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PMA

A hybrid Finite Element - Wave Based Method for coupled acoustic-poroelastic applications

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THEORY

1 Introduction

The importance of noise control has grown a lot over the last years. Not only have the legal regulations tightened, the sound of a product has also become a commercially differentiating factor. Next to that evolution, the attention for fuel efficiency has strongly increased the use of lightweight constructions, which, however, generally leads to lower noise and vibration absorption and insulation. To counter this evolution, trim materials (such as poroelastic foams) are more and more used to have a good NVH performance in a wide frequency range.

In order to reduce and optimise the duration of the design cycle and to reduce the number of (expensive) physical prototypes, Computer Aided Engineering techniques are indispensable in a modern product cycle. In order to develop these virtual prototypes, efficient and accurate numerical prediction techniques are required. Therefore, this poster focuses on an efficient technique to assess the impact of poroelastic foams on the acoustical performance of an acoustic cavity.

2 Conventional prediction techniques

2.1 Finite Element Method and Boundary Element Method

Element based methods (FEM/BEM) require the discretisation of (the boundary of) the system into a large number of small elements. Within each element, simple (mostly polynomial) basis functions provide an approximation for the solution field distribution. However, these functions do not satisfy the dynamic equations exactly, such that a large number of elements is required to account for the high spatial variation of physical quantities, especially at high frequencies. This leads to a practical frequency limit above which these methods are no longer applicable at a reasonable computational cost.

2.2 Ray Tracing Method

If the modal overlap in the system is high enough, concepts as phase lose their physical importance, and acoustic propagation can be considered as a ray, rather than a wave. The assumptions of the Ray Tracing Method are, however, only valid starting from a certain frequency.

2.3 The mid-frequency gap

In between the application range of the element based methods (low-frequency) and the Ray Tracing Method (high-frequency), there exists a mid-frequency gap for which no mature technique exists.

2.4 Trimmed vibro-acoustics: a further restriction

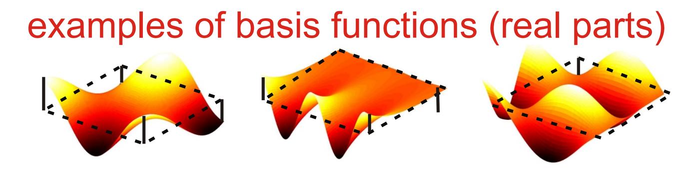
In mid-frequency trimmed vibro-acoustics, the deterministic behaviour of the trim component is very crucial, and cannot yet be modelled in a Ray Tracing context. However, also the application of element based methods to solve trimmed vibro-acoustic models is unfeasable since the computational load will drastically increase as compared to the untrimmed case. Not only will the frequency-dependent material properties impede modal reduction schemes, their near-field behaviour will also require further refinement of the structural and acoustical model to capture the very localised effects. This way, although the method can provide nice results on a trim component level, the full system mid-frequency modelling of vibro-acoustic cavities stays out of reach.

3 A new numerical prediction technique

3.1 The Wave Based Method for acoustics

The Wave Based Method (WBM) is a deterministic method based on an indirect Trefftz approach. In contrast to the FEM, the WBM divides the problem domain in a few large convex subdomains. Within each subdomain the dynamic field variables are expanded using functions which exactly satisfy the governing differential equations. As a result, it is sufficient to minimise the approximation errors on the subdomain boundaries and interfaces. This approach leads to substantially smaller system matrices as compared to the FEM. The smaller system matrices result in a higher computational efficiency, which makes the WBM capable to predict the dynamic response up to the mid-frequency region. Moreover, secondary variables such as velocities, which are important for the accurate acoustic-poroelastic coupling, can be approximated without any loss of accuracy by applying the appropriate differential operators to the wave function set.

examples of basis functions (real parts)



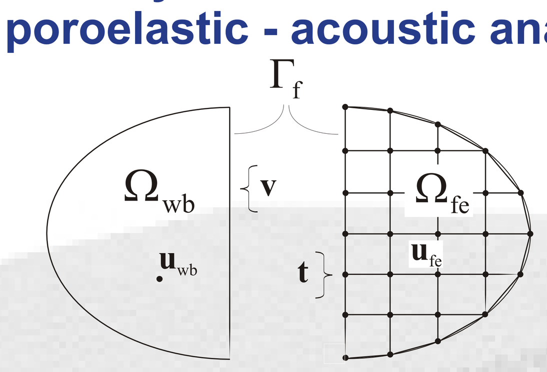
approximation: $p(r) = \sum_{i=1}^{n_i} {}_i(r).p_i + p_q(r)$

boundary errors to zero in an integral sense:

→ $[\mathbf{A}_{aa} \quad)].\{p_{wb}\}=\{f \quad)\}$

↖ *unknown wave function contributions*

3.2 A hybrid Finite Element - Wave Based Approach for coupled poroelastic - acoustic analysis



In order to relax the geometrical (convexity) limitations of the WBM, on the one side, and to relax the problems with full-system simulation of a trimmed cavity, a hybrid FE-WB methodology has been developed for the coupled analysis of poroelastics-acoustics. This hybrid approach benefits from the high computational efficiency of the WBM while retaining the geometrical flexibility of the FEM for modelling the often very thin (multi-)layers.

Two approaches can be followed herein, based on the two most commonly used formulations for the Biot theory, being the (u,U)- and the (u,p)-formulation.

(u,p)

$$\begin{bmatrix} A_{aa}+C_{aa} & 0 & C_{fa} \\ C_{as} & Z_{ss} & Z_{sf} \\ C_{af} & Z_{fs}+Z_{as} & Z_{ff}+Z_{af} \end{bmatrix} \begin{Bmatrix} p_{wb} \\ u_{fe} \\ p_{fe} \end{Bmatrix} = \begin{Bmatrix} f_a+f_{aa} \\ f_s+f_{as} \\ f_f+f_{af} \end{Bmatrix}$$

(u,U)

$$\begin{bmatrix} A_{aa}+C_{aa} & C_{sa} & C_{fa} \\ C_{as} & Z_{ss} & Z_{sf} \\ C_{af} & Z_{fs} & Z_{ff} \end{bmatrix} \begin{Bmatrix} p_{wb} \\ u_{fe} \\ U_{fe} \end{Bmatrix} = \begin{Bmatrix} f_a+f_{aa} \\ f_s+f_{as} \\ f_f+f_{af} \end{Bmatrix}$$

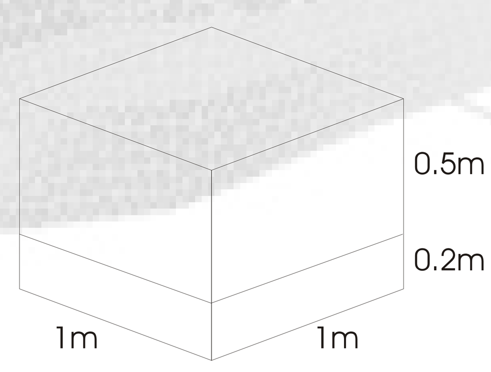
with $p_{wb}, u_{fe}, p_{fe}, U_{fe}$ the degrees of freedom, $Z_{ss}, Z_{sf}, Z_{fs}, Z_{ff}$ and A_{aa} the system matrices, f the RHS vectors. The other matrices are coupling matrices.

APPLICATIONS

4 Examples

The potential of the novel hybrid coupling techniques between the poroelastic FEM and the acoustic WBM is demonstrated in the following example.

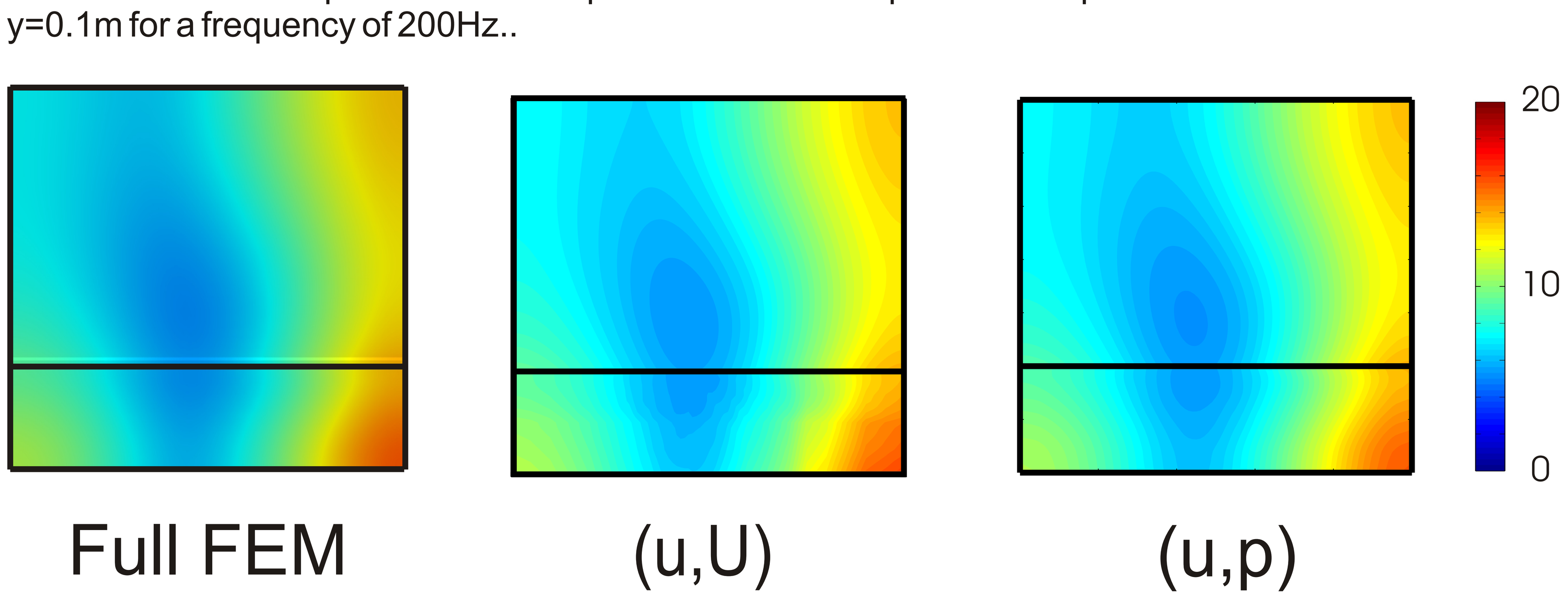
This example consists of an acoustic cavity (1m x1m x 0.7m). In this cavity a polyurethane foam (1m x 1m x 0.2m) is placed at the bottom. The acoustic domain is modelled using the WBM, the poroelastic domain using the FEM, both with a (u,U)- and with a (u,p)-formulation. An acoustic volume source is located in the acoustic domain at coordinates (0.75,0.25,0.3). The source has an amplitude q=1m³/s.



The hybrid FE-WBM for both formulations is compared with pure FE calculations. Both a first visual comparison on a contour level and a convergence study relative to a very fine FEM discretisaton are considered.

4.1 Contours

The figures below show contour plots for the full (u,p)-FEM, the (u,U)- and the (u,p)-based hybrid FE-WBM of the acoustic pressure and the pressure in the fluid phase of the poroelastic material for a slice at y=0.1m for a frequency of 200Hz..



Full FEM (u,U) (u,p)

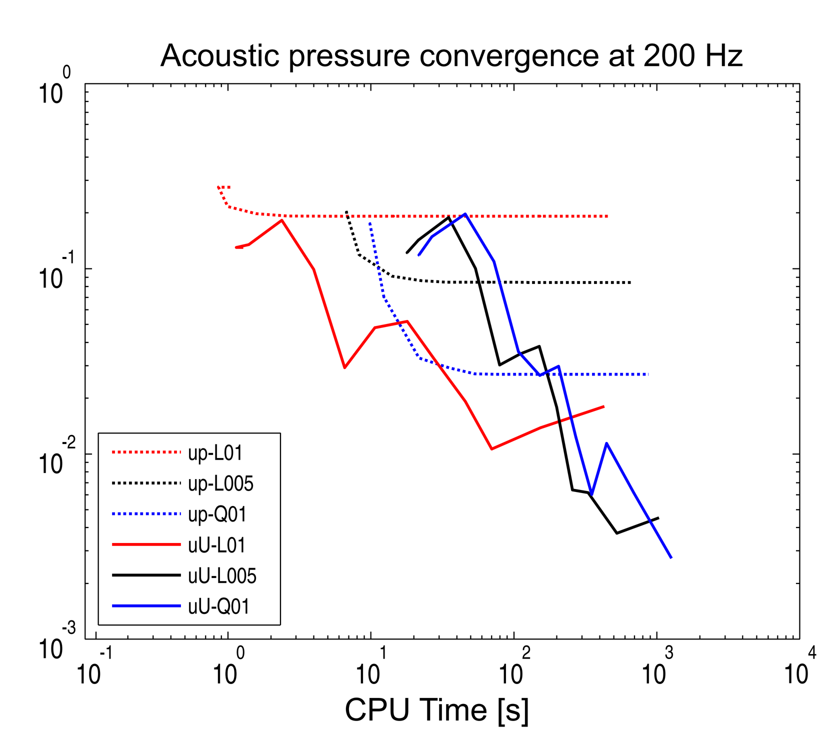
When comparing the contours of the hybrid FE-WBM to those of a purely FEM calculation, a very nice agreement can be seen between the different methods.

4.2 Convergence

One of the goals of this hybrid FE-WB methodology, is to introduce accurate damping models in the efficient acoustic WBM environment. This acoustic pressure is considered in the convergence study. The relative error of the response as compared to a very fine (pure) FE model is calculated in 256 uniformly distributed points and then averaged.

Six different hybrid models are studied, using three different meshes: linear with element sizes of h=0.1m and h=0.05m (L01 and L005) and quadratic with h=0.1m (Q01). On these meshes, both the (u,U)- and the (u,p)-formulation are applied. Subsequent model refinements are performed by adding more functions wave functions to the set.

Although the (u,U)-formulation requires a higher computational effort, these hybrid models clearly converge faster than the models based on the (u,p)- formulation. This is due to the Z_{af} coupling term in the (u,p)-based hybrid FE-WBM. This term contains a derivative of the FEM shape functions, which causes the (u,p)-meshes to lose one order of accuracy. Moreover, because of the low accuracy of the mesh, the convergence curves level out on the accuracy of the FEM part of the hybrid model.



It is however expected that for more refined models, there will be a tradeoff point between the computational efficiency and the loss of accuracy due to the gradient effect for the (u,p)-based hybrid FE-WBM. From this point on, the (u,p)-hybrid FE-WBM will outperform the (u,U)-based approach.

5 Future research topics

Wave function subselection for faster convergence
Multilayer materials and full trim modelling
Hybrid TMM-WBM approaches

References

www.mech.kuleuven.be/mod/wbm/

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