A rational Krylov method based on Hermite interpolation for the nonlinear eigenvalue problem

R. Van Beeumen
joint work with K. Meerbergen, W. Michiels

Abstract

We present a new rational Krylov method for solving the nonlinear eigenvalue problem (NLEP)

\[ A(\lambda)x = 0. \]

The method approximates \( A(\lambda) \) by Hermite interpolation where the degree of the interpolating polynomial and the interpolation points are not fixed in advance. It uses a companion-type reformulation to obtain a linear generalized eigenvalue problem (GEP). This GEP is solved by a rational Krylov method that preserves the structure. As a result, the companion form grows in each iteration and the interpolation points can be dynamically chosen. Each iteration requires a linear system solve with \( A(\sigma) \) where \( \sigma \) is the last interpolation point. We illustrate by numerical examples that the method is fully dynamic and can be used as a global search method as well as a local refinement method. We also compare the method to Newton’s method.

Key words. Numerical linear algebra, Rational Krylov, Newton polynomials, Hermite interpolation, nonlinear eigenvalue problem