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A Data-Driven Constrained Norm-Optimal Iterative Learning Control Framework for LTI Systems

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Abstract—This paper presents a data-driven constrained norm-optimal iterative learning control framework for linear time-invariant systems that applies to both tracking and point-to-point motion problems. The key contribution of this paper is the estimation of the system’s impulse response using input/output measurements from previous iterations, hereby eliminating time-consuming identification experiments. The estimated impulse response is used in a norm-optimal iterative learning controller, where actuator limitations can be formulated as linear inequality constraints. Experimental validation on a linear motor positioning system shows the ability of the proposed data-driven framework to (i) achieve tracking accuracy up to the repeatability of the test setup, (ii) minimize the rms value of the tracking error while respecting the actuator input constraints, (iii) learn energy-optimal system inputs for point-to-point motions.

Index Terms—Iterative learning control (ILC), LTI systems, Energy-optimal point-to-point motions, Precision motion control, Data-driven control.

I. INTRODUCTION

Iterative learning control (ILC) is an open-loop control strategy that aims at improving the tracking performance of a system executing the same task under the same operating conditions [1], [2]. The system input is updated iteratively, i.e. from trial to trial, to improve the accuracy of the desired motion. Using this technique, accurate tracking can be obtained even when the system dynamics are uncertain. While ILC was originally developed for tracking control, it has recently been adapted to solve point-to-point motion problems [3], [4]. Reported applications of ILC include machine tool axes [5] and wafer stage motion systems [6].

When designing an ILC algorithm the aim is to use the error information from previous trials as efficiently as possible in order to achieve a minimal tracking error in as few iterations as possible. The simplest ILC law uses a PID-type learning filter, which consists of a proportional, integral and derivative gain on the tracking error to update the system input. More advanced learning laws [1] include plant inversion methods, $\mathcal{H}_\infty$ design methods and norm-optimal ILC methods [7], [8]. These methods use a plant model and possibly also uncertainty models to ensure (robust) monotonic convergence [9].

Data-driven ILC algorithms [10], [11] estimate a system representation using input/output data obtained during the ILC procedure and hence avoid time-consuming identification experiments prior to the ILC procedure. The data-driven ILC method presented in [10] estimates the system’s frequency response function using the Fourier transform of the previous

iteration’s input and output signal. This method requires a frequency-domain representation of the desired output and is therefore only applicable to tracking control problems. In addition, this method cannot be extended to account for actuator limitations. [11] proposes a data-driven ILC method where the coefficients of a parametric plant model are updated between every two trials. Obviously, this method requires an a priori estimate of the number of poles and zeros.

This paper presents a data-driven constrained norm-optimal ILC framework for linear time-invariant (LTI) systems that is applicable to both tracking and point-to-point motion control problems and allows accounting for actuator limitations. Herein, the key contribution is the estimation of a system’s impulse response using previous iterations’ input/output data without assuming any a priori system information aside from the fact that the plant is LTI. The estimated impulse response is then used for the purpose of norm-optimal ILC.

Since more and more applications require very fast motion control to increase productivity, actuator limitations can no longer be ignored. Traditional norm-optimal ILCs use the analytical solution of an unconstrained convex quadratic program (QP), and hence do not allow actuator constraints to be taken into account explicitly. For this reason, [12] proposes to solve a constrained convex QP at every iteration allowing actuator limitations to be formulated as linear inequality constraints. This way, the model-based norm-optimal ILC framework is extended for systems with actuator constraints. This paper adopts this approach such that the proposed data-driven norm-optimal ILC also applies to LTI systems with actuator constraints.

The paper is organized as follows. Section II presents the data-driven norm-optimal ILC algorithm. Section III discusses the application of norm-optimal ILC to systems with actuator constraints. The experimental validation on a linear motor test setup is presented in section IV. Finally, section V summarizes the conclusions.

II. DATA-DRIVEN NORM-OPTIMAL ILC

This section first discusses the norm-optimal ILC framework and then presents the impulse response estimation based on previous trials’ input/output data.

A. Norm-optimal ILC

Consider a single-input single-output (SISO), discrete-time, LTI system $P(q)$ in Fig. 1 with input $u_j(k)$ and output $y_j(k)$,

Fig. 1. Open-loop discrete-time LTI system $P(q)$.

where k refers to the discrete time instants, subscript $j \in \{0, 1, 2, \dots\}$ denotes the trial number and q is the one-sample advance operator.

In the remainder of this paper, we use the lifted system representation [13], in which the system dynamics of a causal, LTI plant $P(q)$ with relative degree τ are given by:

$$\mathbf{y}_j = \mathbf{P}\mathbf{u}_j. \quad (1)$$

In (1),

$$\mathbf{P} = \begin{bmatrix} p(\tau+1) & 0 & \cdots & 0 \\ p(\tau+2) & p(\tau+1) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ p(\tau+N) & \cdots & p(\tau+2) & p(\tau+1) \end{bmatrix} \quad (2)$$

is the convolution matrix of $P(q)$, a lower-triangular Toeplitz matrix constructed of the system's impulse response $p(k)$, N denotes the number of samples per trial, and

$$\mathbf{u}_j = [u_j(1) \quad u_j(2) \quad \cdots \quad u_j(N)]^T, \quad (3a)$$

$$\mathbf{y}_j = [y_j(\tau+1) \quad y_j(\tau+2) \quad \cdots \quad y_j(\tau+N)]^T, \quad (3b)$$

are $N \times 1$ -dimensional vectors containing the input signal $u_j(k)$, and output signal $y_j(k)$, respectively, during trial j . The output error e_j is defined as $\mathbf{y}_d - \mathbf{y}_j$, where $\mathbf{y}_d$ denotes the desired output.

An ILC updates the system input from trial to trial using previous trials' error information to reduce e_j , as $j \rightarrow \infty$. To this end, the most commonly used ILC update formula is

$$\mathbf{u}_{j+1} = \mathbf{Q}(\mathbf{u}_j + \mathbf{L}e_j), \quad (4)$$

where $\mathbf{Q}$ and $\mathbf{L}$ represent the convolution matrices of the robustness filter $Q(q)$ and learning filter $L(q)$, respectively [1]. In the norm-optimal ILC design framework, the robustness and learning filter are obtained by minimizing the following objective function:

$$J = \hat{e}_{j+1}^T \mathbf{W}_e \hat{e}_{j+1} + \mathbf{u}_{j+1}^T \mathbf{W}_u \mathbf{u}_{j+1} + \delta \mathbf{u}_{j+1}^T \mathbf{W}_{\delta u} \delta \mathbf{u}_{j+1}, \quad (5)$$

where $\delta \mathbf{u}_{j+1} = \mathbf{u}_{j+1} - \mathbf{u}_j$ and $\hat{e}_{j+1} = e_j - \hat{\mathbf{P}} \delta \mathbf{u}_{j+1}$ is the predicted next iteration's output error. The weighting matrices $\mathbf{W}_e$, $\mathbf{W}_u$, and $\mathbf{W}_{\delta u}$ are positive semi-definite and $\hat{\mathbf{P}}^T \mathbf{W}_e \hat{\mathbf{P}} + \mathbf{W}_u + \mathbf{W}_{\delta u}$ is positive definite such that the objective J is strictly convex. Substituting $\hat{e}_{j+1} = e_j - \hat{\mathbf{P}} \delta \mathbf{u}_{j+1}$ in (5), differentiating J with respect to $\mathbf{u}_{j+1}$ and setting this derivative equal to zero [1] yields ILC update formula (4) with

$$\begin{aligned} \mathbf{Q} &= (\hat{\mathbf{P}}^T \mathbf{W}_e \hat{\mathbf{P}} + \mathbf{W}_u + \mathbf{W}_{\delta u})^{-1} (\hat{\mathbf{P}}^T \mathbf{W}_e \hat{\mathbf{P}} + \mathbf{W}_{\delta u}), \\ \mathbf{L} &= (\hat{\mathbf{P}}^T \mathbf{W}_e \hat{\mathbf{P}} + \mathbf{W}_{\delta u})^{-1} (\hat{\mathbf{P}}^T \mathbf{W}_e). \end{aligned} \quad (6)$$

Note that the norm-optimal ILC design method is also applicable to point-to-point motion problems, where the desired output is only defined during the dwell. Choosing

$$\mathbf{W}_e = \begin{bmatrix} \mathbf{0}_{n \times n} & \mathbf{0}_{n \times N-n} \\ \mathbf{0}_{N-n \times n} & \mathbf{I}_{N-n \times N-n} \end{bmatrix}, \quad (7)$$

for example, results in an ILC that minimizes the output error from time sample $n+1$ to sample N only. As shown in [7], the ILC given by (4) and (6) guarantees that $\|e_{j+1}\|_2 \leq \|e_j\|_2$ for all $j = 0, 1, \dots$ when a perfect model is known. In case of model-plant mismatch, increasing the weight $\mathbf{W}_u$ on the input effort can guarantee robust monotonic convergence for a given additive uncertainty [9], [14]. This comes down to trading off nominal performance, represented by the asymptotic tracking error, for robustness to system uncertainty.

The weighting matrix $\mathbf{W}_{\delta u}$ regulates the effect of noise and trial-varying disturbances on the next iteration's input signal and influences therefore also the asymptotic error [15]. By tuning $\mathbf{W}_{\delta u}$, convergence speed is traded off for the effect of noise and trial-varying disturbances on the next iteration's input signal.

Whereas traditional norm-optimal ILC uses a parametric plant model to obtain the convolution matrix $\hat{\mathbf{P}}$, the proposed method obtains an estimated convolution matrix $\hat{\mathbf{P}}$ of the plant $P(q)$ using previous trials' input/output data. The estimated convolution matrix is then used in a norm-optimal ILC.

B. Impulse response estimation based on previous trials' input/output data

The basic idea is to estimate the system's impulse response using a linear combination of previous trials' measurement data. Let $\mathbf{u}_{lc} = l_0 \mathbf{u}_0 + l_1 \mathbf{u}_1 + \cdots + l_j \mathbf{u}_j$ denote a linear combination of previous trials' input signals and $\mathbf{y}_{lc}$ the same linear combination of previous trials' output signals. From (1) follows that the convolution matrix of plant $P(q)$ can be estimated as:

$$\hat{\mathbf{P}} = \mathbf{U}_{lc}^{-1} \mathbf{Y}_{lc}, \quad (8)$$

where $\mathbf{Y}_{lc}$ and $\mathbf{U}_{lc}$ denote the lower-triangular Toeplitz matrices of $\mathbf{y}_{lc}$ and $\mathbf{u}_{lc}$, respectively.

The estimated convolution matrix $\hat{\mathbf{P}}$ is exactly equal to the true convolution matrix $\mathbf{P}$ if the following conditions hold: (i) the plant $P(q)$ is LTI, (ii) $\mathbf{U}_{lc}$ is of full rank, (iii) no measurement noise or other disturbances are present. Under these conditions, a zero output error can be achieved asymptotically by a suitable choice of the weighting matrices ($\mathbf{W}_u = 0$) [1].

C. LTI systems with measurement noise

In practice, however, the measured output is always subject to noise. Consider again an LTI system $P(q)$ with input $u_j(k)$, true output $y_j(k)$, measured output $y_j^m(k) = y_j(k) + n_j(k)$, which is corrupted by zero-mean measurement noise $n_j(k)$ with standard deviation σ_n , and measured output error $e_j^m(k) = y_d(k) - y_j^m(k)$.

Since the true noise-free output y_j is not known, $\hat{\mathbf{P}}$ is estimated using the measured output y_j^m instead. As a result,

the estimated convolution matrix $\hat{P}$ is not equal to the true convolution matrix P :

$$\hat{P} = U_{lc}^{-1} Y_{lc}^m = U_{lc}^{-1} (Y_{lc} + N_{lc}) = P + U_{lc}^{-1} N_{lc}, \quad (9)$$

where N_{lc} denotes the lower-triangular Toeplitz matrix of the linear combination of the previous trials' measurement noise n_j .

From (9) follows that the estimation error $\hat{P} - P$ scales with the noise level and the maximal singular value of U_{lc}^{-1} [16], denoted by $\bar{\sigma}(U_{lc}^{-1})$.

The order of magnitude of the additive uncertainty on the estimated convolution matrix $\hat{P}$ can be estimated as

$$\bar{\sigma}(U_{lc}^{-1}) \sigma_{n_{lc}} = \bar{\sigma}(U_{lc}^{-1}) \|l_0, \dots, l_j\|_2 \sigma_n, \quad (10)$$

where $\sigma_{n_{lc}}$ denotes the standard deviation of n_{lc} . This estimate can be used in the design of a norm-optimal ILC, when trading off robustness to system uncertainty for nominal performance [9].

The aim is to choose the weights $l_0, \dots, l_j$ of the linear combination such that the model estimation error is small. A first attempt to minimize the estimation error might be to average out measurement noise such that $l_i = 1/(j+1)$ for all $i = 0, \dots, j$ yielding $\sigma_{n_{lc}} = \sigma_n/\sqrt{j+1}$. Although $\sigma_{n_{lc}}$ decreases as $j \rightarrow \infty$, the estimation error, given by (10), might increase because of an increase of $\bar{\sigma}(U_{lc}^{-1})$. Since the input signals u_j , for $j = 1, 2, \dots$ result from the iterative learning process and depend on the desired output, there is no freedom to choose these signals such that $\bar{\sigma}(U_{lc}^{-1})$ is small. The first trial's input signal u_0 on the other hand, is free to choose. Our experience shows that the very simple choice $U_{lc} = U_0$, in combination with a well chosen first iteration's input signal u_0 , is preferred.

In case of a pure LTI system with measurement noise, the first iteration's input signal that results in the minimal estimation error is

$$u_0 = [\bar{u} \ 0 \ 0 \ \dots \ 0]^T, \quad (11)$$

where $\bar{u}$ is the upperbound on the input signal. For this input signal, $U_0 = \bar{u} I_N$ and hence $\bar{\sigma}(U_0^{-1}) = 1/\bar{u}$.

In many mechatronic applications, such as positioning systems, nonlinear effects such as dry friction and actuator cogging cannot be ignored [17]. Since the proposed optimal input signal (11) often results in a very small motion, these nonlinearities have a large effect on the estimated convolution matrix. For this reason, a first iteration's input signal that results in a larger motion is preferred to reduce the effects of nonlinearities.

To summarize, during the first iteration ($j = 0$) a suitable input signal is applied to the plant $P(q)$. Then, the convolution matrix $\hat{P}$ is estimated as $U_0^{-1} Y_0^m$ and used in the design of a norm-optimal ILC. Robust monotonic convergence can also be verified using the estimate of the additive uncertainty (10) on the estimated convolution matrix. However, due to nonlinear effects, (10) might underestimate the additive uncertainty on $\hat{P}$. Therefore it is recommended to consider a suitable safety margin on the estimated additive uncertainty.

D. LTI systems with trial-invariant disturbances

In various applications, the output also suffers from trial-invariant disturbances. Consider again an LTI system $P(q)$ with a certain trial-invariant output disturbance $d_j(k) = d(k)$, for all $j = 0, 1, 2, \dots$

Using the lifted-system representation, the system dynamics, including the trial-invariant disturbance $d(k)$, are written as follows:

$$y_j = P u_j + d. \quad (12)$$

Due to the trial-invariant disturbance, the estimated convolution matrix $\hat{P}$ differs from the true convolution matrix P :

$$\hat{P} = U_{lc}^{-1} Y_{lc} = U_{lc}^{-1} (Y_{lc} + D_{lc}) = P + U_{lc}^{-1} D_{lc}. \quad (13)$$

Fortunately, in case of trial-invariant disturbances, the resulting modelling error $U_{lc}^{-1} D_{lc}$ can be eliminated by selecting the weights $l_0, \dots, l_j$ such that $\sum_{i=0}^j l_i = 0$ and therefore $D_{lc} = 0$.

III. NORM-OPTIMAL ILC FOR LTI SYSTEMS WITH ACTUATOR CONSTRAINTS

This section adapts the norm-optimal ILC framework such that actuator limitations can be taken into account. This extension is applicable to both model-based and data-driven norm-optimal ILC [12]. The application to open-loop and closed-loop systems is discussed.

A. Open-loop systems

The unconstrained optimization problem considered in norm-optimal ILC is augmented with a set of linear inequality constraints that represent the actuator limitations of the system. Since the solution of this constrained optimization problem cannot be derived analytically, the following problem is solved numerically at every iteration to obtain the next iteration's input signal u_{j+1} :

$$\begin{aligned} \underset{\delta u_{j+1} \in \mathbb{R}^N}{\text{minimize}} \quad & \hat{e}_{j+1}^T W_e \hat{e}_{j+1} + u_{j+1}^T W_u u_{j+1} \\ & + \delta u_{j+1}^T W_{\delta u} \delta u_{j+1} \end{aligned} \quad (14a)$$

$$\text{subject to} \quad \hat{e}_{j+1} = e_j^m - \hat{P} \delta u_{j+1} \quad (14b)$$

$$u_{j+1} = u_j + \delta u_{j+1} \quad (14c)$$

$$\|u_{j+1}\|_\infty \leq \bar{u} \quad (14d)$$

$$\|du_{j+1}\|_\infty \leq \overline{du}, \quad (14e)$$

where $\bar{u}$ and $\overline{du}$ respectively denote the upperbound on the actuator input $u_{j+1}(k)$ and its first derivative $du_{j+1}(k)$. The same objective function is used as in traditional norm-optimal ILC. The equality constraints (14b) and (14c) represent the system dynamics and the update equation, respectively.

Note that the presented framework is not restricted to tracking control problems. A suitable choice of the weighting matrices allows the ILC to learn energy-optimal point-to-point motions. There may also be other design specifications, such as avoiding the excitation of known structural resonances of the machine. These design specifications can be incorporated by adding appropriate terms to the objective function of the optimization program.

Fig. 2. Closed-loop discrete-time system with actuator constraints.

B. Application to closed-loop systems

Often, ILC is combined with feedback control since an iterative learning controller cannot compensate for nonrepeating disturbances. Consider the closed-loop system in Fig. 2 with actuator constraints $\bar{u}$ and $\bar{du}$, controller $C(q)$, plant $P(q)$, reference signal r_j , actuator input u_j , output y_j and measured output $y_j^m = y_j + n_j$. The proposed ILC algorithm learns the reference signal r_{j+1} for the closed-loop system that results in an output signal y_{j+1} that is as close as possible to the desired output y_d . The difference with the aforementioned open-loop systems is that in the closed-loop case the reference signal r_j is updated in order to track a given desired output y_d , taking into account the constraints on the actuator input u_j , whereas in the open-loop case the actuator input itself is updated.

When the actuator constraints are active, the relation between the reference signal r_j and the output y_j of the closed-loop system in Fig. 2 becomes nonlinear. To avoid this nonlinearity, optimization problem (14) is solved, yielding a desired next iteration's input signal u_{j+1} that respects the actuator constraints and a predicted next iteration's output signal $\hat{y}_{j+1} = y_j^m + \hat{P}\delta u_{j+1}$. Based on the plant $P(q)$, the controller $C(q)$ and the closed-loop configuration, the reference trajectory that leads to the optimized input signal u_{j+1} can be calculated as

$$r_{j+1} = C^{-1}u_{j+1} + \hat{y}_{j+1}, \quad (15)$$

where C denotes the convolution matrix of controller $C(q)$.

IV. EXPERIMENTAL RESULTS

This section presents the experimental validation of the proposed data-driven ILC framework on a permanent magnet linear motor. Subsection IV-A describes the linear motor test setup. Subsection IV-B discusses the experimental results of a tracking problem where the linear motor system is able to track the reference trajectory without saturating the actuator. The results show the ability of the proposed algorithm to achieve tracking control up to the repeatability of the test setup, despite position-dependent disturbances such as actuator cogging [17]. Subsection IV-C presents the experimental results of another tracking problem where the proposed ILC algorithm manages to achieve the minimal rms value of the tracking error taking into account the imposed constraints on the actuator input. Subsection IV-D presents the experimental results of a point-to-point motion problem where positioning accuracy is traded off with input effort.

Fig. 3. Linear motor test setup.

Fig. 4. Desired output $y_d(k)$.

A. Experimental setup

The experimental validation is performed on a current-controlled permanent-magnet linear motor depicted in Fig. 3. The position of the carriage of the linear motor is measured using a linear encoder and fed back to a PI-lead position controller. The position measurement is subject to output noise of which the rms value is estimated to be $0.1 \mu\text{m}$. The limiting factor concerning the asymptotic tracking error, however, is not the measurement noise but the repeatability of the closed-loop system, which is approximately $0.3 \mu\text{m}$ rms. The bounds on the actuator input and its first derivative are selected to be $\bar{u} = 3 \text{ A}$ and $\bar{du} = 1 \text{ A ms}^{-1}$. In this paper, all experiments are performed with a sample time of 2 ms.

B. Unconstrained reference tracking

Fig. 4 shows the desired output $y_d(k)$, which is a forward and backward motion of 5 cm that needs to be executed in 1 s. Because of the position-dependency of the cogging disturbance, the carriage is returned to a home position before every iteration. The carriage reaches the home position within a range of $\pm 0.2 \mu\text{m}$.

During the first trial ($j = 0$) of the learning process, the feedback controller is turned off and the linear motor is actuated directly with a test signal u_0 . Fig. 5 shows the first iteration's input signal u_0 and corresponding output signal y_0^m .

Note that the position-dependent cogging disturbance differs significantly when the test signal u_0 or a learned input signal u_j ($j > 0$) is applied to the linear motor. The position-dependent disturbance cannot be considered as a trial-invariant disturbance and the corresponding estimation error cannot be eliminated using the approach discussed in subsection II-D. Therefore, the convolution matrix of $P(q)$ is estimated as $U_0^{-1}Y_0^m$.

The following choice of weighting matrices resulted in a stable ILC system:

$$W_e = I_N, \quad W_u = 10^{-6}I_N, \quad W_{\delta u} = 2 \cdot 10^{-5}I_N. \quad (16)$$

Fig. 5. First iteration's input and output signal.

	$e_{\max}$ (μm)	e_{rms} (μm)
Feedback only	93	42
Trial 13	1.91	0.62
	-98%	-98.5%

TABLE I

COMPARISON OF THE PEAK VALUE AND RMS VALUE OF THE TRACKING ERROR BEFORE AND AFTER LEARNING FOR A FORWARD AND BACKWARD MOTION OF 5 cm.

Fig. 6 shows the rms value of the tracking error as a function of the trial number and table I compares the peak and rms value of the tracking error before and after learning. Fig. 7 shows the remaining tracking error of the 13th trial. Further reduction of the tracking error is hardly possible due to the limited repeatability of the test setup. These results show that the proposed data-driven ILC algorithm is able to achieve tracking up to the repeatability of the test setup.

C. Constrained reference tracking

The following experimental results show the ability of the proposed method to learn a reference signal leading to the

Fig. 6. Rms value of the tracking error as a function of the iteration number.

Fig. 7. Tracking error of the 13th trial.

Fig. 8. Tracking error e_{11} and actuator input u_{11} of the 11th trial together with the bounds on the actuator input.

minimal rms value of the tracking error taking into account actuator constraints. The desired output is again a forward and backward motion that needs to be executed in 1 s, as in Fig. 4, but the distance is increased to 8 cm such that the constraints on the actuator input become active. The first iteration's input signal u_0 , and the tuning of the weighting matrices $\mathbf{W}_e$, $\mathbf{W}_u$, and $\mathbf{W}_{\delta u}$ are again given by (16).

The algorithm reaches convergence in 11 iterations. Fig. 8 shows the remaining tracking error e_{11} and the actuator input u_{11} of the 11th iteration. Very accurate tracking is achieved at time instants when no actuator constraints are active. When the actuator input hits its bounds, the tracking error is, inevitably, an order of magnitude greater.

D. Learning energy-optimal point-to-point motions

This section discusses the application of the data-driven ILC framework to point-to-point motion problems. Fig. 9 shows the desired output position $y_d(k)$ of the linear motor. Only the desired endpoint of the point-to-point motion is defined in the vector $\mathbf{y}_d$. Therefore, a matrix of the form given by (7) is chosen as weighting matrix for the output error. In order to trade off the required energy to perform the point-to-point motion for the rms value of the positioning error, the weighting matrix on the actuator input is chosen as $\mathbf{W}_u = \lambda_u \mathbf{I}_N$, where λ_u regulates the trade-off between positioning accuracy and input effort.

Fig. 10 shows these two objectives as a function of the iteration number for three different values of the trade-off parameter: $\lambda_u = 0$, $\lambda_u = 5 \cdot 10^{-5}$ and $\lambda_u = 10^{-3}$. The weighting matrix on the change in input signal is again

Fig. 9. Desired output position as a function of time.

Fig. 10. Required energy and rms value of the positioning error as a function of the iteration number.

	e_{max} (μm)	e_{rms} (μm)	$\ u\ _2^2$ (A ²)
$\lambda_u = 0$	2.0	0.8	554
$\lambda_u = 5 \cdot 10^{-5}$	2.1	0.8	517
$\lambda_u = 10^{-3}$	215	30	498

TABLE II

COMPARISON OF THE SQUARED ℓ_2 -NORM OF THE ACTUATOR INPUT SIGNAL, AND THE RMS VALUE AND PEAK VALUE OF THE TRACKING ERROR AFTER CONVERGENCE FOR THREE DIFFERENT VALUES OF λ_u .

chosen as $W_{\delta u} = 2 \cdot 10^{-5} I_N$. As the value of λ_u increases more weight is given to the ℓ_2 -norm of the actuator input signal resulting in a lower input effort at the cost of reduced positioning accuracy.

Table II compares the squared ℓ_2 -norm of the actuator input signal, and the rms and peak value of the tracking error after convergence for the considered values of λ_u . The reference signal for $\lambda_u = 5 \cdot 10^{-5}$ yields 7% less input effort than the reference signal for $\lambda_u = 0$, whereas the positioning accuracy is similar for these two experiments. This is due to the limited repeatability of the linear motor test setup. A further increase of the trade-off parameter λ_u will eventually result in a lower input effort at the cost of reduced positioning accuracy. This is shown by the third experiment, where $\lambda_u = 10^{-3}$.

V. CONCLUSION

This paper investigates the possibility of applying norm-optimal ILC to a system without having any a priori information aside from the fact that the system is LTI. A method to estimate an LTI system's impulse response using previous iterations' input/output data is presented. Together with some extensions of the norm-optimal ILC framework, this constitutes a data-driven ILC framework for LTI systems with actuator limitations that is applicable to both tracking and point-to-point motion problems.

The presented approach is experimentally validated on a linear motor positioning system. The results show the ability of the proposed data-driven ILC framework to (i) achieve tracking accuracy up to the repeatability of the test setup, (ii) minimize the rms value of the tracking error while respecting

the actuator input constraints, (iii) learn energy-optimal system inputs for point-to-point motions.

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