deciles.

This is important when looking at asset pricing, since the size, book-to-market and momentum premia are defined as the difference in returns between the high versus the low quantiles in these spectra. The results hence suggest that the size and momentum premia cannot be explained by the difference in conditional correlation dynamics of the extreme quantiles in the size and momentum spectra. However, the premium on returns of stocks with high versus low book-to-market ratios *does* significantly depend on the (temporary) differential of conditional correlations between these returns and the market return. Differently stated, this could mean that the dynamic nature of correlations nests two ways of pricing correlation dynamics: *between* and *within* portfolio pricing. Between portfolio pricing denotes the pricing of assets as arising from the price of the total market equity portfolio, depending on market equity variance, which on its turn is influenced by the aggregate level of correlations. Within portfolio pricing is the pricing of securities depending on their relative diversification benefit in the portfolio. It depends on the idiosyncratic dynamics of correlations. From a correlation point of view, the results of this paper suggest that conditional pricing of the size and momentum cross-sections is based on a between portfolio pricing, whereas book-to-market is conditionally priced within the equity portfolio.

The peculiar result on the book-to-market panel suggests an interesting way to exploit the book-to-market premium, which could also be observed in the charts of Section 3. Low value stocks have a decreased importance in the market, hence their correlation with the market return will decrease, and this correlation will start behaving more idiosyncratically, compared to other stocks. As a result, a significance idiosyncratic decrease in correlations, could be a proxy for identifying value stocks, which we know outperform on average.

The results also show how complex the risk-return trade-off is in practice, as opposed to the simple and intuitive theoretical expression of the CAPM. A large series of studies have preceded this one to empirically find a positive trade-off—the accurate measure of time-varying risk has been a major ingredient in obtaining this result. By taking further steps in accurately estimating risk, the present paper provides evidence that the risk-return trade-off is not a uniform concept across stocks, but that for different cross-sections, different aspects of correlations matter from a risk perspective.

References

[1] T. Adrian and J. Rosenberg. Stock returns and volatility: Pricing the short-run and long-run components of market risk. *Journal of Finance*, 63:2997–3030, 2008.

[2] L. Alessi, M. Barigozzi, and M. Capasso. Improved penalization for determining the number of factors in approximate static factor models. *Statistics and Probability Letters*, 80:1806–1813, 2010.

[3] A. Ang, R. J. Hodrick, Y. Xing, and X. Zhang. The cross-section of volatility and expected returns. *Journal of Finance*, 61:259–299, 2006.

[4] Andrew Ang and Joseph Chen. Asymmetric correlations of equity portfolios. *Journal of Financial Economics*, 63:443–494, 2002.

[5] J. Bai and S. Ng. Determinig the number of factors in approximate factor models. *Econometrica*, 79:191–221, 2002.

[6] T. G. Bali and R. F. Engle. The intertemporal capital asset pricing model with dynamic conditional correlations. *Journal of Monetary Economics*, 57:377–390, 2010a.

[7] Turan G. Bali and Robert F. Engle. Resurrecting the conditional capm with dynamic conditional correlations. *Working Paper*, 2010b.

[8] G. Bekaert, R. J. Hodrick, and X. Zhang. International stock return comovements. *Journal of Finance*, 64:2591–2626, 2009.

[9] A. Buraschi, P. Porchia, and F. Trojani. Correlation risk and optimal portfolio choice. *Journal of Finance*, 65:393–420, 2010.

[10] J. Y. Campbell. Intertemporal asset pricing without consumption data. *American Economic Review*, 83:487–512, 1993.

[11] J. Y. Campbell. Understanding risk and return. *Journal of Political Economy*, 104:298–345, 1996.

[12] M. M. Carhart. On persistence in mutual fund performance. *Journal of Finance*, 52:57–82, 1997.

[13] J. Chen. Intertemporal capm and the cross-section of stock returns. *Working Paper*, 2002.

[14] J. Driessen, P. J. Maenhout, and G. Vilkov. The price of correlation risk: Evidence from equity options. *Journal of Finance*, 64:1377–1406, 2009.

[15] R. F. Engle. Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics*, 20:339–350, 2002.

[16] E. F. Fama and K. R. French. The cross-section of expected stock returns. *Journal of Finance*, 47:427–465, 1992.

[17] E. F. Fama and K. R. French. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33:3–56, 1993.

[18] M. A. Ferreira and P. M. Gama. Correlation dynamics of global industry portfolios. *Journal of Multinational Financial Management*, 20:35–47, 2010.

[19] K. J. Forbes and R. Rigobon. No contagion, only interdependence: Measuring stock market comovements. *Journal of Finance*, 57:2223–2261, 2002.

[20] E. Ghysels, P. Santa-Clara, and R. Valkanov. There is a risk-return trade-off after all. *Journal of Financial Economics*, 76:509–548, 2005.

[21] W. N. Goetzman, L. Li, and G. Rouwenhorst. Long-term global market correlations. *NBER Working Paper*, 2001.

[22] Y. Hong, J. Tu, and G. Zhou. Asymmetries in stock returns: Statistical tests and economic valuation. *Review of Financial Studies*, 20:1547–1581, 2009.

[23] R. Jagannathan and Z. Wang. The conditional capm and the cross-section of expected returns. *Journal of Finance*, 51:3–53, 1996.

[24] N. Jegadeesh and S. Titman. Returns to buying winners and selling losers: implications for stock market eficiency. *Journal of Finance*, 48:65–92, 1993.

[25] J. Kallberg and P. Pasquariello. Time-series and cross-sectional excess comovement in stock indexes. *Journal of Empirical Finance*, 15:481–502, 2008.

[26] C. N. V. Krishnan, R. Petkova, and P. Ritchken. Correlation risk. *Journal of Empirical Finance*, 16:353–367, 2009.

[27] F. Longin and B. Solnik. Is the correlation in international equity returns constant: 1960-1990. *Journal of International Money and Finance*, 14:3–26, 1995.

[28] F. Longin and B. Solnik. Extreme correlation of international equity markets. *Journal of Finance*, 56:649–676, 2001.

[29] D. N. Politis and J. P. Romano. The stationaty bootstrap. *Journal of the American Statistical Society*, 89:1303–1313, 1994.

[30] J. M. Pollet and M. Wilson. Average correlation and stock market returns. *Journal of Financial Economics*, 96:364–380, 2010.

[31] R. Roll. A critique of the asset pricing theory's tests part i: On past and potential testability of the theory. *Journal of Financial Economics*, 4:129–176, 1977.

[32] W. F. Sharpe. Capital asset prices: a theory of market equilibrium under conditions of risk. *Journal of Finance*, 19:425–442, 1964.

Appendix

Table 6: Main regression estimation controlling for innovations in default spread, term spread and short term interest rates

	γ_κ	γ_χ	γ_ξ	γ_{ddef}	γ_{dterm}	γ_{drrel}	W_α	W_γ
Industry cross-section								
(1)	0.89	4.03*	1.67	-0.02	0.06	-0.06	98.07***	1.24
(2)	0.91	3.32	3.06	-0.01	0.05	-0.06	94.64***	0.98
(3)	0.94	3.21	3.91	-0.01	0.06	-0.05	104.43***	1.03
(4)	0.94	3.52	3.23	-0.02	0.07	-0.05	95.68***	0.89
Size cross-section								
(1)	1.59*	13.94	5.33	-0.02	0.13	0.00	9.66	1.61
(2)	1.51*	29.22**	0.80	-0.02	0.17	0.02	9.22	3.34
(3)	1.49*	28.14**	1.12	-0.02	0.13	0.02	9.80	3.04
(4)	1.50	24.49**	1.53	-0.02	0.17	0.03	9.36	3.51
Book-to-Market cross-section								
(1)	1.65	3.87	9.30	0.06	-0.07	0.05	14.75	0.73
(2)	1.63	2.59	11.50	0.07	-0.06	0.03	15.55	1.09
(3)	1.87	1.84	6.49	0.07	-0.09	0.05	16.29*	0.25
(4)	1.61	-0.18	12.00	0.05	-0.05	0.04	17.70*	1.34
Momentum cross-section								
(1)	1.33	18.91**	-4.73	0.02	0.30	0.16	44.82***	6.38**
(2)	1.77	23.10**	-6.37*	0.03	0.36	0.23	44.64***	8.34**
(3)	1.87	24.19**	-7.07**	0.03	0.34	0.25	49.92***	8.44**
(4)	1.64	15.88**	-5.84	0.04	0.31	0.22	49.34***	7.83**

I add DCC-GARCH conditional covariances of returns with innovations in default spread, term spread and relative interest rate. Each column accounts for a different correlation decomposition procedure, in the order as explained in Section 3. A block-bootstrap two-step-least-squares is applied to estimate the parameters—1000 iterations are considered.

Table 7: Main regression estimation controlling for innovations in inflation rate and output gap

	γ_κ	γ_χ	γ_ξ	γ_{dinf}	γ_{dout}	W_α	W_γ
Industry cross-section							
(1)	0.29	4.62*	1.65	0.91**	0.14	70.07***	1.54
(2)	0.27	3.93	2.92	0.93**	0.14	80.03***	1.38
(3)	0.30	4.01*	3.75	0.93**	0.14	72.86***	1.62
(4)	0.33	4.08*	3.05	0.95**	0.13	78.71***	1.31
Size cross-section							
(1)	1.65	14.33*	4.05	-0.11	0.06	10.12	1.66
(2)	1.71	26.51**	2.82	-0.22	0.05	9.57	3.05
(3)	1.80	27.05**	3.08	-0.19	0.05	9.71	3.05
(4)	1.58	22.34**	3.64	-0.20	0.06	10.86	3.48
Book-to-Market cross-section							
(1)	1.47	4.28	6.91	0.65	-0.24	16.19*	0.66
(2)	1.42	3.64	8.06	0.70	-0.21	16.65*	0.92
(3)	1.61	3.48	4.42	0.66	-0.24*	15.20	0.17
(4)	1.47	0.62	8.67	0.63	-0.21	17.92*	0.99
Momentum cross-section							
(1)	1.37	17.66**	-3.91	-0.56	-0.01	45.99***	5.43*
(2)	1.52	23.26**	-5.23	-0.48	0.00	57.28***	5.84*
(3)	1.60	22.58**	-6.34*	-0.56	-0.01	50.77***	7.27**
(4)	1.46	13.97*	-5.16	-0.60	0.02	50.71***	7.29**

I add DCC-GARCH conditional covariances of returns with innovations in inflation rate and output gap. Each column accounts for a different correlation decomposition procedure, in the order as explained in Section 3. A block-bootstrap two-step-least-squares is applied to estimate the parameters — 1000 iterations are considered.

Table 8: Main regression estimation controlling for innovations in dividend yields

	γ_κ	γ_χ	γ_ξ	γ_{ddiv}	W_α	W_γ
Industry cross-section						
(1)	-3.70*	0.49	1.28	-2.12***	111.31***	2.85
(2)	-3.77*	-0.27	2.55	-2.22***	111.20***	3.45
(3)	-3.86*	-0.45	3.17	-2.21***	118.24***	3.85
(4)	-3.77**	-0.33	2.54	-2.16***	114.85***	3.26
Size cross-section						
(1)	0.69	9.06	2.95	-0.33	9.69	0.76
(2)	0.88	22.71*	1.42	-0.23	11.22	2.37
(3)	1.07	21.99*	1.99	-0.17	9.87	2.13
(4)	0.81	18.17*	1.64	-0.23	10.64	2.07
Book-to-Market cross-section						
(1)	0.16	1.44	8.90	-0.40	15.55	1.28
(2)	0.39	-0.38	11.12*	-0.32	16.47*	1.66
(3)	0.39	-1.80	6.50	-0.38	15.97	0.70
(4)	0.12	-2.87	11.33*	-0.43	17.84*	2.10
Momentum cross-section						
(1)	1.10	16.45**	-4.10	-0.04	50.21***	5.01*
(2)	1.97	22.81**	-5.75*	0.29	48.45***	6.16**
(3)	1.89	23.99**	-6.41*	0.21	45.34***	6.13**
(4)	0.94	13.78*	-5.22	-0.17	45.97***	4.90*

I add DCC-GARCH conditional covariances of returns with innovations in aggregate dividend yields. Each column accounts for a different correlation decomposition procedure, in the order as explained in Section 3. A block-bootstrap two-step-least-squares is applied to estimate the parameters—1000 iterations are considered.

Table 9: Main regression estimation controlling for hedging against financial factors

	γ_{κ}	γ_{χ}	γ_{ξ}	γ_{smb}	γ_{hml}	γ_{umd}	W_{α}	W_{γ}
Industry cross-section								
(1)	0.97	6.06**	3.03	-0.86	4.70**	-1.47*	91.52***	2.24
(2)	1.00	5.43**	4.23	-0.89	4.58**	-1.44*	86.68***	2.02
(3)	1.01	5.38**	5.05	-0.92	4.64**	-1.47*	85.79***	1.97
(4)	1.02	5.59**	4.42	-0.91	4.66**	-1.43*	88.41***	1.98
Size cross-section								
(1)	2.34*	10.84	7.86	-0.00	3.89**	-1.14	10.37	1.36
(2)	2.21*	23.08**	3.74	-0.20	3.31*	-1.03	10.45	2.49
(3)	2.17*	22.40**	3.53	-0.23	3.34*	-1.13*	10.19	2.43
(4)	2.25*	19.04*	4.65	-0.12	3.45*	-1.14*	9.66	1.93
Book-to-Market cross-section								
(1)	2.16	3.93	11.91*	-0.93	5.29***	0.26	3.79	1.32
(2)	2.27	3.09	15.26**	-1.03	5.80***	0.24	4.01	2.19
(3)	2.50	1.92	9.55	-1.14	5.23**	0.27	3.98	0.75
(4)	2.24	-0.14	15.84**	-0.76	5.87***	0.37	3.77	2.58
Momentum cross-section								
(1)	1.44	21.09**	-1.96	-2.32	3.90	-1.25**	47.69***	5.01*
(2)	1.63	25.19**	-4.04	-2.03	3.70	-1.29**	46.03***	5.32*
(3)	1.40	27.16*	-3.80	-1.83	3.71	-1.27*	57.20***	4.13
(4)	1.67	17.21**	-3.28	-2.19	3.83*	-1.24**	48.42***	6.80**

I add DCC-GARCH conditional covariances of returns with the financial factors of size, book-to-market and momentum. Each column accounts for a different correlation decomposition procedure, in the order as explained in Section 3. A block-bootstrap two-step-least-squares is applied to estimate the parameters — 1000 iterations are considered.

Table 10: Main regression estimation controlling for innovations in market volatility

	γ_κ	γ_χ	γ_ξ	$\gamma_{\tilde{\sigma}}$	W_α	W_γ
Industry cross-section						
(1)	1.27	4.45*	1.66	0.52	93.30***	1.11
(2)	1.32	3.75	3.12	0.50	95.89***	0.76
(3)	1.48	3.89*	4.11	0.58	93.92***	0.86
(4)	1.33	3.88*	3.33	0.53	104.41***	0.81
Size cross-section						
(1)	1.26	14.19*	4.73	-0.05	8.18	1.59
(2)	1.35	28.60**	2.10	0.03	8.94	3.73
(3)	1.32	28.71**	2.14	-0.02	9.05	3.60
(4)	1.38	22.58**	3.44	0.00	7.94	3.07
Book-to-Market cross-section						
(1)	3.02	4.52	9.03	1.11	15.07	0.54
(2)	3.20	3.98	11.00	1.16	16.50*	0.81
(3)	2.94	2.55	6.28	0.96	14.85	0.18
(4)	3.14	-0.68	11.61*	1.17	15.05	1.46
Momentum cross-section						
(1)	2.22*	19.09**	-3.73	0.62*	45.32***	6.92**
(2)	2.47**	22.28**	-5.28	0.57	47.43***	8.13**
(3)	2.35*	23.58**	-6.04*	0.57*	46.18***	8.29**
(4)	2.37*	15.41**	-4.52	0.57	44.43***	7.41**

I add DCC-GARCH conditional covariances of returns with innovations in market volatility. Each column accounts for a different correlation decomposition procedure, in the order as explained in Section 3. A block-bootstrap two-step-least-squares is applied to estimate the parameters — 1000 iterations are considered.

Table 11: Main regression estimation controlling for innovations in average correlations

	γ_κ	γ_χ	γ_ξ	$\gamma_{\tilde{\rho}}$	W_α	W_γ
Industry cross-section						
(1)	1.55	4.57*	2.09	1.83*	84.75***	0.87
(2)	1.56	3.68	3.57	1.82*	89.91***	0.54
(3)	1.55	3.91	4.31	1.89*	88.44***	0.78
(4)	1.57	4.20*	3.70	1.79*	85.59***	0.75
Size cross-section						
(1)	1.79*	14.04	4.85	0.22	9.96	1.42
(2)	1.80*	28.74**	2.10	0.13	10.13	3.22
(3)	1.78*	27.90**	2.29	0.13	9.02	2.94
(4)	1.81*	23.80**	3.09	0.12	9.04	2.83
Book-to-Market cross-section						
(1)	1.12	2.73	8.44	0.09	11.75	0.76
(2)	1.19	1.93	10.77*	0.25	13.81	1.21
(3)	1.36	1.24	5.54	0.24	12.06	0.26
(4)	1.16	-1.20	11.42*	0.25	12.21	1.62
Momentum cross-section						
(1)	1.58	17.52**	-4.23	1.05	45.62***	5.69*
(2)	1.73	22.20**	-5.62*	0.76	43.46***	6.84**
(3)	1.69	23.39**	-6.32*	0.87	52.86***	6.58**
(4)	1.68	14.31**	-5.44	0.98	46.34***	7.76**

I add DCC-GARCH conditional covariances of returns with innovations in the aggregate level of correlations. Each column accounts for a different correlation decomposition procedure, in the order as explained in Section 3. A block-bootstrap two-step-least-squares is applied to estimate the parameters — 1000 iterations are considered.

Table 12: Variance decomposition of conditional covariance

Decile	1	2	3	4	5	6	7	8	9	10
Common correlation components: size										
(1)	0.91	0.91	1.03	0.94	0.91	0.84	0.71	0.68	0.33	0.36
(2)	0.50	0.59	0.81	0.94	0.97	1.00	1.05	1.00	0.69	0.33
(3)	0.50	0.56	0.77	0.88	0.93	1.01	1.01	1.04	0.72	0.28
(4)	0.53	0.61	0.83	0.95	0.97	0.98	1.07	1.00	0.69	0.31
Common correlation components: book-to-market										
(1)	0.00	0.50	0.86	0.78	0.96	0.81	0.99	0.89	0.81	0.28
(2)	0.12	0.75	0.97	0.77	0.91	0.91	0.82	0.88	0.75	0.40
(3)	0.12	0.94	1.15	0.63	0.82	1.22	0.56	0.75	0.66	0.44
(4)	0.06	0.70	0.92	0.83	0.92	0.88	0.91	0.95	0.78	0.39
Common correlation components: momentum										
(1)	0.76	0.79	0.92	0.93	0.85	0.86	0.84	0.44	0.24	0.16
(2)	0.67	0.69	0.68	0.84	0.96	0.96	0.90	0.70	0.60	0.66
(3)	0.78	0.69	0.54	0.62	0.81	0.94	0.73	0.71	0.75	0.86
(4)	0.64	0.71	0.71	0.87	0.97	0.99	0.90	0.68	0.50	0.49
Idiosyncratic correlation components: size										
(1)	0.07	0.06	0.03	0.11	0.12	0.20	0.22	0.22	0.48	0.62
(2)	0.29	0.23	0.12	0.09	0.07	0.08	0.08	0.06	0.26	0.59
(3)	0.27	0.23	0.11	0.09	0.07	0.11	0.07	0.07	0.29	0.66
(4)	0.28	0.22	0.11	0.09	0.07	0.08	0.07	0.06	0.27	0.50
Idiosyncratic correlation components: book-to-market										
(1)	1.00	0.47	0.29	0.15	0.07	0.16	0.05	0.13	0.12	0.47
(2)	0.90	0.26	0.09	0.15	0.05	0.11	0.14	0.21	0.14	0.34
(3)	0.97	0.38	0.25	0.15	0.05	0.20	0.12	0.18	0.15	0.33
(4)	0.93	0.33	0.11	0.12	0.04	0.14	0.10	0.18	0.13	0.37
Idiosyncratic correlation components: momentum										
(1)	0.46	0.18	0.19	0.07	0.11	0.15	0.16	0.50	0.66	0.87
(2)	0.41	0.23	0.25	0.09	0.07	0.07	0.06	0.28	0.36	0.57
(3)	0.48	0.23	0.29	0.12	0.11	0.07	0.09	0.33	0.52	0.92
(4)	0.47	0.23	0.20	0.07	0.06	0.07	0.05	0.28	0.45	0.75

For each decile, I look at the ratio $\mathrm{var}(\chi_{i|t}\sigma_{i|t}\sigma_{m|t})/\mathrm{var}((\chi_{i|t}+\xi_{i|t})\sigma_{i|t}\sigma_{m|t})$ in the first three panels, and the ratio $\mathrm{var}(\xi_{i|t}\sigma_{i|t}\sigma_{m|t})/\mathrm{var}((\chi_{i|t}+\xi_{i|t})\sigma_{i|t}\sigma_{m|t})$ in the last three panels.