Lyapunov based design of robust linear-feedback for time-optimal periodic quadcopter motion

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1 Introduction

In recent years, Unmanned Aerial Vehicles (UAV), such as quadcopters, have received increasing attention. In the research world, they are an interesting platform for e.g. testing new control algorithms. An interesting challenge is the robust control of quadcopters. This presentation discusses how the period lyapunov differential equations can be used to obtain approximate robust control for a time-optimal periodic quadcopter flight task. The results are limited to computer simulation for now, but the KU Leuven quadcopter platform is available for demonstration in the future.

2 Quadcopter model

A quadcopter typically has 4 rotors, of which 2 rotate clock-wise, and the other 2 rotate anti-clock-wise. The state vector of a quadcopter consists of the 3D postion, velocity, orientation and rotation speeds of the body, and of the spinning speeds of the 4 rotors. Orientation is parametrised by quaternions. This amounts to a total of 17 states. The control inputs to the system are torques applied on the 4 rotos. The aerdoynamic forces and torques are modelled according to [4].

3 Problem formulation

3.1 Flight scenario

In the investigated flight scenario, the quadcopter must meet two waypoints A and B in a periodic time-optimal fashion. The system is free to choose at what time these waypoint shall be met. A hard constraint is present in the form of an impermeable vertical cylinder that prohibits a line-of-sight connection between A and B.

3.2 Robust linear feedback control

To robustify the non-linear system in an approximate fashion, a Lyapunov based approach is taken [1]. In this formalism, the original system is augmented with extra states P that satisfy the following linear Lyapunov differential equations:

$$P(t) = A(t).P(t) + P(t).A^T(t) + B(t).B^T(t) \quad P(0) = P(T) \tag{1}$$

where A and B are linearisations of the system dynamics with respect to states and disturbances respectively. P can be interpreted as an uncertainty ellipsoid on the states of the original system. Robustification is obtained by adding a P-weighted term to constraints.

To obtain a linear feedback control, original controls of the system are explicitized as $u = \bar{u} + \bar{K}\left[(x - \bar{x}) + w\right]$, where bar quantities become parameters of the optimal control problem and w models measurement noise.

An invariant appears in the system due to the use of quaternions, requiring modifications to the periodicity constraints [2].

4 Numerical approach

The resulting optimal control problem is treated numerically by a direct approach, using a collocation scheme on fully implicit model equations. A sparsity-exploiting interior point method is used to solve the resulting non-linear problem. The python interface of CasADi [3], is used as a development framework.

References

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