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A Smoothed GMS Friction Model for Moving Horizon Friction State and Parameter Estimation

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Abstract—This paper presents a smoothed friction model that closely approximates the Generalized Maxwell-Slip (GMS) model, a multi-state friction model known to describe all essential friction characteristics in presliding and sliding motion. In contrast to the GMS model, which consists of a switching structure to accommodate for its hybrid nature, the Smoothed GMS (S-GMS) model consists of an analytic set of differential equations well suited for on-line state and parameter estimation, such as in Moving Horizon Estimation (MHE). Efficient on-line state and parameter estimation is essential for model-based friction compensation in order to track friction characteristics changes in time and space. Moreover, MHE is known to better handle model nonlinearities, disturbances and constraints than Extended Kalman Filter (EKF). This paper discusses the implementation of the EKF and MHE estimators for both the GMS and the S-GMS friction models. The benefit of the combination of MHE and S-GMS model is shown.

I. INTRODUCTION

Model-based friction compensation uses a friction model to predict friction force and compensate for it. Accurate friction compensation is obtained provided that the friction model structure includes all major friction characteristics, and accurate estimates of the states and the model parameters are available through state and parameter estimation. Moreover, the need exists to track on-line changes in friction dynamics during normal machine operation.

Most existing identification methods are off-line methods that typically use several sets of tailored input signals, where each set excites certain elements of the friction behavior and hence generates data to estimate a subset of the friction model parameters. Off-line friction identification for the LuGre and Maxwell-slip models using dynamic non-linear regression is discussed in [1], [2]. Auto-tuning of feedforward friction compensation based on gradient methods has been used with a simplified LuGre model without Stribeck effect [3]. Friction estimation based on the Elasto-Plastic model [4] has been investigated [5]. The Generalized Maxwell-Slip (GMS) model [6] is known to be able to describe all essential friction characteristics, which is not the case for the above mentioned models. Identification and compensation of friction based on the GMS model are implemented in [7], [8]. Adaptive friction compensation based on a linearly-parametrized Stribeck function in the GMS model is proposed [9], yet with an extra robustifying term to account for unmodeled dynamics.

Gradient-based optimization techniques for on-line state and parameter estimation, such as the Moving Horizon Estimation (MHE) algorithm [10], [11], are efficient provided that the derivatives of the model with respect to the states and parameters are continuous. This is not the case for the GMS model due to the switching state conditions between presliding and

sliding. To overcome this limitation, a smoothed (analytic) version of the multi-state GMS (S-GMS) friction model is developed in this paper. The benefit of the MHE algorithm in combination with the S-GMS instead of the GMS friction model is shown. Moreover, it is also shown that the MHE algorithm outperforms the Extended Kalman Filter (EKF) state estimation.

The paper is organized as follows. First, in section II, the GMS friction model is briefly reviewed before the S-GMS model is formulated, and its properties are compared to the GMS model. In section III, friction force estimation with both the GMS and S-GMS models is implemented via an Extended Kalman Filter (EKF) for state estimation and a Moving Horizon Estimator (MHE) for state and parameter estimation. Simulation results are discussed in section IV, with an evaluation of the estimation performance. Finally, in section V, conclusions and future work are presented.

II. FRICTION MODELING

This section briefly presents the GMS friction model before introducing the smoothed GMS version (S-GMS).

A. The GMS Friction Model

The GMS model [6] is a multi-state dynamic friction model relating the velocity $\dot{q}$ of the object to the friction force y_f depending on an internal state vector $z \in \mathbb{R}^{n_z}$. The GMS model consists of a friction force equation $y_f = h(z, \dot{q})$ and a differential state equation $\dot{z} = f(z, \dot{q})$.

Each state variable z_i represents the displacement of an element i that can switch between a sticking and a slipping mode:

- where the element i sticks:

$$\frac{dz_i}{dt} = \dot{q} \quad \text{until} \quad z_i = \text{sgn}(\dot{q})z_i^{s+}(\dot{q}), \quad (1)$$

- then the element i slips:

$$\frac{dz_i}{dt} = C_i \left(\text{sgn}(\dot{q}) - \frac{z_i}{z_i^{s+}(\dot{q})} \right), \quad (2)$$

until $\dot{q}$ goes through zero.

$z_i^{s+}(\dot{q}) = \frac{\nu_i}{\kappa_i} s_+(\dot{q})$ is a positive steady-state deflection corresponding to a normalized fraction ν_i (with $\sum \nu_i = 1$) of the positive velocity weakening (Stribeck) function $s_+(\dot{q}) = f_C + (f_S - f_C)e^{-(\dot{q}/v_s)^2}$, with f_S the static force, f_C the Coulomb force, and v_s the Stribeck velocity. κ_i is the stiffness of element i . In sliding mode (2), $C_i = \frac{\nu_i}{\kappa_i} C$ is the attraction parameter, which is a gain that determines how fast $|z_i|$ converges to $z_i^{s+}(\dot{q})$.

Fig. 1. Smooth sigmoidal transition functions η_{A_i} and η_{B_i} between sticking and slipping mode for element i . The element i sticks when $\eta_{A_i} \eta_{B_i} \simeq 0$ and slips when $\eta_{A_i} \eta_{B_i} \simeq 1$.

The resulting friction force is the summation of the n_z stiffness forces $\kappa_i z_i$ and micro-viscous damping $\sigma_i \dot{z}_i$, plus a viscous term:

$$y_f = \sum_{i=1}^{n_z} (\kappa_i z_i + \sigma_i \dot{z}_i) + \sigma_v \dot{q}. \quad (3)$$

B. The Smoothed GMS Model Formulation

The smoothed GMS model (S-GMS) aims to preserve the benefits of the GMS model, while having an analytic formulation of the differential state equations. Hence, the S-GMS is a multi-state model that combines the state equations (1) and (2) into one state equation for each element i . The following analytic multi-state GMS model is proposed:

$$\frac{dz_i}{dt} = \dot{q} - \eta_i(z_i, \dot{q}) \dot{q}_+ \frac{z_i}{z_i^{s+}(\dot{q})} \quad (4)$$

where $\dot{q}_+$ represents the absolute value of the velocity $\dot{q}_+ = |\dot{q}|$. The discontinuity at $\dot{q} = 0$ is allowed, as the velocity is not a state variable of the friction model.¹

In (4), $\eta_i(z_i, \dot{q}) \in (0, 1)$ is a smoothed transition function between presliding ($\eta_i \simeq 0$) and sliding ($\eta_i \simeq 1$) regime for each element i . η_i consists of two multiplicative terms:

$$\eta_i(z_i, \dot{q}) = \eta_{A_i}(z_i, \dot{q}) \eta_{B_i}(z_i, \dot{q}) \quad (5)$$

where the transition function η_{A_i} , shown in Fig. 1(a), vanishes when $|z_i| < \zeta z_i^{s+}$ (presliding), and reaches unity otherwise (sliding):

$$\eta_{A_i}(z_i, \dot{q}) = 1 \mp \frac{1}{2} \tanh \left[\lambda \left(\frac{z_i}{z_i^{s+}(\dot{q})} \pm \zeta \right) \right] \quad (6)$$

A factor $\zeta \in [0.9, 1)$ is introduced to avoid ringing in sliding regime when $|z_i| \approx z_i^{s+}$. The factor λ sets the transition sharpness of (6), and is usually set to insure a normalized slope of $\sim 10^3$, as will be discussed in section II-D.

The reset function η_{B_i} , shown in Fig. 1(b), vanishes at velocity reversal (onset of presliding) when $\text{sgn}(\dot{q}) \neq \text{sgn}(z_i)$:

$$\eta_{B_i}(z_i, \dot{q}) = \frac{1}{2} + \frac{1}{2} \tanh \left(\gamma \frac{z_i}{z_i^{s+}(\dot{q})} \frac{\dot{q}}{v_s} \right), \quad (7)$$

where γ sets the transition sharpness.

¹Should the velocity be a state variable of the friction model, a smoothed approximation of the absolute velocity could be, e.g. $\dot{q}_+ = \sqrt{\dot{q}^2 + \epsilon}$ with ϵ an arbitrarily small positive constant.

C. Properties of the S-GMS model

The S-GMS model is a multi-state friction model, similar to the GMS model. The state equation (4) is however closely related to the single-state Elasto-Plastic (EP) friction model [4], [5]. Hence, the S-GMS model can also be viewed as a multi-state EP model. The EP model uses a piecewise continuous transition function (of class $\mathcal{C}^1$) between the presliding and sliding regime. On the other hand, the S-GMS model introduces an analytic transition function, which is infinitely differentiable (also known as smooth, or $\mathcal{C}^\infty$). Furthermore, the analytic multi-state S-GMS model (4) is completely free of any conditional branching (e.g. if-then-else statements).

The S-GMS model has the same set of parameters as the GMS model, except for the additional smoothing parameters λ , ζ , γ (see section II-D), and the attraction parameter C (discussed below), which is absent in the S-GMS model. The parameters common to both models have the same value in both models.

The multi-state representation of the GMS and S-GMS models allows the presliding hysteresis behavior to be shaped through the elements' stiffnesses distribution (κ_i, ν_i) , while guarantying the nonlocal memory effect (Fig. 2(a)) and the nondrifting property (Fig. 2(b)). No noticeable differences between the S-GMS and GMS models can be observed in Fig. 2(a) and 2(b).

The S-GMS model does not include an explicit attraction parameter C . While the transition behavior (Fig. 2(d)) in response to a positive/negative oscillating velocity is still similar between the S-GMS and GMS models, the frictional lag (Fig. 2(c)) obtained with an oscillating positive velocity around the Stribeck velocity looks different for both models (especially when $f_s \gg f_C$). In practice however, there is no easy way to identify the parameter C in the GMS model, and it is more considered a design parameter. Its absence in the S-GMS model does not affect the ability of the model to accurately predict frictional behavior in both presliding and sliding regime.

The main added value of the S-GMS over the GMS or over a multi-EP model is that the S-GMS model provides a completely smooth and differentiable set of ODEs, without conditional switching with regard to the state variables and model parameters. This feature makes the S-GMS model well suited for on-line gradient-based estimation techniques such as the EKF or, even more, the MHE for both state and parameter estimation in an optimization framework [12].

D. Influence of the smoothing parameters

Three smoothing parameters (λ, ζ, γ) are needed to set the smoothed transition function η_i (5). λ and γ set the transition slope of η_{A_i} and η_{B_i} respectively. They should be large enough to ensure a sufficiently sharp transition when $|z_i|$ reaches ζz_i^{s+} (set by λ in (6)) and at velocity reversal (set by γ in (7)). A normalized transition slope of $\pm 10^3$ for η_{A_i} and η_{B_i} , i.e.

$$z_i^{s+} \frac{d\eta_{A_i}}{dz_i} \Big|_{|z_i|=\zeta z_i^{s+}} \approx z_i^{s+} v_s \frac{d\eta_{B_i}}{dz_i \dot{q}} \Big|_{\dot{q}=0} \approx \pm 10^3, \quad (8)$$

gives a good tradeoff. Too small a value gives a sloppy transition for η_i , which results in a corrupted value of $\dot{z}_i$ in

Fig. 2. The friction behavior of the GMS and S-GMS friction models. (a) Hysteresis with nonlocal memory during presliding. (b) The nondrifting property, which is the response to an oscillatory force input, after sliding, whose amplitude remains under the breakaway limit. (c) Frictional lag with an oscillating positive velocity. (d) Periodic-motion friction with sliding. The friction parameters are $n_z = 8$ elements, $\kappa_i = \frac{i}{n_z} \cdot 10^7 \text{N}$, $\nu_i = \frac{2i}{n_z(n_z+1)}$, $f_S = 3\text{N}$, $f_C = 2\text{N}$, $v_s = 1\mu\text{m/s}$. The smoothing parameters of the S-GMS model are $\lambda = 1000$, $\gamma = 10$, $\zeta = 0.99$.

the sliding regime of element i , and an overestimated value of z_i (Fig. 3(a) and 3(b)). The factor $\zeta \in [0.9, 1)$ is introduced to avoid ringing in sliding regime when $|z_i| \simeq z_i^{s+}$. A good tuning rule is to set ζ , once λ has been set, so that η_{A_i} approaches one with a desired precision when $z_i = \pm z_i^{s+}$, e.g. $\eta_{A_i}(z_i^{s+}) = 1 - 10^{-3}$, as shown in Fig. 1(a). The influence of these three smoothing parameters on the shape of the presliding hysteresis behavior is summarized in Fig. 3.

III. THE ESTIMATION FRAMEWORK

This section first presents the general friction force estimation framework. Then, the Extended Kalman Filter (EKF) for state estimation and the Moving Horizon Estimator (MHE) for state and parameter estimation are formulated. The EKF and the MHE are both implemented for the GMS and the S-GMS friction models.

A. The Friction Observer

The estimation problem considers the following generic friction model, discretized at time instant t_k :

$$z_k = f(z_{k-1}, \dot{q}_{k-1}) + w_{k-1}, \quad w_k \sim \mathcal{N}(0, Q_k), \quad (9)$$

$$y_k = h(z_k, \dot{q}_k) + v_k, \quad v_k \sim \mathcal{N}(0, R_k), \quad (10)$$

where w_k is the Gaussian system model uncertainty and v_k is the Gaussian measurement noise with covariance Q_k and R_k , respectively. For the GMS and S-GMS models, the function $f(\cdot)$ implements the discrete-time version of state equations (1-2) and (4), respectively. Function $h(\cdot)$ implements the friction force equation (3).

A model of a moving mass m with friction force y_f is represented in Fig. 4. The mass m is driven through a feedback controller K . Based on the available control input u , which

Fig. 3. Effect of the smoothing parameters λ (a), γ (b), and ζ (c) in the S-GMS compared to the GMS model. Plots (a), (b), and (c) are a magnification of the closing of a minor loop in (d). (a) and (b) show that a decreasing value for λ and γ overestimate the friction force, and create a local memory effect, which means that a minor loop does not close at a major loop. On the other hand, a decreasing value for ζ in (c) underestimates the friction force. The friction parameters are the same as in Fig. 2.

represents the actuator force, and the position q , one can estimate the friction states z by minimizing an error criterion [block $\mathcal{Z}$] between the estimated friction $\hat{y}_f = h(\hat{z}, \dot{\hat{q}})$ [block $\mathcal{F}$] and the available measured friction based on the control input $\tilde{y}_f = u - m\ddot{q}$ [block $\mathcal{Y}$] assuming a known mass m and good estimates of velocity $\dot{\hat{q}}$ and acceleration $\ddot{\hat{q}}$ [block $\mathcal{V}$].

B. Extended Kalman Filter (EKF) formulation

The EKF [13] prediction step (or time update) and correction step (or measurement update) are formulated as follows:

Prediction step:

$$\hat{z}_k^- = f(\hat{z}_{k-1}^+, \tilde{q}_{k-1}), \quad (11a)$$

$$P_k^- = F_{k-1}P_{k-1}^+F_{k-1}^T + Q_{k-1}, \quad \text{and} \quad (11b)$$

correction step:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1}, \quad (12a)$$

$$\hat{z}_k^+ = \hat{z}_k^- + K_k(\tilde{y}_k - h(\hat{z}_k^-, \tilde{q}_k)), \quad \text{and} \quad (12b)$$

$$P_k^+ = (I - K_k H_k) P_k^-, \quad (12c)$$

where the Jacobian matrices F_k and H_k are defined as $F_k = \left. \frac{\partial f}{\partial z} \right|_{\hat{z}_k^+, \tilde{q}_k}$ and $H_k = \left. \frac{\partial h}{\partial z} \right|_{\hat{z}_k^-, \tilde{q}_k}$, respectively.

At time instant t_k , the variance of the estimated state $\hat{z}_{i,k}$ is given by the corresponding diagonal element of P_k^+ , i.e. $\sigma_{\hat{z}_{i,k}}^2 = P_{ii,k}^+$, and the variance of the estimated friction $\hat{y}_f$ is given by

$$\sigma_{\hat{y}_f,k}^2 = \sum_{i=1}^{n_z} \sum_{j=1}^{n_z} \kappa_i \kappa_j P_{ij,k}^+. \quad (13)$$

C. Moving Horizon Estimator (MHE) formulation

The main feature of MHE [10] is to look into the past to solve an optimization problem over a finite-time horizon. In addition to state trajectory estimation over the horizon, parameter estimation can be included as a constant optimization variable for the considered horizon. The MHE formulation offers a natural way to handle constraints on states and parameters. The MHE of the system states z_k and model parameters p corresponds to solving the following nonlinear constrained least-squares problem:

$$\min_{z_k, p} \left\| \begin{bmatrix} z_{\ell-N} - \bar{z}_{\ell-N} \\ p - \bar{p} \end{bmatrix} \right\|_{P_{\ell-N}^{-1}}^2 \quad (14a)$$

$$+ \sum_{k=\ell-N}^{\ell-1} \|f(z_k, p, \tilde{q}_k) - z_{k+1}\|_{Q_k^{-1}}^2 \quad (14b)$$

$$+ \sum_{k=\ell-N+1}^{\ell} \|h(z_k, p, \tilde{q}_k) - \tilde{y}_k\|_{R_k^{-1}}^2, \quad (14c)$$

$$\text{s.t.} \quad -\beta z^{s+}(\tilde{q}_k) \leq z_k \leq \beta z^{s+}(\tilde{q}_k), \quad (14d)$$

$$-p_{\min} \leq p \leq p_{\max}. \quad (14e)$$

In a real-time embedded implementation, this constraint optimization problem is solved at each single time instant considering a finite-time horizon of N discrete-time instant $k = \ell - N + 1, \dots, \ell$, where ℓ indicates the current time instant. The first term in the objective function (14a), referred to as the arrival cost, summarizes all available information prior to the data horizon in $(\bar{z}_{\ell-N}, \bar{p})$, and is weighted through the covariance matrix $P_{\ell-N}$. The second term (14b) is the weighted squared difference between the estimated state and model prediction, taking into account the model uncertainty Q_k . Finally, the third term (14c) is the weighted squared difference between the available measured friction $\tilde{y}$ and the estimated friction $\hat{y} = h(\cdot)$, taking into account the measurement uncertainty R_k . Natural bounds on states (14d) and parameters (14e) are added. A parameter $\beta > 1$ is included to take into account that the friction state trajectory can exceed the Stribeck curve (Fig. 2(c)).

D. Numerical Implementation Issues

1) *Real-time iterations for MHE*: Solving the optimization problem (14) requires to perform many Newton-type iterations of a sequential quadratic programming (SQP) method to reach convergence. To reduce the computational burden, only a limited or even a single Newton-type iteration is performed at each time instant ℓ . Then, at the next time instant, the oldest measurement is discarded, and a new one is introduced, while the previously estimated states are used as initial values for the next Newton-type iteration [11], [14]. In this work, the MATLAB `fmincon` function involving an interior-point method has been used to solve two Newton-type iterations at each time instant ℓ .

Fig. 4. A moving mass m , subject to friction y_f , is driven by a position feedback controller K . The friction observer comprises four parts: block $\mathcal{V}$, a velocity and acceleration observer; block $\mathcal{Y}$, a measured friction estimation $\tilde{y}_f$ based on the control input u minus an inertial term $m\ddot{q}$; block $\mathcal{F}$, the estimated friction $\hat{y}_f$, a function of the estimated states $\hat{z}$ and the velocity input $\dot{\hat{q}}$; and block $\mathcal{Z}$, a state observer to get the estimated state vector $\hat{z}$ from the velocity input $\dot{\hat{q}}$ and a measurement update correction between the measured friction $\tilde{y}_f$ and the estimated friction $\hat{y}_f$.

2) *On the arrival cost of MHE*: By summarizing past information before $k = \ell - N$, the arrival cost ensures stability of the MHE algorithm [15], especially in case of a short horizon. Typically [15], the arrival cost uses an EKF recursion to update the arrival state $\bar{z}_{\ell-N}$ and its covariance matrix $P_{\ell-N}$.

3) *S-GMS versus GMS*: The analytical expression of the S-GMS model (4) is much easier to implement in a gradient-based estimation framework than expressions (1)-(2) of the GMS model. In the EKF, integration of state equation (11a) and evaluation of Jacobian F are straightforward for the S-GMS model, while it requires quite a few conditional branching steps for the GMS model. For the GMS model, in addition to the state vector, a binary state condition vector is needed to indicate the mode of the elements, i.e. presliding (1) or sliding (2), in order to determine the state equation to be solved at each time instant. To this end, $2n_z + 1$ conditions have to be checked at each time instant: the value of each state z_i (below or above its breakaway limit), the condition of each element i , and the sign of the velocity. Things are even getting worse in the MHE as the binary state conditions needed for the GMS model are not part of the optimization variable set (z_k, p) . Thus, a-priori rules have to be set resulting in non-optimal solution for the MHE/GMS compared to the MHE/S-GMS as shown in the next section.

Moreover, thanks to its analytic set of ODE's, the S-GMS model can efficiently benefit from advanced explicit or implicit integration schemes. Implicit integration is needed for stiff equations, which can arise when the stiffness κ_i of some elements is high in presliding.²

²In friction model equations, it happens that numerical stiffness is closely related to mechanical stiffness.

TABLE I
GMS & S-GMS FRICTION PARAMETERS

Model	Parameter	Value	Unit
GMS & S-GMS	n_z	4	-
	κ	$[12, 2.5, 0.71, 0.79] \cdot 10^6$	N/m
	ν	$[0.067, 0.056, 0.047, 0.83]$	-
	σ	$[0, 0, 0, 0]$	Ns/m
	σ_v	20	Ns/m
	f_S	4.5	N
	f_C	4	N
GMS	v_s	10^{-3}	m/s
	C	50	N/s
	λ	10^3	-
S-GMS	ζ	0.98	-
	γ	10^2	-

IV. SIMULATION RESULTS

This section presents the numerical validation results of three different friction estimators:

- the EKF(z) state estimator,
- the MHE(z) state estimator and
- the MHE(z, p) state and parameter estimator.

Each one of them is implemented for both the GMS and the S-GMS friction models. The estimation framework (Fig. 4) is applied on a simulated mechanical system with system friction φ using the GMS model and parameters listed in Table I. These friction parameters have been identified based on experimental input-output data from an XY positioning table using estimation techniques from [7], [16].

For the numerical validation of the EKF and MHE, a sine reference trajectory q_r with an amplitude of $200\mu\text{m}$ and a frequency of 5 Hz (see Fig. 5) is applied to the position controller of the simulated motion system (Fig. 4). This trajectory is relevant to study presliding at velocity reversal, and sliding regime slightly above the Stribeck velocity $v_s = 10^{-3}$ m/s. The estimated velocity $\hat{q}$ is obtained by taking the finite-difference derivative of the measured position, sampled at 0.1ms, combined with a second order low-pass filter with a cutoff frequency at 500 Hz.

All estimators have been initialized with the parameters listed in Table I, at the exception of the stiffness vector κ , which has been 1) underestimated and 2) overestimated by 20% compared to the system friction parameters. The set of estimated parameters p in the MHE(z, p) is $p = \{\kappa\}$. Prediction steps are evaluated at a system update rate of 0.1ms, and correction steps (EKF) or optimization steps (MHE) are performed at a measurement update rate of 2ms. The measurement noise is assumed to be known, and EKF covariance matrix $R \in \mathbb{R}$ (scalar) is $R = 0.1^2$. The state noise EKF covariance matrix $Q \in \mathbb{R}^{n_z \times n_z}$ is $Q = \text{diag}((10^{-2}\nu_1)^2, \dots, (10^{-2}\nu_{n_z})^2)$. The same covariance matrices are used in the MHE formulation, provided that they are extended to the complete horizon. An MHE horizon of $N = 4$ is used.

For each of the estimation schemes, the Normalized Root-Mean-Square Error (NRMSE) between the system friction and the estimated friction, over one period of the sine trajectory, is used to evaluate and compare the different estimation schemes. Results are summarized in Table II. The EKF estimators with the GMS and S-GMS models perform in a similar way, for

Fig. 5. Simulated sine trajectory with position, velocity, and friction force. The window in the friction force at time $t = 0.15\text{s}$ represents the onset of presliding (at velocity reversal) and the window at time $t = 0.18\text{s}$ represents the onset of full sliding. Both windows are magnified in Fig. 6-7 to show details of friction estimation.

both under and overestimated parameters p . The MHE with GMS model behave worse than the EKF for underestimated parameters p , and better than the EKF for overestimated parameters p . On the other hand, the MHE with S-GMS model behave better than all the EKF and the MHE with GMS model, even though the system friction model is based on a GMS friction model. Moreover, the MHE with S-GMS model for state and parameter estimation is also providing a better estimate than the MHE for state estimation only. However, the improvement is not so significant in this study due to a short horizon of the MHE and an observed slow convergence rate of parameters p .

Fig. 6 (with underestimated parameters) and Fig. 7 (with overestimated parameters) show the evolution of friction state estimation at the onset of presliding (a-c) and full sliding (b-d) for the case of GMS-based state estimators (a-b) and S-GMS-based state estimators (c-d).

A general conclusion from NRMSE values presented in Table II and evolution of friction state estimation in Fig. 6-7 is that the MHE with S-GMS-based friction model outperforms the EKF estimators (both GMS and S-GMS models) and the MHE with GMS-based friction model. The benefit of the MHE over the EKF is explained by the ability of the MHE to cope better with nonlinearities in an optimization framework, provided that the MHE uses a smoothed (analytic) model representation.

V. CONCLUSION AND FUTURE WORK

This paper introduced a smoothed formulation of the GMS model (S-GMS) including a set of analytic ODE's without conditional branching with regard to the state variables. The S-GMS model offers the possibility to use implicit ODE integration schemes and reliable Jacobian evaluation. Moreover, a smoothed model representation makes it well suited for on-line gradient-based state and parameter estimation methods solving a series of optimization problems, such as in moving horizon estimation algorithms.

Simulated results of friction state estimation based on EKF and MHE for both the GMS and S-GMS models have been presented. They show the benefit of using the S-GMS model within the MHE optimization framework for state and parameter estimation.

Fig. 6. Simulated system friction (in solid blue line) with the GMS model, estimated friction with the EKF (in dashed green line) and estimated friction with the MHE (in dash-dot red line). Parameter κ is underestimated by 20%. Measurement update is done every 2ms (circles). Graphs (a) and (b) show the estimated friction with GMS-based estimators. Graphs (c) and (d) show the estimated friction with S-GMS-based estimators.

TABLE II
NORMALIZED ROOT-MEAN-SQUARE ERROR (NRMSE)

κ		EKF(z)	MHE(z)	MHE(z, p)
1) -20%	GMS	1.71	2.59	2.52
	S-GMS	1.65	1.51	1.49
2) +20%	GMS	1.52	1.09	1.23
	S-GMS	1.56	0.96	0.93

NRMSE [%] between the system friction (GMS model) and the estimated friction is evaluated for three different friction estimators: EKF(z) state estimator, MHE(z) state estimator, MHE(z, p) state and parameter estimator. Each estimator implements either the GMS or the S-GMS model. Estimators initialization have been done with two sets of values for κ : 1) underestimated and 2) overestimated by 20% compared to the system parameters. Results show the benefit of the MHE algorithm using the S-GMS model.

The MHE framework will be further extended to estimate and update additional friction parameters. On-line MHE algorithms for state and parameter estimation based on the S-GMS friction model will be implemented with ACADO [12], a toolkit for automatic code generation and dynamic optimization, and will be tested on the ETEL XY positioning table in a real-time application to compensate for friction.

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REFERENCES

- [1] D. Rizo and S. Fasseis, "Friction Identification Based upon the LuGre and Maxwell Slip Models," *IEEE Trans. on Control Systems Technology*, vol. 17, no. 1, pp. 153–160, 2009.
- [2] A. Amthor, T. Hausotte, P. Li, C. Ament, and G. Jaeger, "Friction Identification and Compensation on Nanometer Scale," in *Proc. IFAC World Congress*, 2008.

Fig. 7. Simulated system friction (in solid blue line) with the GMS model, estimated friction with the EKF (in dashed green line) and estimated friction with the MHE (in dash-dot red line). Parameter κ is overestimated by 20%. Measurement update is done every 2ms (circles). Graphs (a) and (b) show the estimated friction with GMS-based estimators. Graphs (c) and (d) show the estimated friction with S-GMS-based estimators.

- [3] F. Altpeter, M. Grunenberg, P. Myszkowski, and R. Longchamp, "Auto-Tuning of Feedforward Friction Based on the Gradient Method," in *Proc. American Control Conference*, 2000.
- [4] P. Dupont, V. Hayward, B. Armstrong, and F. Altpeter, "Single State Elastoplastic Friction Models," *IEEE Trans. on Automatic Control*, vol. 47, no. 5, pp. 787–792, 2002.
- [5] V. Hayward, B. Armstrong, F. Altpeter, and P. Dupont, "Discrete-Time Elastoplastic Friction Estimation," *IEEE Trans. on Control Systems Technology*, vol. 17, no. 3, pp. 688–696, 2009.
- [6] F. Al-Bender, V. Lampaert, and J. Swevers, "The Generalized Maxwell-Slip Model: A Novel Model for Friction Simulation and Compensation," *IEEE Trans. on Automatic Control*, vol. 50, no. 11, pp. 1883–1887, 2005.
- [7] Z. Jamaludin, H. Van Brussel, and J. Swevers, "Quadrant Glitch Compensation using Friction Model-Based Feedforward and an Inverse-Model-Based Disturbance Observer," in *Proc. IEEE Intl Workshop on Advanced Motion Control*, 2008.
- [8] C. Casanova, E. De Pieri, U. Moreno, and E. Castelan, "Friction Compensation in Flexible Joints Robot with GMS Model: Identification, Control and Experimental Results," in *Proc. IFAC World Congress*, 2008.
- [9] I. Nilkhamhang and A. Sano, "Model-based Adaptive Friction Compensation for Accurate Position Control," in *Proc. IEEE Conf. on Decision & Control*, 2008.
- [10] D. Robertson, J. Lee, and J. Rawlings, "A Moving Horizon-based approach for Least-Squares State Estimation," *AIChE Journal*, vol. 42, pp. 2209–2224, 1996.
- [11] P. Kuehl, M. Diehl, T. Kraus, J. Schloeder, and H. G. Bock, "A Real-Time Algorithm for Moving Horizon State and Parameter Estimation," *Computers and Chemical Engineering*, vol. 35, pp. 71–83, 2011.
- [12] B. Houska, H. Ferreau, and M. Diehl, "ACADO Toolkit - An Open-Source Framework for Automatic Control and Dynamic Optimization," *Optimal Control Applications and Methods*, vol. 32, no. 3, pp. 298–312, 2011, <http://acadotoolkit.org>.
- [13] D. Simon, *Optimal State Estimation*. Wiley, 2006.
- [14] M. Diehl, H. Bock, J. Schloeder, R. Findeisen, Z. Nagy, and F. Allgöwer, "Real-Time Optimization and Nonlinear Model Predictive Control of Processes governed by Differential-Algebraic Equations," *Journal of Process Control*, vol. 12, pp. 577–585, 2002.
- [15] C. Rao, J. Rawlings, and D. Mayne, "Constrained State Estimation for Nonlinear Discrete-Time Systems: Stability and Moving Horizon Approximations," *IEEE Trans. on Automatic Control*, vol. 48, no. 2, pp. 246–258, 2003.
- [16] M. Boegli, T. De Laet, J. De Schutter, and J. Swevers, "A Split-Horizon Scheme for On-line Friction Parameter Estimation," in *Proc. IEEE/ASME Intl Conf. on Advanced Intelligent Mechatronics*, 2011.