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Author contact	Jan.Swevers@kuleuven.be + 32 (0)16 32 25 40
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A Smoothed GMS Friction Model suited for Gradient-Based Friction State Estimation

Max Boegli, Tinne De Laet, Joris De Schutter, and Jan Swevers

Abstract—The multi-state Generalized Maxwell-Slip (GMS) friction model is known to describe all essential friction characteristics in presliding and sliding motion. It is also known that due to its switching state conditions between presliding and sliding, gradient-based parameter and state estimation methods cannot be implemented efficiently. Efficient on-line state and parameter estimation is essential for model-based friction compensation in order to track changes of friction characteristics in time and space. This paper presents a Smoothed GMS (S-GMS) friction model with an analytic set of differential equations well suited for gradient-based estimation techniques. Friction state estimation is implemented with an Extended Kalman Filter (EKF) to validate the S-GMS model numerically and experimentally, and compare its behavior with the GMS model.

I. INTRODUCTION

Both model-free and model-based friction compensation techniques exist. Model-free techniques include iterative learning control and disturbance observers. Model-based friction compensation use a friction model to predict friction forces and hence compensate for them. Accurate friction compensation is obtained provided that the friction model structure includes all major friction characteristics and accurate estimates of the states and the model parameters are available, e.g. through experimental identification.

Off-line friction identification for the LuGre [1] and Maxwell-slip models using dynamic non-linear regression are discussed in [2], [3]. Auto-tuning of feedforward friction compensation based on gradient methods has been used with a simplified LuGre model without Stribeck effect [4]. Friction estimation based on the Elasto-Plastic model has been investigated [5], [6]. The Generalized Maxwell-slip (GMS) model [7] is known to be able to describe all essential friction characteristics, which is not the case for the above mentioned models. Identification and compensation of friction based on the GMS model are discussed in [8], [9], [10]. Most existing identification methods are off-line methods that typically use several sets of tailored input signals, where each set excites certain elements of the friction behavior and hence generates data to estimate a subset of the friction model parameters.

A need exists to track on-line changes in friction dynamics during normal machine operation. Gradient based on-line

state and parameter estimation techniques [11], [12] are efficient provided that the second derivative of the model to respectively the states and parameters is continuous. This is not the case for the GMS model due to the switching state conditions between presliding and sliding, and therefore it is troublesome to implement the GMS model inside an estimation framework. To overcome this limitation, a smoothed (analytic) version of the multi-state GMS (S-GMS) friction model is developed.

The paper is organized as follows. First, in section II, the GMS friction model is briefly recalled before the S-GMS model is formulated and its properties are compared to the GMS model. In section III, friction state estimation with both the GMS and S-GMS models is implemented through the Extended Kalman Filter (EKF). Simulation and experimental results are discussed in section IV with an evaluation of the estimation performance. Finally, in section V, conclusion and future work are presented.

II. FRICTION MODELING

This section briefly presents the GMS friction model before introducing the smoothed GMS version (S-GMS).

A. The GMS Friction Model

The GMS model [7] is a multi-state dynamic friction model relating the velocity $\dot{q}$ of the object to the friction force y_f depending on an internal state vector $z \in \mathbb{R}^{n_z}$. The GMS model consists of a friction force equation $y_f = h(z, \dot{q})$ and a differential state equation $\dot{z} = f(z, \dot{q})$.

Each state variable z_i represents the displacement of an element i . Each element i can switch between a sticking and a slipping mode:

- when the element i sticks:

$$\frac{dz_i}{dt} = \dot{q} \quad \text{until} \quad z_i = \text{sgn}(\dot{q})z_i^{s+}(\dot{q}), \quad (1)$$

- then the element i slips:

$$\frac{dz_i}{dt} = C_i \left(\text{sgn}(\dot{q}) - \frac{z_i}{z_i^{s+}(\dot{q})} \right), \quad (2)$$

until $\dot{q}$ goes through zero.

$z_i^{s+}(\dot{q}) = \frac{\nu_i}{\kappa_i} s_+(\dot{q})$ is a positive steady-state deflection corresponding to a normalized fraction ν_i (with $\sum \nu_i = 1$) of the positive velocity weakening (Stribeck) function $s_+(\dot{q}) = f_C + (f_S - f_C)e^{-(\dot{q}/v_s)^2}$, with f_S the static force, f_C the coulomb force, and v_s the Stribeck velocity. κ_i is the stiffness of element i . In sliding mode (2), $C_i = \frac{\nu_i}{\kappa_i} C$ is the attraction parameter, which is a gain that determines how fast $|z_i|$ converges to $z_i^{s+}(\dot{q})$.

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The resulting friction force is the summation of the n_z stiffness forces $\kappa_i z_i$ and micro-viscous forces $\sigma_i \dot{z}_i$ plus a viscous term:

$$y_f = \sum_{i=1}^{n_z} (\kappa_i z_i + \sigma_i \dot{z}_i) + \sigma_v \dot{q}. \quad (3)$$

B. The Smoothed GMS Model Formulation

The smoothed GMS model (S-GMS) aims to preserve the benefits of the GMS model while having an analytic formulation of the differential state equations. Hence, the S-GMS is a multi-state model that combines the state equations (1) and (2) into one state equation for each element i . The following analytic multi-state GMS model is proposed:

$$\frac{dz_i}{dt} = \dot{q} - \eta_i(z_i, \dot{q}) \dot{q}_+ \frac{z_i}{z_i^{s+}(\dot{q})}, \quad (4)$$

where $\dot{q}_+$ represents the absolute value of the velocity $\dot{q}_+ = |\dot{q}|$. The discontinuity at $\dot{q} = 0$ is allowed as the velocity is not part of the state variables¹.

In (4), $\eta_i(z_i, \dot{q}) \in (0, 1)$ is a smoothed transition function between presliding ($\eta_i \simeq 0$) and sliding ($\eta_i \simeq 1$) regime for each element i . η_i consists of two multiplicative terms:

$$\eta_i(z_i, \dot{q}) = \eta_{A_i}(z_i, \dot{q}) \eta_{B_i}(z_i, \dot{q}), \quad (5)$$

where the transition function η_{A_i} , shown in Fig. 1(a), vanishes when $|z_i| < \zeta z_i^{s+}$ (presliding) and reaches unity otherwise (sliding):

$$\eta_{A_i}(z_i, \dot{q}) = 1 - \frac{1}{2} \tanh \left[\lambda \left(\frac{z_i}{z_i^{s+}(\dot{q})} + \zeta \right) \right] \quad (6a)$$

$$+ \frac{1}{2} \tanh \left[\lambda \left(\frac{z_i}{z_i^{s+}(\dot{q})} - \zeta \right) \right]. \quad (6b)$$

A factor $\zeta \in [0.9, 1)$ is introduced to avoid ringing in sliding regime when $|z_i| \approx z_i^{s+}$. The factor λ sets the transition sharpness of (6), and is usually set to insure a normalized slope of $\sim 10^3$, as will be discussed in section II-D.

The reset function η_{B_i} , shown in Fig. 1(b), vanishes at velocity reversal (onset of presliding) when $\text{sgn}(\dot{q}) \neq \text{sgn}(z_i)$:

$$\eta_{B_i}(z_i, \dot{q}) = \frac{1}{2} + \frac{1}{2} \tanh \left(\gamma \frac{z_i}{z_i^{s+}(\dot{q})} \frac{\dot{q}}{v_s} \right), \quad (7)$$

where γ sets the transition sharpness.

C. Properties of the S-GMS Model

The S-GMS model is a multi-state friction model, similar to the GMS model. If the state equation (4) reduces to one single-state and $\eta = 1$ the S-GMS model boils down to the LuGre model [1]. The state equation (4) is however closely related to the single-state Elasto-Plastic (EP) friction model [5], [6]. Hence, the S-GMS model can also be viewed as a

¹If the velocity would be part of the state variables, a smoothed approximation of the absolute velocity, e.g. $\dot{q}_+ = \sqrt{\dot{q}^2 + \epsilon}$ with ϵ an arbitrarily small positive constant, could be used.

Fig. 1. Smooth sigmoidal transition functions η_{A_i} and η_{B_i} between sticking and slipping mode for element i . The element i sticks when $\eta_{A_i} \eta_{B_i} \simeq 0$ and slips when $\eta_{A_i} \eta_{B_i} \simeq 1$.

multi-state EP model. The EP model uses a piecewise continuous transition function (of class C^1) between the presliding and sliding regime. On the other hand, the S-GMS model introduces an analytic transition function, which is infinitely differentiable (also known as smooth or C^∞). Furthermore, the analytic multi-state S-GMS model (4) is completely free of any conditional branching (e.g. if-then-else statements).

The S-GMS model has the same set of parameters as the GMS model, except for the smoothing parameters λ , ζ , γ (see section II-D), and the attraction parameter C (discussed below). For the parameters common to both model, their values can simply be copied.

The multi-state representation of the GMS and S-GMS models allows the presliding hysteresis behavior to be shaped through the elements' stiffnesses distribution (κ_i, ν_i), while guarantying the nonlocal memory effect (Fig. 2(a)) and the nondrifting property (Fig. 2(b)). No noticeable differences between the S-GMS and GMS can be observed in Fig. 2(a).

The S-GMS does not include an explicit attraction parameter C . While the transition behavior (Fig. 2(d)) in response to a positive/negative oscillating velocity is still similar between the S-GMS and GMS models, the frictional lag (Fig. 2(c)) obtained with an oscillating positive velocity around the Stribeck velocity looks different for both models (especially when $f_S \gg f_C$). In practice however, there is no easy way to identify the parameter C in the GMS model and it is more considered as a design parameter. Its absence in the S-GMS does not affect the ability of the model to accurately predict frictional behavior in both presliding and sliding regime.

The main added value of the S-GMS over the GMS and a multi-EP model is that the S-GMS provides a completely smooth and differentiable set of ODEs, without conditional switching with regard to the state variables and model parameters. This feature makes the S-GMS well suited for on-line gradient-based estimation techniques such as the EKF or the Moving Horizon Estimation (MHE) for both state and parameter estimation in an optimization framework [12].

Fig. 2. The friction behavior of the GMS and S-GMS friction models. (a) Hysteresis with nonlocal memory during presliding. (b) The nondrifting property, which is the response to an oscillatory force input, after sliding, whose amplitude remains under the breakaway limit. (c) Frictional lag with an oscillating positive velocity. (d) Periodic-motion friction with sliding. For all four behaviors, the friction parameters are $n_z = 8$ elements, $\kappa_i = \frac{1}{n_z} 10^7 \text{N}$, $\nu_i = \frac{2i}{n_z(n_z+1)}$, $f_s = 3\text{N}$, $f_c = 2\text{N}$, $v_s = 1\mu\text{m/s}$, $C = 20\text{N/s}$ (GMS only). The smoothing parameters of the S-GMS model are $\lambda = 100$, $\gamma = 10$, $\zeta = 0.97$.

D. Influence of the Smoothing Parameters

Three smoothing parameters (λ, ζ, γ) are needed to set the smoothed transition function η_i (5). λ and γ set the transition slope of η_{A_i} and η_{B_i} respectively. They should be large enough to ensure a sufficiently sharp transition when $|z_i|$ reaches ζz_i^{s+} (set by λ in (6)) and at velocity reversal (set by γ in (7)). A normalized transition slope of $\pm 10^3$ for η_{A_i} and η_{B_i} , i.e.

$$z_i^{s+} \frac{d\eta_{A_i}}{dz_i} \Big|_{|z_i|=\zeta z_i^{s+}} \approx z_i^{s+} v_s \frac{d\eta_{B_i}}{dz_i \dot{q}} \Big|_{\dot{q}=0} \approx \pm 10^3, \quad (8)$$

gives a good tradeoff. A too small value gives a sloppy transition for η_i , which results in a corrupted value of $\dot{z}_i$ in the sliding regime of element i and an overestimated value of z_i (Fig. 3(a) and 3(b)). The factor $\zeta \in [0.9, 1)$ is introduced to avoid ringing in sliding regime when $|z_i| \simeq z_i^{s+}$. A good tuning rule is to set ζ , once λ has been set, so that η_{A_i} approaches one with a desired precision when $z_i = \pm z_i^{s+}$, e.g. $\eta_{A_i}(z_i^{s+}) = 1 - 10^{-3}$, as shown in Fig. 1(a). The influence of these three smoothing parameters on the shape of the presliding hysteresis behavior is summarized in Fig. 3.

III. THE ESTIMATION FRAMEWORK

This section first presents the general friction estimation scheme for a mechatronic system. Then, two friction state estimators, based on the GMS and S-GMS friction models, are implemented through an Extended Kalman Filter (EKF) to demonstrate the pertinence of the S-GMS model. The analytic representation of the S-GMS model offers a natural,

Fig. 3. Effect of the smoothing parameters λ (a), γ (b), and ζ (c) in the S-GMS compared to the GMS model. Plots (a), (b), and (c) are a magnification of the closing of a minor loop indicated with a rectangle in (d). (a) and (b) show that a decreasing value for λ and γ overestimate the friction force, and create a local memory effect, which means that a minor loop does not close at a major loop. On the other hand, a decreasing value for ζ in (c) underestimates the friction force. Initial smoothing parameters for all plots are $\lambda = 1000$, $\gamma = 10$, $\zeta = 0.99$. All others friction parameters are the same as in Fig. 2.

robust and straightforward way to implement it in a gradient-based estimation framework.

A. The Friction Observer

A model of a moving mass m with friction force y_f is represented in Fig. 4. The mass m is driven through a linear time-invariant feedback controller K . Based on the available control input u , which represents the actuator force, and the position q , one can estimate the friction states z by minimizing an error criterion [block $\mathcal{Z}$] between the estimated friction $\hat{y}_f = h(\hat{z}, \hat{q})$ [block $\mathcal{F}$] and the available measured friction based on the control input $\tilde{y}_f = u - m\ddot{q}$ [block $\mathcal{Y}$] assuming a known mass m and good estimates of velocity $\hat{q}$ and acceleration $\hat{\ddot{q}}$ [block $\mathcal{V}$].

The estimation problem consider the following generic friction model, discretized at time instant t_k :

$$z_k = f(z_{k-1}, \dot{q}_{k-1}) + w_{k-1}, \quad w_k \sim \mathcal{N}(0, Q_k), \quad (9)$$

$$y_k = h(z_k, \dot{q}_k) + v_k, \quad v_k \sim \mathcal{N}(0, R_k), \quad (10)$$

where w_k is the Gaussian system model uncertainty and v_k is the Gaussian measurement model uncertainty with covariance Q_k and R_k respectively. For the GMS and S-GMS models, the function $f(\cdot)$ implements the discrete-time version of the state equations (1-2) and (4), respectively. The function $h(\cdot)$ implements the friction force equation (3).

B. Extended Kalman Filter (EKF) Formulation

Formulation of the EKF [13] is given below for the prediction step (or time update) and the correction step (or measurement update).

Fig. 4. A moving mass m , subject to friction y_f , is driven by a position feedback controller K . The friction observer comprises four parts: block $\mathcal{V}$, a velocity and acceleration observer; block $\mathcal{Y}$, a measured friction estimation $\tilde{y}_f$ based on the control input u minus an inertial term $m\ddot{q}$; block $\mathcal{F}$, the estimated friction $\hat{y}_f$, a function of the estimated states $\hat{z}$ and the velocity input $\dot{q}$; and block $\mathcal{Z}$, a state observer to get the estimated state vector $\hat{z}$ from the velocity input $\dot{q}$ and a measurement update correction between the measured friction $\tilde{y}_f$ and the estimated friction $\hat{y}_f$.

Prediction step:

$$\hat{z}_k^- = f(\hat{z}_{k-1}^+, \tilde{q}_{k-1}), \quad (11)$$

$$P_k^- = F_{k-1} P_{k-1}^+ F_{k-1}^T + Q_{k-1}, \quad \text{and} \quad (12)$$

correction step:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1}, \quad (13)$$

$$\hat{z}_k^+ = \hat{z}_k^- + K_k (\tilde{y}_k - h(\hat{z}_k^-, \tilde{q}_k)), \quad \text{and} \quad (14)$$

$$P_k^+ = (I - K_k H_k) P_k^-, \quad (15)$$

where the Jacobian matrices F_k and H_k are defined as

$$F_{k-1} = \left. \frac{\partial f}{\partial z} \right|_{\hat{z}_{k-1}^+, \tilde{q}_{k-1}}, \quad \text{and} \quad (16)$$

$$H_k = \left. \frac{\partial h}{\partial z} \right|_{\hat{z}_k^-, \tilde{q}_k}. \quad (17)$$

At time instant t_k , the variance of the estimated state $\hat{z}_{i,k}$ is given by the corresponding diagonal element of P_k^+ , i.e. $\sigma_{\hat{z}_{i,k}}^2 = P_{ii,k}^+$, and the variance of the estimated friction $\hat{y}_f$ is given by

$$\sigma_{\hat{y}_f,k}^2 = \sum_{i=1}^{n_z} \sum_{j=1}^{n_z} \kappa_i \kappa_j P_{ij,k}^+. \quad (18)$$

C. Numerical Implementation

The analytical expression of the S-GMS model (4) is much easier to implement in a gradient-based estimation framework than expressions (1)-(2) of the GMS model. Integration of state equation (11) and evaluation of Jacobian F (16) are straightforward for the S-GMS, while it requires quite a few conditional branching for the GMS model. For the GMS model, in addition to the state vector, a binary

TABLE I

GMS & S-GMS FRICTION PARAMETERS USED IN STATE ESTIMATION

Model	Parameter	Value	Unit
GMS & S-GMS	n_z	4	-
	κ	$[12, 2.5, 0.71, 0.79] \cdot 10^6$	N/m
	ν	$[0.067, 0.056, 0.047, 0.83]$	-
	σ	$[0, 0, 0, 0]$	Ns/m
	σ_v	20	Ns/m
	f_S	4.5	N
	f_C	4	N
GMS	v_s	10^{-3}	m/s
	C	50	N/s
	λ	10^3	-
	ζ	0.97	-
	γ	10^2	-

state condition vector is needed to indicate the mode of the element, i.e. presliding (1) or sliding (2), in order to determine the state equation to be solved at each time instant. To this end, $2n_z + 1$ conditions have to be checked at each time instant: the value of each state z_i (below or above its breakaway limit), the condition of each element i , and the sign of the velocity. The switching structure (due to the conditional statements) of the GMS model makes it hard to use more advanced integration schemes than basic explicit ODE integration using the Euler scheme.

Thanks to its analytic equation, the S-GMS model can however be straightforwardly integrated using more advanced explicit and implicit integration schemes. Implicit integration is needed for stiff equations, which can arise when the stiffness κ_i of some elements is high in presliding².

IV. MODEL VALIDATION WITH STATE ESTIMATION

This section presents the results of state estimation based on both the EKF GMS and EKF S-GMS estimators. The state estimation framework (Fig. 4) is applied on 1) a simulated mechanical system with a GMS friction model and 2) a set of experimental input-output data from an XY positioning table (Fig. 7).

A. Simulation Results

Friction state estimation based on the EKF GMS and the EKF S-GMS is evaluated with a simulated mechanical system model as represented in Fig. 4. The reference trajectory is a sine motion of $200\mu\text{m}$ at 5 Hz as shown in Fig. 5. This trajectory is relevant to study presliding at velocity reversal and sliding regime slightly above the Stribeck velocity $v_s = 10^{-3}$ m/s. The simulated system friction is modeled with the friction parameters listed in Table I using the GMS model.

The EKF state estimators (with GMS and S-GMS models) are evaluated with three different sets of friction parameters: a set of parameters with identical values to the simulated system parameters, a subset of parameters (κ , f_S , f_C) with values 10% below (underestimated), and 10% above (overestimated) the values of the simulated system parameters.

²In friction model equations, it happens that numerical stiffness is closely related to mechanical stiffness.

Fig. 5. Simulated sine trajectory with position, velocity, and friction force. The window in the friction force at time $t = 0.15$ s represents the onset of presliding (at velocity reversal) and the window at time $t = 0.18$ s represents the onset of full sliding. Both windows are magnified in Fig. 6 to show details of friction estimation.

Fig. 6. Simulated system friction (in solid blue line) with the GMS model, estimated friction with the GMS EKF (in dashed green line) and estimated friction with the S-GMS EKF (in dash-dot red line). Estimated frictions are shown with their standard deviation $\sigma_{\hat{y}_f}$ ranges. Measurement update is done every 1ms (circles). Graphs (a) and (b) show the estimated friction with 10% underestimated friction parameters from the system parameters, Graphs (c) and (d) show the estimated friction with 10% overestimated friction parameters from the system parameters, Graphs (a) and (c) magnify the onset of presliding at $t = 0.15$ s while graphs (b) and (d) magnify the onset of full sliding at $t = 0.18$ s.

Prediction steps are evaluated at a system update rate of 0.1ms and measurement updates are performed with four different measurement update rates: three with a rate of 0.1, 1, 10 ms and one without measurement update. Two different estimated velocity inputs are considered: an ideal one with the simulated system velocity ($\tilde{q} = \dot{q}$) and a more realistic one with a second order low-pass filtered velocity at a cutoff frequency of 500 Hz. The phase-lag of the low-pass filtered velocity is mainly seen at velocity reversal when facing high stiffness of the presliding.

For each of the estimation schemes, the Normalized Root-Mean-Square Error (NRMSE) between the system friction and the estimated friction is evaluated for both the GMS and the S-GMS friction estimators over one period of the sine

TABLE II
NORMALIZED ROOT-MEAN-SQUARE ERROR (NRMSE)

NRMSE [%] between the system friction and the estimated friction is evaluated for both the GMS and the S-GMS friction. Comparison is done for three sets of parameters: identical, underestimated, and overestimated compared to the system parameters. For each set of parameters, NRMSE is shown for four different EKF measurement update rates: 0.1, 1, 10 ms and no measurement update. Also, two different velocity inputs are considered: the simulated velocity and the estimated velocity resulting from low-pass filtering the position.

Estimator parameters	Measurement update rate	NRMSE [%]			
		Simulated Velocity GMS	Simulated Velocity S-GMS	Estimated Velocity GMS	Estimated Velocity S-GMS
Identical to system parameters	n/a	0	0.88	1.65	1.15
	10 ms	0.22	0.53	0.92	0.94
	1 ms	0.31	0.38	0.43	0.46
	0.1 ms	0.32	0.33	0.34	0.35
Under-estimated parameters	n/a	6.82	7.16	7.02	7.21
	10 ms	5.38	4.34	5.53	4.60
	1 ms	1.91	1.12	1.94	1.16
	0.1 ms	0.68	0.40	0.69	0.42
Over-estimated parameters	n/a	6.81	6.56	7.00	6.58
	10 ms	5.17	5.19	5.14	4.98
	1 ms	1.84	3.56	1.86	3.60
	0.1 ms	2.47	2.47	2.48	2.48

trajectory. Results are summarized in Table II. In the case of estimators parameters identical to the system parameters, the NRMSE is overall slightly higher for the S-GMS than for the GMS based estimator, since the system friction is based on a GMS model. In the case of overestimated parameters the NRMSE from the S-GMS is still slightly higher than for the GMS-based estimator while in the case of underestimated parameters the NRMSE from the S-GMS is slightly lower than than for the GMS-based estimator. The model structure difference between the GMS and S-GMS explain the different behavior towards a model parameters mismatch. However, the overall NRMSE values are quite close between the GMS and S-GMS estimators. Fig. 6 shows some screenshots from the state estimation at the onset of presliding (a-c) and full sliding (b-d) for the case of underestimated (a-b) and overestimated (c-d) parameters.

A general conclusion from NRMSE values presented in Table II and screenshots of Fig. 6 is that the S-GMS-based friction estimator has similar performance to the GMS-based friction estimator, while offering a better implementation (section III-C).

B. Experimental Validation

Friction state estimation is validated with experimental input-output data from an XY positioning table (Fig. 7) used in optical wafer inspection. The reference trajectory is a sine motion of $200\mu\text{m}$ at 1Hz. The estimated velocity $\tilde{q}$ is obtained by taking the finite-difference derivative of the measured position, sampled at 0.1ms, combined with a second order low-pass filter with a cutoff frequency at 500 Hz. Friction parameters listed in Table I have been estimated using techniques from [8], [14], and used in the EKF state estimators. Fig. 8 shows the experimental friction

Fig. 7. Direct-drive XY positioning table for wafer inspection

Fig. 8. Experimental friction estimation. Control input minus inertial term and cogging (in solid blue line), estimated friction with the GMS EKF (in dashed green line) and estimated friction with the S-GMS EKF (in dash-dot red line). Estimated frictions are shown with their standard deviation $\sigma_{\hat{y}_f}$ ranges. Measurement update is done every 5ms. The lag observed at velocity reversal ($t = 2$ s) for the estimated frictions is due to the phase delay of the estimated velocity. Behavior of both EKF (GMS & S-GMS) are similar.

estimation with both the GMS and S-GMS friction model state estimators. Measurement update (14) is done every 5ms. The lag observed at velocity reversal ($t = 2$ s) for the estimated frictions is due to the phase delay of the estimated velocity. As in simulation, the behavior of the S-GMS-based estimator is similar to the GMS-based estimator.

V. CONCLUSION AND FUTURE WORK

A. Conclusion

This paper introduced a smoothed formulation of the GMS model (S-GMS) with a set of analytic ODE's without conditional branching with regard to the state variables. This smoothed model representation makes it well suited for on-line gradient-based state estimation methods. It offers the possibility to use implicit ODE integration schemes and reliable Jacobian evaluation.

Simulated and experimental results of friction state estimation based on an Extended Kalman Filter for both the GMS and S-GMS friction models show the pertinence and relevance of the S-GMS model in terms of friction behavior representation and numerical implementation.

B. Future Work

A smoothed model formulation makes it not only well suited for state estimation but also for parameter estimation within a Moving Horizon Estimation (MHE) framework. Moreover, MHE formulation is a natural way to handle constraints on states and parameters. An on-line MHE algorithm for state and parameter estimation based on the S-GMS friction model will be implemented with ACADO [12], a toolkit for automatic code generation and dynamic optimization, and tested on the ETEL XY positioning table (Fig. 7) in a real-time application to compensate for friction.

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