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Children's Use of Addition to Solve Two-Digit Subtraction Problems

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Abstract

Subtraction problems of the type $M - S = ?$ can be solved with various mental calculation strategies. We investigated fourth- to sixth-graders' use of the subtraction by addition strategy, first by fitting regression models to the reaction times of 32 two-digit subtractions. These models represented 3 different strategy use patterns: the use of direct subtraction, subtraction by addition, and switching between the 2 strategies based on the magnitude of the subtrahend. Additionally, we compared performance on problems presented in 2 presentation formats, i.e., a subtraction format ($81 - 37 = .$) and an addition format ($37 + . = 81$). Both methods converged to the conclusion that children of all three grades switched between direct subtraction and subtraction by addition based on the combination of two features of the subtrahend: If the subtrahend was smaller than the difference, direct subtraction was the dominant strategy; if the subtrahend was larger than the difference, subtraction by addition was mainly used. However, this performance pattern was only observed when the numerical distance between subtrahend and difference was large. These findings indicate that theoretical models of children's strategy choices in subtraction should include the nature of the subtrahend as an important factor in strategy selection.

Children's Use of Addition to Solve Two-Digit Subtraction Problems

Current reform movements in elementary mathematics education stress that instruction should no longer focus on solving school mathematics exercises quickly and accurately by means of the school-taught standard strategies (i.e., *routine expertise*). In contrast, mathematical tasks should be solved efficiently, creatively, and flexibly with a variety of meaningfully acquired strategies (i.e., *adaptive expertise*). Consequently, more attention should be paid to the insightful, varied, and flexible use of strategies when teaching how to solve mathematical problems (Baroody & Dowker, 2003; Hatano, 2003; Kilpatrick, Swafford, & Findell, 2001; Verschaffel, Greer, & De Corte, 2007). Strategy flexibility has been investigated from a cognitive psychological and/or a mathematics educational perspective in several subdomains of mathematics, such as numerosity judgment (e.g., Luwel, Verschaffel, Onghena, & De Corte, 2003), mental calculation (e.g., Blöte, Klein, & Beishuizen, 2000), computational estimation (e.g., Star & Rittle-Johnson, 2009), and solving equations (e.g., Star & Rittle-Johnson, 2008). In the current study, we examined children's flexible use of mental subtraction strategies, more specifically their flexible use of the subtraction by addition strategy.

The variety and flexibility in the way children solve symbolically presented subtraction problems ($M - S = .$) has received a lot of attention from researchers in the last four decades (e.g., Barrouillet, Mignon, & Thevenot, 2008; Cowan et al., 2011; Dowker, 2009; Fuson, 1992; LeFevre, DeStefano, Penner-Wilger, & Daley, 2006; Robinson, 2001; Selter, 2001; Siegler, 2003; Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009a; Woods, Resnick, & Groen, 1975). Several studies have shown that children can develop various strategies to mentally solve multi-digit subtraction problems (e.g., Beishuizen, 1993; Carpenter, Franke, Jacobs, Fennema, & Empson, 1997; Torbeyns, De Smedt, Stassens, Ghesquière, & Verschaffel, 2009). One way to classify these mental subtraction strategies is by looking at

the operation that underlies the solution process, which can be either subtraction or addition. (For other ways of classifying mental subtraction strategies, see for example Peltenburg, van den Heuvel-Panhuizen, & Robitzsch, 2012). In this way, two types of strategies can be distinguished¹: (1) *direct subtraction strategies*, in which the subtrahend is directly subtracted from the minuend (e.g., $75 - 43 = .$ by “ $75 - 40 = 35$, $35 - 3 = 32$ ”), and (2) *subtraction by addition strategies*², in which one determines how much needs to be added to the subtrahend to get to the minuend (e.g., $75 - 43 = .$ by “ $43 + 30 = 73$ and $73 + 2 = 75$, so the answer is $30 + 2 = 32$ ”). This subtraction by addition strategy is considered to be particularly efficient on problems with a large subtrahend (S) compared to the difference (D) (e.g., $93 - 88 = .$, where S and D are 88 and 5, respectively), because the subtraction by addition strategy then requires fewer and/or smaller calculation steps than a direct subtraction strategy (e.g., Torbeyns, De Smedt, Stassens, et al., 2009). In contrast, direct subtraction is assumed to be more efficient than subtraction by addition when the subtrahend is small compared to the difference (e.g., $93 - 5$, where S and D are 5 and 88, respectively).

While previous studies have shown that adults use the subtraction by addition strategy frequently, efficiently (i.e., fast and accurately), and flexibly (i.e., mainly, but not exclusively on problems with a relatively large subtrahend) (Peters, De Smedt, Torbeyns, Ghesquière, & Verschaffel, 2010a, 2010b; Torbeyns, De Smedt, Peters, Ghesquière, & Verschaffel, 2011; Torbeyns, Ghesquière, & Verschaffel, 2009), well-documented evidence on primary school children's self-reported use of the subtraction by addition strategy when mentally solving two-digit subtraction problems is limited (Blöte et al., 2000; De Smedt, Torbeyns, Stassens, Ghesquière, & Verschaffel, 2010; Selter, 2001; Torbeyns, De Smedt, Ghesquière, et al., 2009a). For example, Torbeyns, De Smedt, Ghesquière, et al. (2009a) asked second-, third-, and fourth-graders to mentally solve two-digit subtractions in two tasks. In the *Spontaneous Strategy Use* task, the children could solve each item with the strategy they preferred. This

task included 15 two-digit subtraction problems and five of them were designed to elicit the subtraction by addition strategy through a relatively large subtrahend (as in $43 - 38 = .$). Findings revealed that these children hardly applied the subtraction by addition strategy spontaneously. In the *Variability on Demand* task, the children were asked to generate up to five different ways to solve 4 two-digit subtraction problems, two of them being specifically designed to trigger the subtraction by addition strategy (e.g., $81 - 79 = .$). Surprisingly, the frequency of subtraction by addition strategies did not differ from the first task, not even after asking children to provide up to five possible strategies to solve a problem. The authors concluded that the subtraction by addition strategy was not included in most children's strategy repertoire.

De Smedt et al. (2010) tried to stimulate the use of subtraction by addition in third-grade children through an implicit and an explicit learning environment. Participants were divided over the two learning environments, both involving four training sessions. In the implicit learning environment, children were confronted with an unusually large number of two-digit problems with a relatively large subtrahend (e.g., $92 - 88 = .$), which were assumed to trigger the use of subtraction by addition. In this way, the authors aimed to stimulate children's spontaneous discovery and further development of the subtraction by addition strategy. Children in the explicit learning environment were instructed to solve each problem once with their preferred strategy and once with subtraction by addition, which was demonstrated and explained at the beginning of each training session. None of the children from the implicit learning environment reported using the subtraction by addition strategy after two training sessions, at the end of the training, or in the retention session one month later. In the explicit learning environment, only 6 % of the children reported the use the subtraction by addition strategy halfway the training, only 11 % reported subtraction by addition by the end of the training sessions, and only 10 % reported it one month later. These low percentages lead the

authors to conclude that – even in the explicit learning environment – third-grade children experienced great difficulties with picking up the subtraction by addition strategy and integrating it into their strategy repertoire.

One important limitation of the studies reviewed above is that they relied exclusively on verbal protocol data to unravel children's strategy use. It can be argued that such verbal protocols may be less suited to identify the subtraction by addition strategy in multi-digit subtraction, because this strategy may be executed in a very fast and quasi-automatic way (Kirk & Ashcraft, 2001; Russo, Johnson, & Stevens, 1989). For example, when solving a problem such as $93 - 88$, children may not be aware of the calculation steps they executed, or they may have difficulties in articulating precisely how they found the answer (and therefore report another strategy they can easily explain). Furthermore, children may hide the use of a particular strategy because they think it is not valued, or even not allowed, according to the socio-mathematical classroom norms (Yackel & Cobb, 1996). This may have resulted in an underestimation of the number of subtraction by addition strategies in the available studies on multi-digit subtraction in children.

In this respect, we would like to point out an intriguing inconsistency between the reaction time data and children's verbal reports in De Smedt et al. (2010). The majority of the third-graders in that study only reported the use of the direct subtraction strategy for the three kinds of problems involved in the study, namely problems with relatively small, medium, and large subtrahends. Against the background of a rational task analysis, this consistent use of direct subtraction should have led to an increase in the reaction times from problems with relatively small subtrahends (e.g., $81 - 7 = .$) over problems with medium-sized subtrahends (e.g., $81 - 43 = .$) to problems with relatively large subtrahends (e.g., $81 - 79 = .$), because subtracting a large subtrahend requires more and/or larger calculation steps than subtracting a small or a medium-sized subtrahend (Peters et al., 2010b; Woods et al., 1975). However, the

observed reaction time patterns were not in line with this prediction: Problems with relatively small subtrahends as well as problems with relatively large subtrahends were solved faster than problems with medium-sized subtrahends. These reaction time data are very hard to reconcile with the idea that only one strategy (i.e., direct subtraction) was used on these problems and they suggest the involvement of different strategies. This opens the possibility that the subtraction by addition strategy might have been used more frequently than suggested by the children's verbal reports, particularly on problems with relatively large subtrahends. This observation asks for the application of other methods of inferring participants' strategy use that do not rely on verbal reports. Two such methods have already been successfully used in both single- and multi-digit arithmetic with children and/or adults: the regression-based approach (Groen & Poll, 1973; Woods et al., 1975) and the manipulation of presentation formats (Campbell, 2008; Peters et al., 2010a, 2010b).

The Present Study

In this study, we examined the use of the subtraction by addition strategy on two-digit subtraction problems of fourth- to sixth-grade primary school children by means of two non-verbal research methods, namely the regression-based approach and the manipulation of presentation formats.

Starting from Groen and Poll (1973), we first calculated three linear regression models for the reaction times of problems presented in the standard subtraction format (see also Peters, De Smedt, Torbeyns, Ghesquière, & Verschaffel, 2012). These regression models represented three different strategy use patterns: the use of the direct subtraction strategy, the use of the subtraction by addition strategy, and switching between both strategies based on the relative magnitude of the subtrahend ($S < D$ vs. $S > D$). If children use the direct subtraction strategy (*DS-Model*), reaction times should be best predicted by the size of the subtrahend (S), because it takes longer to subtract a larger number from the minuend (e.g., $83 - 79 = .$) than to subtract

a smaller number from that minuend (e.g., $83 - 4 = .$). If children use the subtraction by addition strategy (*SBA-Model*), reaction times should be best predicted by the size of the difference (D), because it takes more time to determine how much needs to be added to get at a given number when the difference between both numbers is large ("How much needs to be added to 4 to have 83?") than when it is small ("How much needs to be added to 79 to have 83?"). If participants switch between both strategies depending on the magnitude of the subtrahend (*Switch-Model*), reaction times should be best predicted by the minimum of subtrahend and difference ($\min[D, S]$): For problems with the subtrahend smaller than the difference (e.g., $83 - 4 = .$ and $84 - 38 = .$), reaction times are expected to increase with the size of the subtrahend, because these problems can be easily solved by means of the direct subtraction strategy. In contrast, for problems with the difference smaller than the subtrahend (e.g., $83 - 79 = .$ and $84 - 46 = .$), reaction times are expected to increase with the size of the difference, because these problems can be solved easily by means of the subtraction by addition strategy. Based on the above-mentioned post-hoc analysis of the results of De Smedt et al. (2010), we expected that problems with relatively small *and* problems with relatively large subtrahends would be solved faster than problems with medium-sized subtrahends, suggesting that children switch between direct subtraction and subtraction by addition depending on the magnitude of the subtrahend. We therefore predicted that the *Switch-model* would provide the best fit to the reaction time data of the subtractions presented in the standard subtraction format (= Hypothesis 1).

Second, we hypothesised that children's choices between direct subtraction and subtraction by addition would depend not only on the magnitude of the subtrahend ($S < D$ vs. $S > D$), but also on the numerical distance between S and D (large vs. small). More specifically, we predicted that the magnitude of S would only influence the strategy choice process between direct subtraction and subtraction by addition when the numerical distance

between S and D was sufficiently large (i.e., in problems such as $83 - 79 = .$ or $83 - 4 = .$), and not when this distance was rather small (i.e., in problems such as $84 - 46 = .$ or $84 - 38 = .$). In the latter case the computational advantage of using one of the two strategies is arguably marginal (compare solving $84 - 38 = .$ by means of $84 - 30 - 8 = 54 - 8 = 50 - 4 = 46$, with solving $84 - 38 = .$ by means of $38 + 2 = 40$; $40 + 40 = 80$; $80 + 4 = 84$, so the answer is $2 + 40 + 4 = 46$).

To test this latter hypothesis, we used a second non-verbal method in which we compared participants' reaction times on four different problem types (i.e., the combination of the magnitude of S [$S < D$ vs. $S > D$] and the numerical distance between S and D [small vs. large]) in two presentation formats, namely the traditional subtraction format ($M - S = .$) and the (unusual) addition format ($S + . = M$) (see also Campbell, 2008; Peters et al., 2010a, 2010b). Following Campbell (2008), it can be hypothesised that if subtractions are solved by addition, problems presented in the addition format (e.g., $79 + . = 83$) should be solved faster than those in the traditional subtraction format (e.g., $83 - 79 = .$). This should be the case because the subtraction by addition strategy can be easily performed when a problem is presented in the addition format, while a mental *re*-representation is needed when the problem is presented in the subtraction format (and vice versa for performing the direct subtraction strategy for problems presented in the addition format). So, if children are systematically switching between direct subtraction and subtraction by addition depending on the combination of the magnitude of S and the numerical distance between S and D (or the *nature of the subtrahend*, as we will call the combination of these two features of the subtrahend from now on), a three-way interaction between the magnitude of S , the presentation format, and the numerical distance between S and D should be found (= Hypothesis 2). More specifically, the large-distance $S > D$ problems (such as $83 - 79 = .$) will be solved faster in the addition format compared to the same problems in subtraction format, whereas large-

distance $S < D$ problems (such as $83 - 4 = .$) will lead to slower response times in the addition format than in the subtraction format. On the other hand, this magnitude $\times$ format interaction will be minimal or even completely absent for the problems with a small distance between S and D (such as $84 - 46 = .$ or $84 - 38 = .$), because this small distance does not yield a clear computational advantage for either direct subtraction or subtraction by addition.

Method

Participants

Participants were 177 typically developing fourth- to sixth-grade children from two primary schools located in Flanders (Belgium). These children completed two short paper-and-pencil subtests to check whether they were able to solve subtraction problems presented in the (unusual) $S + . = M$ format. The first subtest included all 32 possible non-tie single-digit subtraction problems with the minuend larger than 10 (e.g., $6 + . = 11$). In the second subtest, 36 problems from the number domain 20-100 were presented (e.g., $37 + . = 81$). All problems were presented in the horizontal format, which is the standard format for doing mental subtraction in Flanders (Belgium)³. For the single-digit subtractions, fourth-graders had 90 seconds to solve the task; for fifth- and sixth-graders a time-limit of 60 seconds was applied. For the two-digit subtractions, fourth-graders had a time-limit of 120 seconds, and fifth- and sixth-graders had 90 seconds to solve the task. The administration of this paper-and-pencil task was group based.

We only wanted to include children who demonstrated mastery of subtraction problems in the addition format. We therefore excluded 10 children who made one or more conceptual error(s) (e.g., giving 17 as the answer for $6 + . = 11$), as well as all children who performed within the weakest 10% of their grade on this task ($n = 25$). From this remaining sample, we randomly selected 81 children for participation in two sessions of an individually administered computer task one month later. All children came from medium to high socio-

economic backgrounds. Due to illness, absence, or technical problems, only 72 out of the 81 selected children finalised the two sessions of the computer task (23 fourth-graders, 26 fifth-graders and 23 sixth-graders). Their mean ages (with *SD* in parentheses) were 9 years and 9 months (3 months) in fourth grade, 10 years and 10 months (4 months) in fifth grade, and 11 years and 9 months (3 months) in sixth grade.

Materials and Procedure

All 72 participants had to mentally solve 32 horizontally presented subtraction problems which had a two-digit minuend larger than 30 and required borrowing. Each problem was presented both in the subtraction format (e.g., $52 - 4 = .$) and in its corresponding addition format ($4 + . = 52$), yielding a total set of 64 trials. They were divided into four problem types, based on the combination of the magnitude of S compared to D ($S < D$ or $S > D$) and the numerical distance between S and D (small or large). Small-distance problems were defined by S and D differing by less than 10, whereas in the large-distance problems S and D differed by at least 10 *and* either S or D was a one-digit number. This resulted in the following problem types: (a) *large-distance $S < D$ problems*, with subtrahends smaller than 10 (e.g., $83 - 4 = .$ and $8 + . = 34$); (b) *large-distance $S > D$ problems*, with differences smaller than 10 (e.g., $77 - 68 = .$ and $37 + . = 42$); (c) *small-distance $S < D$ problems* (e.g., $92 - 44 = .$ and $36 + . = 75$); and (d) *small-distance $S > D$ problems* (e.g., $32 - 17 = .$ and $29 + . = 53$) (see Appendix A for an overview of all problems).

As already mentioned, problems were administered on two consecutive days. Each day, children solved 32 experimental problems: 16 problems in the subtraction format and 16 problems in the addition format. Problems that only differed in presentation format (e.g., $31 - 28 = .$ and $28 + . = 31$) were divided over the two sessions. In each session, the items were presented into three blocks of 10 or 11 problems, assigned in a pseudo-random manner with the following three constraints: (a) items in the same presentation format could not occur

on more than two consecutive trials; (b) items from the same problem type could also not occur on more than two consecutive trials (e.g., $31 - 28 = .$ and $68 + . = 77$); and (c) complementary items (e.g., $31 - 28 = .$ and $3 + . = 31$) could not appear in the same block. Each session started with two practice problems per presentation format to familiarise participants with the task administration. Task instructions stressed both accuracy and speed.

The task was administered individually in the presence of an experimenter on a DELL portable computer that was put in a quiet room. Each trial started with an asterisk that appeared for 1000 ms in the centre of the screen. Next, the item was presented in the middle of the screen. The characters were 2×2 cm large, black, separated by adjacent spaces and presented against a white background. The item remained on the screen until the child uttered the answer. Time started to run when the item appeared on the screen, and ended when the experimenter pushed the spacebar of an external keyboard that was connected to the computer. Next, the experimenter entered the answer via the keyboard after which the next trial was initiated. No feedback was given.

Results

All 72 children solved 64 problems in the computer task, resulting in a total of 4608 trials. We excluded 61 trials due to incorrect task administration (1.32 %), and 436 more trials (9.46 %) were removed because they were answered incorrectly. Most of these incorrect answers were examples of the typical errors that are observed when solving mental subtractions (e.g., Beishuizen, 1993): Answers that differed only in tens from the correct answer (e.g., $24 + . = 53$ answered with 39 instead of 29; in 41 % of all incorrect answers), answers that differed in only 1 unit from the correct answer (e.g., $44 + . = 92$ answered with 47 instead of 48; in 11 % of all incorrect answers), and answers resulting from the smaller-from-larger bug (e.g., $52 - 48$ answered with 16 instead of 4; in 16 % of all incorrect

answers). In only 2 % of all incorrect answers the addition operation was used incorrectly (e.g., $32 - 17$ answered with 49, or $3 + . = 31$ answered with 34).

A closer inspection of the remaining data further revealed that 3 children (1 fourth-grader and 2 fifth-graders) did not provide at least 2 out of 8 correct answers within one or more of the eight problem types. We excluded these three children's trials from further analyses, as it was not possible to reliably estimate their reaction times in the regression models and in the ANOVA. This resulted in a total of 4017 trials for the reaction time analyses (i.e., 90.97 % of all trials of the 69 remaining participants).

Results are presented in two parts. The first part involves the evaluation of the regression models that predict children's reaction times, whereas in the second part we compare children's reaction times in the two presentation formats for the four different problem types (For an overview of the mean accuracy rates for these different problem types, we refer to Appendix B). All analyses were carried out by means of SAS Version 9.3.

Regression analyses for problems in subtraction format

We predicted the reaction times of the 32 problems presented in the subtraction format by three regression models representing different strategy use patterns (see Peters et al., 2012; Woods et al., 1975). As stated before, these three models represented the use of direct subtraction (*DS-Model*), the use of subtraction by addition (*SBA-Model*), and switching between both strategies based on the magnitude of the subtrahend (*Switch-Model*). Reaction times were predicted based on either the size of the subtrahend (*DS-Model*), the size of the difference (*SBA-Model*), or the minimum of subtrahend and difference (*Switch-Model*).

All regression models were fitted to the mean reaction time, averaged across individuals, of each problem per grade. These analyses revealed that in each grade the model in which children switched between direct subtraction and subtraction by addition based on the

magnitude of the subtrahend (*Switch-Model*) provided the best fit to the data (Table I), explaining more than 70% of the variance in each grade.

Insert Table I about here

These regression analyses aggregated performance over the various participants per grade, and may thus mask strategy patterns at the individual level. Therefore, we also predicted the reaction times by the three models for each participant individually. In all three grades, the model representing a switch between direct subtraction and subtraction by addition based on the magnitude of the subtrahend (*Switch-Model*) provided the best fit to the reaction times for nearly all children (see Table II).

Insert Table II here

The results of the regression analyses are thus in line with Hypothesis 1, which stated that the regression model that represents children switching between direct subtraction and subtraction by addition depending on the magnitude of the subtrahend (i.e., the *Switch-Model*) would fit best to the reaction time data.

Comparison of reaction times between the two presentation formats

The mean reaction times per presentation format, grade, and problem type are depicted in Table III. We performed a $2 \times 2 \times 2 \times 3$ repeated measures ANOVA on the reaction time data with magnitude of S ($S < D$ vs. $S > D$), numerical distance between S and D (small vs. large) and presentation format (subtraction vs. addition) as within-subject factors and grade (fourth, fifth, and sixth) as a between-subjects factor. Tukey-Kramer adjustments were used for post-hoc comparisons.

Insert Table III about here

There was no main effect of presentation format, $F(1, 66) = 1.94, p = .17$, or magnitude, $F(1, 66) = 0.03, p = .87$, but there was a significant effect of numerical distance, $F(1, 66) = 308.74, p < .01$: Large-distance problems, such as $31 - 3 = .$ or $28 + . = 31$, were

solved faster ($M = 5194$ ms) than small-distance problems, such as $32 - 15 = .$ or $46 + . = 84$ ($M = 8698$ ms). There was also a significant magnitude $\times$ distance interaction, $F(1, 264) = 4.94, p = .03$, suggesting that the speed difference from the large-distance to the small-distance problems is larger for the $S > D$ problems (from 5027 ms to 8842 ms, i.e., 3815 ms) than for the $S < D$ problems (from 5360 ms to 8555 ms, i.e., 3195 ms).

The interaction between magnitude and presentation format was significant as well, $F(1, 264) = 26.23, p < .01$: $S > D$ problems in the addition format were solved 908 ms faster than the same problems in the subtraction format ($p < .01$), whereas $S < D$ problems were solved 519 ms faster in the subtraction than in the addition format ($p = .04$). Most importantly, the magnitude $\times$ format $\times$ distance interaction was significant, $F(1, 264) = 8.53, p < .01$ (see Figure 1). Post-hoc tests revealed that, as predicted in Hypothesis 2, the above-mentioned magnitude $\times$ format interaction was significant for the large-distance problems, but not for the small-distance problems. The large-distance $S > D$ problems in the addition format (e.g., $28 + . = 31$) were solved faster than the same problems in the subtraction format (e.g., $31 - 28 = .$) ($p < .01$), whereas the large-distance $S < D$ problems were solved faster in the subtraction (e.g., $31 - 3 = .$) than in the addition format (e.g., $3 + . = 31$) ($p = .04$).

Insert Figure 1 about here.

There was a significant main effect of grade, $F(2, 66) = 5.82, p < .01$: Sixth-graders ($M = 5466$ ms) were significantly faster than fifth-graders ($M = 7364$ ms) and fourth-graders ($M = 8008$ ms), whereas the latter two groups did not differ from each other. There was also a significant grade $\times$ distance interaction, $F(2, 264) = 4.37, p = .01$, showing that fourth-graders were only slower than sixth-graders on small-distance problems, such as $32 - 15 = .$ or $17 + . = 32$ ($p < .01$). Furthermore, the grade $\times$ magnitude $\times$ distance interaction was significant, $F(2, 264) = 3.32, p = .04$, indicating that the magnitude $\times$ distance interaction was larger for fourth-graders than for the two other grades. Finally, the four-way interaction

between grade, magnitude, format and distance was not significant, $F(2, 264) = 2.79, p = .06$.

A closer inspection of this interaction revealed that the magnitude $\times$ format $\times$ distance interaction was similar across the three grades, which indicates that the effect as predicted in Hypothesis 2 was observed in each grade.

Discussion

In the present study we investigated fourth- to sixth-grade primary school children's use of the subtraction by addition strategy when mentally solving two-digit subtraction problems. Previous studies on children's use of this strategy showed that they rarely use subtraction by addition spontaneously on two-digit subtractions (De Smedt et al., 2010; Dowker, 2009; Torbeyns, De Smedt, Ghesquière, et al., 2009a). However, these findings were based on verbal protocol methods, which may be less suited to identify the subtraction by addition strategy (Kirk & Ashcraft, 2001; Russo et al., 1989). In this study, we conducted regression analyses in which reaction times were predicted based on different task characteristics, and we contrasted speed between problems presented in different formats. Both non-verbal methods converged to the conclusion that children solve two-digit subtraction problems by switching between direct subtraction and subtraction by addition depending on the combination of the magnitude of S and the numerical distance between S and D (or the *nature of the subtrahend*).

Based on Woods et al. (1975), we first carried out three linear regression analyses per grade. The regression models predicted reaction times by three different problem characteristics: the size of the subtrahend, the size of the difference, and the minimum of subtrahend and difference. The minimum of subtrahend and difference was the best predictor of the reaction time data in all three grades. This result suggests that fourth- to sixth-grade children switched between the direct subtraction and the subtraction by addition strategy based on the magnitude of the subtrahend: The direct subtraction strategy was used when the

subtrahend was smaller than the difference, whereas the subtraction by addition strategy was used when the subtrahend was larger than the difference.

Second, we compared reaction times on problems presented in the standard subtraction format with its corresponding addition format, based on Campbell (2008). We expected to find significant reaction time differences between the two formats: If a problem is solved faster in the addition format than in the subtraction format, this suggests that this problem is likely to be solved by means of subtraction by addition – the reaction time saving is expected to occur because presenting the problem in the addition format saves the mental *re-arrangement* of the problem elements into the addition format (and vice versa for performing the direct subtraction strategy for problems in the addition format). Furthermore, we predicted that these reaction time differences would be moderated by the numerical distance between subtrahend and difference, because the computational advantage of one strategy over the other is very clear for problems with a large numerical distance between subtrahend and difference (such as $83 - 4 = .$ and $83 - 79 = .$), but not for problems with a small numerical distance between subtrahend and difference (such as $84 - 38 = .$ and $84 - 46 = .$). In line with this second hypothesis, we found an interaction between the magnitude of the subtrahend, the presentation format, and the numerical distance between subtrahend and difference: When the numerical distance between the subtrahend and the difference was large, $S < D$ problems were solved faster in the subtraction than in the addition format, while $S > D$ problems were solved faster in the addition than in the subtraction format. When the numerical distance was small, there were no format effects. This three-way interaction suggests that children switched between the two strategies when solving two-digit subtractions. Moreover, they show that this switching only takes place when the numerical distance between subtrahend and difference is large. These results echo those of Peters et al. (2010b) observed in adults.

Both the linear regression analyses and the comparison of the two presentation formats indicated that the selection pattern between direct subtraction and subtraction by addition strategies was similar in all three grades. Further research with both younger and older children is definitely needed to shed further light on possible developmental trends in strategy choice between direct subtraction and subtraction by addition strategies. One option would be to include a measure of mathematical ability in future studies, to see whether children with higher mathematical ability adapt their strategy choice more to the nature of the subtrahend compared to children with lower mathematical ability (e.g., Baroody, 2003; Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009).

Interestingly, our findings are not in line with Peters et al. (2012), in which children's use of subtraction by addition was investigated with the same research method but in the number domain up to 20. In that study Flemish third- to sixth-grade children consistently used a direct subtraction strategy to solve problems presented in subtraction format, whereas in the current study children switched between direct subtraction and subtraction by addition according to the numbers in the problem. A first explanation for these different results might be found in the way subtraction is taught in primary school in Flanders. In the number domain up to 20 children typically receive only direct instruction and intense practice in the direct subtraction strategy: They learn to decompose the subtrahend into two parts and to sequentially subtract these two parts from the minuend (for example solving $13 - 5 = .$ by first taking away 3 to get to 10 and then taking away the remaining 2 to arrive at 8). This procedure is practiced so often from first grade on that it becomes automatized for most children. By contrast, in the number domain 20-100 Flemish pupils are typically confronted with a wider variety of solution methods. The above-mentioned version of the direct subtraction strategy remains the pivotal strategy for doing mental calculation in this number domain (e.g. solving $73 - 25 = .$ by first subtracting 20 and then 5), but once this strategy is well mastered some attention is also given

to so-called “clever” calculation strategies (including the subtraction by addition strategy) and from third grade on children also receive systematic instruction in the written subtraction algorithm. This difference in the instructional culture and practice between subtracting in the number domain up to 20 and the domain 20-100 may provide a first explanation for the contrasting results in the two number domains. A second explanation may be found in differences in strategy switch cost in these two number domains. It might be that in the number domain up to 20 children preferred to use only direct subtraction because of the relatively high cognitive costs involved in switching to subtraction by addition (Lemaire & Lecacheur, 2010; Luwel, Onghena, Torbeyns, Schillemans, & Verschaffel, 2009). It could be argued that the relative cost of switching becomes smaller when the size of the numbers in the subtraction problem increases. In other words, the advantage of switching to subtraction by addition for problems such as $93 - 88 =$. may be much more striking than for problems such as $13 - 8 =$. Further research is needed to explore the possible role of these two factors in the observed differences in strategy choices between the number domains up to 20 and 20-100.

Our results are also not in line with previous studies in which Flemish second to fourth-graders hardly reported the use of the subtraction by addition strategy to solve two-digit subtraction problems, not even when asked to generate several strategies for a problem (Torbeyns, De Smedt, Ghesquière, et al., 2009a) or after explicit instruction about this strategy (De Smedt et al., 2010). In contrast, our results suggest that fourth- to sixth-graders do apply the subtraction by addition strategy to solve two-digit subtractions and, moreover, that they apply this strategy flexibly, as they select it particularly on those problems for which it is, according to a rational task analysis, most suited. Apart from the differences in participant characteristics between the studies, the contrasting findings may be due to the differences in data-collection techniques (i.e., verbal reports vs. non-verbal methods). Because of the questionable validity of verbal protocol data to detect strategy use on problems such as

the ones presented in the current study (especially on problems with relatively small subtrahends or differences), we are more inclined to give merit to the non-verbal data of the present study. However, further research involving a direct comparison between verbal and non-verbal methods is needed to further unravel these contradicting results.

The current data are in line with research on children's strategy choices in other mathematical domains, such as computational estimation (Lemaire and Lecacheur, 2002) and word problem solving (e.g., Verschaffel & De Corte, 1990), that showed the influence of various kinds of number characteristics on children's strategy selection. At a more general level, our findings also fit with theoretical strategy selection models such as Siegler's SCADS* model (Siegler & Araya, 2005; see also Verschaffel et al., 2009), in which it is stated that people – including children – make flexible strategy choices when confronted with cognitive tasks, based on their knowledge about the efficiency of a particular strategy for a particular problem type. Our results reveal that children, like adults, rely on the nature of the subtrahend when flexibly choosing between direct subtraction and subtraction by addition to solve two-digit subtractions. Apparently, in the initial stage of the solution process, the given numbers in the subtraction problem are involved in a comparison process that leads to the selection of either a direct subtraction or a subtraction by addition strategy. Clearly, this comparison does not involve precise and deliberate calculations with these given numbers, but relies on fast and (quasi-)automatic estimation processes. Although several studies have highlighted the importance of magnitude comparison skills for successful mathematical development (De Smedt, Verschaffel, & Ghesquière, 2009; Gilmore, McCarthy, & Spelke, 2007; Holloway & Ansari, 2009), future research should more systematically investigate the nature and role of these comparison processes in multi-digit subtraction. Another issue for further research is whether children's strategy choice process only takes into account the nature of the subtrahend, or whether this process is rather co-determined by children's

personal knowledge about how good they actually are at performing the various strategies, and/or by children's estimations of the value of the different strategies in the particular context wherein they are supposed to demonstrate their performance (Verschaffel et al., 2009).

We now turn to some limitations of the present study and make suggestions for further research. A first limitation relates to the way in which large-distance problems have been operationalized in the present study. We only used large-distance problems that involved a single-digit number either as subtrahend (as in $83 - 4 = .$) or as difference (as in $83 - 79 = .$). Based on a rational task analysis, we assumed that if children did not use the subtraction by addition strategy on these extreme problems, they certainly would not use it for problems with less extreme differences between subtrahend and difference, such as $84 - 38 = .$ However, this specific problem set limits the generalizability of our findings to subtraction in the number domain 20-100 *with* single-digit subtrahends or differences. It remains to be determined whether similar strategy choices are observed when confronting children with large-distance problems including only two-digit subtrahends and differences, such as $71 - 58 = .$, a question that should be explored in future studies.

A second limitation of our study is that we only applied two non-verbal methods to investigate children's strategic behaviour. Besides a direct comparison between the non-verbal research methods with verbal protocol methods (see above), it would be interesting to include other non-verbal methods as well. More specifically, we should look for methods that focus on strategy identification at an item level, since this was not possible with the methods we used in the current study. Eye-movement data, for example, might help to provide a better understanding about children's strategy use when doing subtraction, and they may also be very helpful in determining the validity of the various verbal and non-verbal methods. They even may provide a unique view on children's strategy choice processes that precede their

actual strategy execution. An alternative might be presenting participants with worked examples that represent different subtraction strategies and asking them to compare and discuss the different solution methods on different problem types. This might be another interesting way to get more insight into the reason why a particular strategy is used by children and provide extra information about children's knowledge about the different strategies and when to use them (e.g., Star & Rittle-Johnson, 2009).

Finally, in this study we did not include children who performed badly on the pre-test with problems presented in the addition format, because we wanted to be sure that our participants would be able to solve subtraction problems presented as $S + . = M$. This inclusion criterion may have led to an overestimation of the flexibility of children's strategy choices for two-digit subtraction. The question thus remains whether low-performing children and children with mathematical difficulties will show the same strategy pattern as the children involved in the present study. Interestingly, a recent study by Peltenburg et al. (2012) showed that Dutch special education pupils reported using the subtraction by addition strategy in more than 50% of the problems with a relatively large subtrahend which required borrowing (e.g., $61 - 59 = .$). However, this remarkably high percentage of the subtraction by addition strategy may have been due to the fact that two thirds of the presented problems were word problems, half of them even reflecting a typical adding-on situation (e.g., "The album has space for 51 cards. 49 are already included. How many more cards can be added?"). It has been repeatedly shown that such word problems elicit a lot of subtraction by addition strategies among elementary school children (Carpenter & Moser, 1982; De Corte & Verschaffel, 1987; Riley, Greeno, & Heller, 1983). Further research is thus needed to shed light on the flexible use of the subtraction by addition strategy on symbolically presented subtraction problems by children with mathematical difficulties.

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Footnotes

¹ People can also use a third strategy, the so-called *indirect subtraction* strategy in which they determine how much needs to be subtracted from the minuend to get to the subtrahend (e.g., $75 - 43 = .$ by $75 - 30 = 45$ and $45 - 2 = 43$; so the answer is $30 + 2 = 32$) (De Corte & Verschaffel, 1987). This indirect subtraction strategy may be particularly efficient on problems with relatively large subtrahends (e.g., $81 - 79$). However, previous studies on people's strategy use in subtraction revealed that participants use this strategy only very rarely or not even at all (Beishuizen, Van Putten, & Van Mulken, 1997; De Smedt et al., 2010; Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009b; Van Lieshout, 1997).

² Several authors have reported the use of these addition-based strategies, but different terms are used to denote them, such as the forward strategy (Brissiaud, 1994), solving subtractions by means of addition (Beishuizen, 1997), the short jump strategy (Blöte et al., 2000), the adding up strategy (Selter, 2001), or indirect addition (Torbeyns, Ghesquière, et al., 2009). In this article, we will call this strategy *subtraction by addition*, whereas *direct subtraction* will be used to refer to the common subtraction strategy where the subtrahend is directly taken away from the minuend (e.g., solving $71 - 57 = .$ by $71 - 50 = 21$, and $21 - 7 = 14$).

³ As in many other European countries (e.g., The Netherlands, Germany), the horizontal format is the standard format for doing subtraction in Flanders (Belgium). Flemish children learn to subtract in this way in the first two grades of elementary school, before they are introduced to the vertical columnar algorithm, which they only learn after mastering subtractions in the horizontal format in the number domain up to 100. Even after the introduction of the columnar algorithm in third grade, children are still often presented with problems presented in the horizontal format. This differs from North-America, where the vertical format is the dominant one.

Appendix A*Overview of the 32 two-digit subtraction items, presented in subtraction format*

<i>S</i> < <i>D</i> problems		<i>S</i> > <i>D</i> problems	
Large-distance	Small-distance	Small-distance	Large-distance
31 – 3 = .	32 – 15 = .	32 – 17 = .	31 – 28 = .
34 – 8 = .	43 – 18 = .	43 – 25 = .	34 – 26 = .
42 – 5 = .	51 – 25 = .	51 – 26 = .	42 – 37 = .
52 – 4 = .	53 – 24 = .	53 – 29 = .	52 – 48 = .
71 – 2 = .	75 – 36 = .	75 – 39 = .	71 – 69 = .
77 – 9 = .	81 – 37 = .	81 – 44 = .	77 – 68 = .
83 – 4 = .	84 – 38 = .	84 – 46 = .	83 – 79 = .
93 – 5 = .	92 – 44 = .	92 – 48 = .	93 – 88 = .

Note. *S* = Subtrahend, *D* = Difference

Appendix B*Mean percentage correct (and standard deviations) per format, grade and problem type*

	<i>S</i> < <i>D</i>		<i>S</i> > <i>D</i>	
	Large-distance	Small-distance	Small-distance	Large-Distance
	(e.g., $31 - 3 = .$)	(e.g., $32 - 15 = .$)	(e.g., $32 - 17 = .$)	(e.g., $31 - 28 = .$)
<hr/> Subtraction Format				
Fourth grade	92.90 (25.76)	86.19 (34.60)	84.07 (36.70)	91.85 (27.44)
Fifth grade	94.71 (22.43)	84.06 (36.70)	82.21 (38.33)	88.35 (32.16)
Sixth grade	98.36 (12.73)	90.76 (29.04)	89.67 (30.51)	93.99 (23.83)
<hr/> Addition Format				
Fourth grade	96.15 (19.28)	82.61 (38.01)	84.70 (36.10)	94.51 (22.85)
Fifth grade	94.23 (23.37)	87.92 (32.67)	87.44 (33.22)	95.19 (21.45)
Sixth grade	96.15 (19.28)	86.41 (34.36)	88.04 (32.54)	97.27 (16.35)

Table I.

Model specifications of the linear regression models for problems presented in subtraction format

Model	Model Specifications			R ²	Effect Estimates	
	DF	<i>F</i> -value	<i>p</i> -value		Intercept	Parameter
Fourth grade						
<i>DS</i> -Model	1, 30	1.09	.3042	0.0351		
<i>SBA</i> -Model	1, 30	0.07	.7989	0.0022		
<i>Switch</i> -Model	1, 30	77.43	<.0001	0.7208	5399.20	149.69
Fifth grade						
<i>DS</i> -Model	1, 30	2.63	.1151	0.0807		
<i>SBA</i> -Model	1, 30	0.15	.6987	0.0051		
<i>Switch</i> -Model	1, 30	70.09	<.0001	0.7003	5348.68	123.41
Sixth grade						
<i>DS</i> -Model	1, 30	0.72	.4019	0.0235		
<i>SBA</i> -Model	1, 30	0.15	.6997	0.0050		
<i>Switch</i> -Model	1, 30	153.38	<.0001	0.8364	3676.02	105.00

Table II.

Frequency table of the best fitting model per pupil (per grade) for the subtraction format

Best fitting model	<i>DS</i> -Model	<i>SBA</i> -Model	<i>Switch</i> -Model	None
Fourth grade	1	0	19	2
Fifth grade	2	0	18	4
Sixth grade	1	0	20	2

Table III.

Mean reaction times (and standard deviations) for correct responses only in ms per format, grade and problem type

		$S < D$		$S > D$	
		Large-distance (e.g., $31 - 3 = .$)	Small-distance (e.g., $32 - 15 = .$)	Small-distance (e.g., $32 - 17 = .$)	Large-Distance (e.g., $31 - 28 = .$)
<hr/>					
Subtraction Format					
Fourth grade	5390 (4710)	9257 (4309)	10944 (6375)	6299 (4443)	
Fifth grade	4937 (2196)	9249 (6188)	9110 (3882)	6567 (5054)	
Sixth grade	3942 (1779)	6761 (3449)	7039 (3187)	4182 (2553)	
Addition Format					
Fourth grade	6250 (4109)	10448 (6781)	10664 (6016)	4878 (4664)	
Fifth grade	6236 (3515)	8553 (4923)	8494 (4119)	4824 (2590)	
Sixth grade	4664 (2823)	6866 (4723)	6680 (3073)	3245 (1498)

Figure 1. Graph showing the three-way interaction between format, magnitude and numerical distance. * $p < .05$. ** $p < .01$