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Multi-objective optimal control of an air-to-water heat pump for residential heating

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Abstract

The current study investigates the optimal operation of an air-to-water heat pump system. To this end, the control problem is formulated as a classic optimal control or dynamic optimization problem. As conflicting objectives arise, namely minimizing energy cost while maximizing thermal comfort, the optimization problem is tackled from a multi-objective optimization perspective. The adopted system model incorporates the building dynamics and the heat pump characteristics. Because of the state-dependency of the coefficient of performance (COP), the optimal control problem (OCP) is nonlinear. If the COP is approximated by a constant value, the OCP becomes convex which is easier to solve. The current study investigates how this approximation affects the control performance. The optimal control problems are solved using the freely available Automatic Control And Dynamic Optimization toolkit ACADO (Houska et al. 2011). It is found that the lower the weighting factor for thermal discomfort is, the higher the discrepancy is between the nonlinear and convex OCP formulations. For a weighting factor resulting in a quadratic mean difference of 0.5°C between the zone temperature and its reference temperature, the difference in electricity cost amounts to 4% for a first scenario with fixed electricity price, and up to 6% for a second scenario with a day and night variation in electricity price.

1 Introduction

Heat pumps are devices that extract heat from the environment and deliver this heat at a higher temperature at supply side. The efficiency of a heat pump is characterized by its coefficient of performance (COP) which is amongst others a function of both the source and the supply temperature. The smaller the temperature difference to be bridged, the higher the COP. Heat pumps are therefore ideally suited for combination with low temperature heat emission systems such as floor heating. At the source side, the COP of an air-source heat pump can be increased by extracting heat from the ventilation exhaust air (e.g. Sakellari and Lundqvist 2005), or even further, by integrating solar air collectors to preheat the air (e.g. Badescu 2002a,b, 2003). If the ambient air is directly used without any form of preheating, the ambient temperature will directly influence the COP. The question arises when it is optimal to operate an air-source heat pump given the diurnal variation of the ambient air temperature. The answer is not straightforward,

as a trade-off is expected between operating at times with higher ambient temperature (and thus higher COP) versus operating at times with lower ambient temperature (and thus higher building losses). Moreover, when energy costs are envisaged, the day-night variation of the electricity price will influence the answer. These questions motivate the research on heat pump control in a system's perspective.

Current heat pump systems are controlled by a heating curve which determines the set point for the supply or return water temperature as a function of the ambient temperature. This heating curve is tuned such that in stationary conditions the heat loss of the building is exactly compensated. This control strategy is suitable for fast reacting systems such as high temperature radiators in buildings with a small heat capacity. For heat pumps combined with a low-temperature heating system, e.g., floor heating, this control is not suited because of the high thermal inertia of the heat emission system. These slowly reacting systems require a controller with knowledge of the system dynamics and the future heating demand in order to satisfy the thermal comfort requirements. An optimal control strategy can satisfy the thermal comfort requirements based on knowledge of the system dynamics and the future weather prediction. Moreover, if the COP of the heat pump and the time dependency of the electricity price are taken into account, the degrees of freedom can be used to satisfy the thermal comfort requirements at the lowest energy cost.

Research on optimal control of low-temperature heat emission systems such as floor heating and concrete core activation started about two decades ago and revealed growing research interest (e.g. Chen 2002, Cho and Zaheer-Uddin 2003, Gruber et al. 2000, Gwerder and Tödtli 2005, Karlsson and Hagentoft 2011, Kummert and André 2005, Zaheer-Uddin et al. 1997). In most cases, the focus is on the building level and the heat production efficiency is assumed to be fixed. This assumption holds for conventional boilers on gas or heating oil, but not for heat pumps. The earliest investigations on optimal control of heat pump systems, on the other hand, take the temperature dependency of the heat pump COP into account, but neglect the building dynamics (Rink et al. 1988, Zaheer-Uddin 1992, Zaheer-Uddin and Fazio 1988, Zaheer-Uddin et al. 1988). More recently, the optimal control of heat pump systems with floor heating was studied from an integrated system point of view, i.e., taking both the heat pump COP and the building dynamics into account (Bianchi 2006, Wimmer 2005). In these works, on-off heat pump operation is considered. The optimization takes the dependency of the COP on the ambient air temperature into account, but not the dependency on the supply water temperature. For modulating heat pumps, however, Karlsson and Fahlén (2008) emphasize the impact of the supply water temperature on the COP in the comparison of different control strategies with respect to energy cost. Therefore, the question arises how sensitive the optimal control solution is with respect to the COP model used in the optimization.

The current paper investigates the optimal control problem (OCP) for a modulating air-to-water heat pump in a residential building, which minimizes the weighted sum of the predicted electricity cost and thermal discomfort, by taking into account:

- the system dynamics of the heat pump and the building,
- the predicted outside temperature,
- the COP dependency on the outside temperature and the supply water temperature,
- the electricity price.

The OCP is nonlinear due to the division of two optimization variables in the objective function, namely the heat pump thermal power and the state dependent COP. A concise overview of the existing methods for solving nonlinear OCP, illustrated for chemical processes, is given by

(Srinivasan et al. 2003). The methods can be classified into two broad categories, namely the direct and the indirect methods. Indirect methods yield an analytical, closed-loop formulation for the control input (Kirk 1998). The analytical expression allows to gain insight into the factors determining the optimal control strategy, i.e. which parameters and variables influence the control trajectory and how. Indirect methods have been applied in various fields of optimal control of dynamic processes, e.g. minimal fuel consumption of car engines (Saerens et al. 2010), chemical processes (Logist et al. 2009) and heat exchanger operation (Badescu 2004). Rink et al. (1988) used the indirect method to analyse the optimal control of a heat pump connected to a storage tank for space heating. However, as this method is restricted to problems with only a few states, the storage tank was represented by a single temperature and a static building model was used. Contrary to indirect methods, direct methods allow to solve more complex nonlinear problems. Still, these nonlinear solvers have higher, and thus more expensive, hardware requirements compared to convex solvers which can be embedded in standard software.

As the nonlinearity arises from the COP entering the objective function, a convex formulation is obtained if the efficiency of the heat production is assumed to be constant. The resulting objective function is then equivalent to minimizing the weighted sum of the delivered thermal energy and the thermal discomfort. This paper compares the electricity cost and thermal discomfort level obtained with the nonlinear OCP which incorporates the heat pump characteristics, to the convex OCP which makes abstraction of the heat production system. This way, the added value of optimizing the heat pump system from a systems perspective (i.e., taking both building dynamics and heat production efficiency into account) versus optimizing the heat production at the building level alone, is quantified.

The control task is tackled from a multi-objective perspective. The benefit of this approach is twofold. First, it allows to consider only trade-off or so-called Pareto-optimal solutions, i.e., solutions which satisfy the user defined thermal comfort level with the lowest energy cost (or heating demand in the convex OCP case). Second, it is expected that the lower the required comfort level, the higher the flexibility to shift heat pump operation towards time periods with high ambient air temperatures or low electricity prices. As a consequence, the energy savings potential of system level optimization compared to building level optimization is also expected to depend on the required thermal comfort level. By means of the multi-objective approach this sensitivity of the savings potential on the required thermal comfort level can be easily assessed. The second section describes the formulation of the nonlinear OCP. The third and fourth sections analyze and compare the optimal heat pump operation resulting from the nonlinear and the convex OCP. The third section studies the scenario with a constant electricity price, whereas the fourth section deals with the scenario with a day-night pricing scheme. Both the nonlinear and convex OCPs are formulated and solved in the C++ toolkit Automatic Control and Dynamic Optimization (ACADO) (Houska et al. 2009, 2011). ACADO provides a general framework for using a large variety of algorithms for direct optimal control, including model predictive control, state and parameter estimation and robust optimization.

2 Formulation of the nonlinear optimal control problem

Optimal control deals with problems in which a time variable control profile is sought for a dynamic system such that a certain optimality criterion is met. To this end, an optimization problem is solved, which comprises (i) the definition of the objective function, (ii) the system dynamics, (iii) the state and control constraints and (iv) initial and/or boundary conditions. For the investigated case of a heat pump connected to a residential floor heating system, schematically represented in Figure 1, the control objectives are minimizing thermal discomfort and energy cost. Evaluation of these two criteria requires a building model which expresses the

Figure 1: Schematic representation of the system comprising the air-coupled heat pump, the floor heating system and the controlled building zone, with indication of the building model states and inputs.

relation between the zone temperature and the heating power on the one hand, and a heat pump model which relates the heating power to the electricity consumption, on the other hand. These models should be as simple as possible, while capturing all control relevant information. This section starts with the description of the building model and heat pump model used in this study, followed by the definition of the objective function, to end with the state and input constraints.

2.1 Dynamic building model

The work of Bianchi (2006) shows that a simple second order model is suited for the purpose of optimal control of a residential floor heating system. The two model states are the return water temperature T_{wr} [°C] and the zone temperature T_z [°C], both indicated in Figure 1. The inputs to the model are the heat pump thermal power $\dot{Q}_{hp}$ [W] and the ambient air temperature T_{amb} [°C]. In this study the supply water temperature, denoted by T_{ws} , is added as extra state to account for the dynamics of the hydraulic circuit at heat pump side. As also indicated by the study of Karlsson and Fahlén (2008), this is required to correctly assess the coefficient of performance (COP) of the heat pump (see Section 2.2). The resulting model equations, represented in state space formulation, are given by Eq.(1).

$$\begin{bmatrix} \dot{T}_{w,s} \\ \dot{T}_{w,r} \\ \dot{T}_z \end{bmatrix} = \begin{bmatrix} -\frac{\dot{m}_w c_w}{C_{w,s}} & \frac{\dot{m}_w c_w}{C_{w,s}} & 0 \\ \frac{\dot{m}_w c_w}{C_{w,r}} & -\frac{\dot{m}_w c_w + \kappa_{wz}}{C_{w,r}} & \frac{\kappa_{wz}}{C_{w,r}} \\ 0 & \frac{\kappa_{wz}}{\kappa_b \tau_b} & -\frac{\kappa_{wz} + \kappa_b}{\kappa_b \tau_b} \end{bmatrix} \begin{bmatrix} T_{ws} \\ T_{wr} \\ T_z \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{C_{w,s}} \\ \frac{1}{\tau_b} & 0 \end{bmatrix} \begin{bmatrix} T_{amb} \\ \dot{Q}_{hp} \end{bmatrix} \quad (1)$$

The different parameters represent the following: $\dot{m}_w$ [$\frac{\text{kg}}{\text{s}}$] is the circulating mass flow of the water circuit, c_w [$\frac{\text{J}}{\text{kgK}}$] is the specific heat capacity of water, $C_{w,s}$ [$\frac{\text{J}}{\text{K}}$] is the thermal capacity of the volume of water at temperature T_{ws} , $C_{w,r}$ [$\frac{\text{J}}{\text{K}}$] is the thermal capacity of the volume of water at temperature T_{wr} , κ_{wz} [$\frac{\text{W}}{\text{K}}$] is the thermal conductivity between water circuit and house, κ_b [$\frac{\text{W}}{\text{K}}$] is the thermal conductivity between house and surroundings, τ_b [h] is the time constant of the building and T_{amb} [°C] is the ambient air temperature. The values for the model parameters are given in Table 1.

Table 1: Building model parameters

$\dot{m}_w$ ($\frac{\text{kg}}{\text{s}}$)	c_w ($\frac{\text{J}}{\text{kgK}}$)	$C_{w,s}$ ($\frac{\text{J}}{\text{K}}$)	$C_{w,r}$ ($\frac{\text{J}}{\text{K}}$)	$\kappa_{w,z}$ ($\frac{\text{W}}{\text{K}}$)	κ_b ($\frac{\text{W}}{\text{K}}$)	τ_b (h)
0.663	4185	2.006e5	4.185e6	1100	260	240

Figure 2: Schematic representation of an air-to-water heat pump.

2.2 Heat pump model

Figure 2 gives a simplified representation of the operation of an air-to-water heat pump. The compressor drives a thermodynamic cycle which extracts heat $\dot{Q}_{amb}$ from the ambient air at evaporator side and supplies the heat $\dot{Q}_{hp}$ to the building at condensor side. The first law of thermodynamics yields following relation between the thermal power delivered to the building $\dot{Q}_{hp}$, the thermal power extracted at evaporator $\dot{Q}_{amb}$ and the compressor power P_{hp} :

$$\dot{Q}_{hp} = \dot{Q}_{amb} + P_{hp} \quad (2)$$

The heat pump coefficient of performance (COP) is defined as the ratio between the delivered thermal power $\dot{Q}_{hp}$ and the required compressor power P_{hp} :

$$COP = \frac{\dot{Q}_{hp}}{P_{hp}} \quad (3)$$

The second law of thermodynamics sets an upper bound for the COP, which is referred to as the COP_{Carnot} . This upper bound solely depends on the heat source temperature T_{source} and the heat supply temperature T_{supply} , both expressed in Kelvin.

$$COP_{Carnot} = \frac{T_{supply}}{T_{supply} - T_{source}} \quad (4)$$

The COP of a real heat pump is lower due to the irreversibilities in the compressor, the heat exchangers and the expansion valve. The actual COP can be estimated by means of a detailed first-principles heat pump model (see e.g., Badescu 2002b, Jin and Spitler 2003). These models are however highly nonlinear, require a large number of state variables and are thus too complex for incorporation in an optimal control formulation. An alternative is to represent the heat pump characteristics by polynomial fits based on experimental data (see e.g., Gayeski 2010, Shao et al. 2004). This approach has been adopted here. The temperature dependency of the COP is described by a quadratic fit in T_{amb} and T_{ws} as indicated by Eq.(5). The coefficients,

Figure 3: Catalogue data for the COP of a modulating air-to-water heat pump as a function of the source temperature T_{amb} and the supply temperature T_{ws} at maximal compressor speed (Daikin 2006). The quadratic fit is presented in dashed lines.

listed in Table 2, are derived from catalogue data of a residential air-to-water heat pump (Daikin 2006). The difference between the catalogue data and the quadratic fit, both depicted in Figure 3, is maximally 3%. Note that the catalogue only tabulates performance data at maximal compressor speed. The dependency of the COP on the compressor speed, which influences the irreversibilities in the compressor and heat exchangers, is not included in the fit.

$$COP(T_{amb}, T_{ws}) = COP_0 + \alpha_1 T_{amb} + \alpha_2 T_{ws} + \alpha_3 T_{amb}^2 + \alpha_4 T_{ws}^2 + \alpha_5 T_{amb} T_{ws}. \quad (5)$$

Table 2: Heat pump model coefficients

COP_0 (-)	α_1 (°C ⁻¹)	α_2 (°C ⁻¹)	α_3 (°C ⁻²)	α_4 (°C ⁻²)	α_5 (°C ⁻²)
8.24	1.58e-1	-1.95e-1	1.01e-3	1.48e-3	-2.33e-3

Substituting Eq.(5) into Eq.(3), yields following expression for the compressor power P_{hp} required to deliver a certain thermal power $\dot{Q}_{hp}$ to the building:

$$P_{hp} = \frac{\dot{Q}_{hp}}{COP(T_{amb}, T_{ws})} \quad (6)$$

2.3 Objective function

The aim is to find the $\dot{Q}_{hp}$ -profile which optimizes the operation with respect to both thermal comfort and energy cost. The objective function is defined as a weighted sum of these conflicting objectives:

$$\min_{\dot{Q}_{hp}(t)} (1 - K) \int_0^{t_{end}} c_{el}(t) \left(\frac{\dot{Q}_{hp}(t)}{COP(T_{amb}(t), T_{ws}(t))} \right) dt + K \int_0^{t_{end}} (T_z(t) - T_{z,ref})^2 dt \quad (7)$$

The first term in Eq. (7) represents the predicted electricity cost, with the factor c_{el} [$\frac{\text{€}}{\text{kWh}}$] representing the time varying electricity price and $\dot{Q}_{hp}$ [W] the thermal heat pump power. The COP

takes the temperature dependency of both the ambient air and the supply water into account. The second term in Eq. (7) represents the thermal discomfort which is calculated as the sum of squared differences between the zone temperature T_z [°C] and the reference zone temperature $T_{z,ref}$ [°C]. The latter is assumed constant in this study. K [-] is a weighting factor between 0 and 1 that determines the relative weight of each of the objectives. For $K = 1$, the optimal control profile minimizes the zone temperature deviation from the reference temperature, regardless of the corresponding electricity consumption. For $K = 0$, the heat pump operation minimizes the electricity cost under the given boundary conditions and state and input constraints, without tracking the zone reference temperature.

2.4 Periodic boundary conditions

This research focuses on the dominant phenomena on a 24h time scale. To study the impact of the diurnal variation of T_{amb} and c_{el} on the optimal $\dot{Q}_{hp}$ -profile, the optimization is performed for a 24 hour time interval. Inspired by this daily periodicity, free periodic boundary conditions are imposed on the state variables, with t_0 [h] and t_{end} [h], 0 a.m. and 12 p.m. respectively;

$$T_{ws}(t_0) = T_{ws}(t_{end}) \quad (8)$$

$$T_{wr}(t_0) = T_{wr}(t_{end}) \quad (9)$$

$$T_z(t_0) = T_z(t_{end}) \quad (10)$$

The initial and final states are imposed to be equal and their specific value is determined by the optimization. If these constraints were relaxed and replaced by initial conditions only, the optimal solution would deplete the energy stored in the building thermal mass. The choice for these periodic boundary condition eliminates this boundary effect. This allows to limit the time interval to 24 hours even if the dominant time constant of the building is substantially larger. The choice for t_0 (and thus t_{end}) is in theory arbitrary. In practice, it makes sense to chose this time point when the day-by-day variations are expected to be smallest. As the main disturbances arise from solar gains and internal gains, a proper choice is a time point during the night. Moreover, from computational point of view it is interesting to define the periodic boundary condition far away from the time intervals with large transient behavior. Therefore it is safer to locate t_0 rather at the beginning of the night than at the start of the day. As such this approach allows studying the trends that an optimal heat pump controller exhibits for several scenarios, e.g., a typical cold or a typical mild winter day. For the implementation in an online framework such as Model Predictive Control, the control profile found can be used as a tracking reference.

2.5 Input constraints

As the considered heat pump is of the modulating type, the thermal power $\dot{Q}_{hp}$ can be modulated to match the building load. This is done by regulating the speed of the compressor through a frequency inverter. $\dot{Q}_{hp}$ is limited by the maximum compressor power $P_{hp,max}$ [W]. Note that the maximum for $\dot{Q}_{hp}$ depends on the COP and thus on the T_{amb} and T_{ws} .

$$0 \leq \frac{\dot{Q}_{hp}(t)}{COP(T_{amb}(t), T_{ws}(t))} \leq P_{hp,max} \quad (11)$$

Eq.(11) is based on the assumption that the compressor power P_{hp} is able to modulate continuously between zero and $P_{hp,max}$. In practice, the heat pump has to switch off below a given

threshold as the compressor losses become too large. This constraint could be taken into account in a post-processing step, e.g., by applying pulse width modulation if the proposed compressor power is below this threshold.

2.6 Discretization of the optimal control problem

The continuous optimal control problem has to be discretized for a direct numerical solver. The control variable is discretized using a piece-wise constant function, i.e. $\dot{Q}_{hp}$ is constant during one discretization time step. The smaller the discretization time step, the smaller the difference between the continuous and the discretized optimal control profiles. Small discretization time steps, however, involve more control variables to be optimized over the given time horizon, which significantly increases the computation time. The formulations discussed in this paper adopt a discretization time step of 30 minutes, which is found to be sufficiently small to approximate the continuous optimal control profile. Given the time horizon of 24 hours, this yields 48 control variables to be optimized. The freely available Automatic Control And Dynamic Optimization toolkit ACADO (Houska et al. 2011), used for this study, discretizes the OCP with the *multiple shooting* method developed by Bock and Plitt (1984). While the control variables are discretized with a fixed discretization time step, i.e., 30 minutes, the state variables are discretized with a variable time step such that the discretization error remains below a user-defined value. For this purpose, a standard *Runge -Kutta 45* integrator is used. The resulting optimization problem is solved with a Sequential Quadratic Programming approach using an *exact Hessian* approximation (Nocedal and Wright 2000).

3 Nonlinear versus convex OCP given a fixed electricity price

The control strategy obtained by solving the OCP with the nonlinear objective function, given by Eq. (7), is compared to the control strategy obtained by solving the OCP with following convex objective function, which minimizes the weighted sum of thermal energy and thermal discomfort.

$$(1 - K) \int_0^{t_{end}} \dot{Q}_{hp}(t) dt + K \int_0^{t_{end}} (T_z(t) - T_{z,ref})^2 dt. \quad (12)$$

In the case of a constant electricity price, the energy cost term in the objective function of the nonlinear OCP, Eq. (7), is proportional to the electricity consumption. This scenario therefore enables to assess the added value of minimizing electricity consumption versus minimizing thermal energy. Analogously, the energy cost term in Eq. (12) is proportional to the electricity consumption in the case of a constant COP. In other words, this scenario also enables to assess the added value of taking the temperature dependency of the COP into account.

3.1 Scenario

The simulations are run at the following scenario settings: (i) free periodic boundary conditions on the three state variables, (ii) a time horizon of 24 hours, (iii) a control time step of 30 minutes and (iv) a fixed desired zone temperature $T_{z,ref}$ of 20°C. The electricity price is constant and the ambient air temperature varies sinusoidally around 0°C with an amplitude of 5°C, as shown in Figure 4. The sinusoidal profile is assumed to represent the major trend of a *typical* diurnal variation of the ambient air temperature. The presented methodology is however not restricted to periodical profiles and can be used for any ambient air temperature profile.

Figure 4: The predicted ambient air temperature for the first scenario.

3.2 Optimal control profiles

Figure 5 shows the optimal profiles of the control variables for three values of the weighting factor K . At $K = 1.0$ the profiles obtained with the nonlinear OCP and the convex OCP are equal, since the desired temperature $T_{z,ref}$ is tracked as well as possible with the given system and for the given boundary conditions. The optimal controller tries to exactly compensate the instantaneous heat loss to the environment by the heat emitted by the floor heating. Maximum power is delivered at around 3 a.m., while the ambient air temperature reaches a minimum at 4 a.m. The time difference of approximately one hour compensates for the thermal inertia of the heating system. At $K = 0.5$ the optimal profiles obtained with the nonlinear and the convex OCP formulations differ significantly. With the nonlinear formulation, more heat is delivered when the ambient air temperature is high. This is beneficial for the COP at which the heat is produced. On the other hand it inevitably causes the zone temperature to drop when the ambient air temperature is low and to rise when the ambient air temperature is high. The optimal profile obtained with the convex formulation for $K = 0.5$ hardly differs from the optimal profile at $K = 1.0$. The heating power of the heat pump is determined such that the temperature difference between zone temperature and desired zone temperature is constant. At $K = 0.005$ the weight of the thermal discomfort term in the objective function is very small. The total amount of heating power is significantly reduced for both formulations at the expense of thermal comfort.

3.3 Pareto fronts

The optimality of multi-objective solutions can be examined by a Pareto front, which visualizes the value of the objectives as a function of the weighting factor. In this case the Pareto front depicts the electricity consumption and thermal discomfort for different weighting factors K . Figure 6 shows the Pareto fronts obtained with the nonlinear and convex OCP. The Pareto front for the nonlinear OCP lies closer to the origin than the one of the convex OCP. This means that solutions obtained with the convex OCP can be improved in both objectives: thermal discomfort and consumed electricity.

Figure 7 zooms into the Pareto fronts for a weighting factor between $K = 0.075$ and $K = 1.0$. The electricity consumption with the nonlinear OCP formulation is lower than for the convex formulation, even for very small levels of thermal discomfort. These savings result from an optimal timing of the heating power and not from a decrease in delivered heat. With the convex OCP formulation, a reduction of the electricity consumption results in a significantly higher level of thermal discomfort.

Figure 5: The optimal heat pump thermal power profiles resulting from the nonlinear OCP formulation (left) and the convex OCP formulation (right), for three different weighting factors ($K = 1, 0.5, 0.005$) between energy cost and thermal discomfort for the case with fixed electricity price.

3.4 Quantification of the difference between nonlinear and convex OCP

The added value of the nonlinear OCP formulation versus the convex one, is evaluated by comparing solutions with the same level of thermal discomfort. Table 3 tabulates for different values of the weighting factor K , the value for the thermal discomfort, given by the second term in Equations (7) and (12), and the corresponding quadratic mean difference between the zone temperature and the reference temperature ΔT_{av} [$^{\circ}\text{C}$], which is calculated as:

$$\Delta T_{av} = \sqrt{\frac{\left(\int_0^{t_{end}} (T_z - T_{z,ref})^2 dt\right)}{t_{end}}}. \quad (13)$$

The last column of Table 3 gives the electricity consumption reduction achieved by the nonlinear OCP relative to the convex OCP, as a function of the weighting factor K . The relative reduction increases as the relative weight of the thermal discomfort term decreases. This is logic as the less stringent the comfort requirements, the more the nonlinear OCP will see the opportunity to shift the heat production to the time of day with high source temperatures. A high relative weight of the thermal discomfort term, on the contrary, allows less time-shifting of heat production since delivering less heat when the ambient air temperature is low and more heat when the ambient air temperature is high, inevitably results in a higher deviation from the reference zone temperature. Thanks to the large thermal inertia of the heating system, however, electricity consumption reduction is possible even if only small zone temperature deviations are allowed. For the investigated scenario and a weighting factor resulting in a quadratic mean temperature difference ΔT_{av} of 0.5°C , the nonlinear OCP formulation yields a 4% electricity reduction compared to the convex one. This illustrates the added value of a nonlinear OCP formulation and more specifically the importance of taking the temperature dependency of the COP into account in the objective function. Note that the possible reduction strongly depends on the amplitude of

Figure 6: The Pareto fronts of the nonlinear and convex OCP for the case with fixed electricity price. Values for weighting factors: $K = 0.005, 0.0075, 0.01, 0.015, 0.02, 0.035, 0.05, 0.075, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$ and $K = 1.0$.

Figure 7: The Pareto fronts of the nonlinear and convex OCP for the case with fixed electricity price. Values for weighting factors: $K = 0.075, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$ and $K = 1.0$.

the ambient air temperature during the projected time horizon. If the ambient air temperature is constant, both formulations generate exactly the same Pareto front.

4 Nonlinear versus convex OCP given a variable electricity price

In this scenario, the electricity price $c_{el}(t)$ depends on the time of day. The price profile is depicted in Figure 8. The off-peak price of 0.09€ per kWh sets in at 9 p.m. and is available until 7 a.m.. The on-peak electricity price is 0.15€ per kWh. The price profile is incorporated in the objective function of the nonlinear OCP, given by Eq. (7). The results obtained for the nonlinear OCP are compared to those obtained with the convex formulation with Eq. (12) as objective function. This scenario thus enables to study the difference between minimizing electricity cost versus minimizing thermal energy in the case of a time dependent electricity price. In other words, the added value of taking both the COP temperature dependency and the electricity price time dependency into account, is evaluated.

4.1 Scenario

The simulations are run at the following scenario settings: (i) free periodic boundary conditions on the three differential state variables, (ii) a time horizon of 24 hours, (iii) a control time step

K [—]	Therm. disc. [K ² h]	ΔT_{av} [°C]	Elec. cons. 1 [kWh]	Elec. cons. 2 [kWh]	Rel. red. [%]
1.0	0.0	0.0	37.20	37.20	0.0
0.9	0.04	0.004	36.20	37.06	2.32
0.8	0.10	0.065	35.88	36.99	2.99
0.6	0.21	0.094	35.62	36.89	3.44
0.4	0.46	0.14	35.37	36.73	3.69
0.2	2.12	0.30	34.77	36.21	3.81
0.1	9.49	0.63	33.65	35.12	4.17
0.05	38.2	1.26	31.62	33.06	4.35
0.01	601	5.0	21.38	22.69	5.76
0.005	1543	8.0	14.96	16.24	7.82

Table 3: Results as a function of the weighting factor K for the case with fixed electricity price; *Therm. disc* is the thermal discomfort with the nonlinear OCP for the given weighting factor K , ΔT_{av} is the corresponding quadratic mean difference between the zone temperature and the reference temperature, *Elec. cons. 1* is the associated electricity consumption with the nonlinear OCP, *Elec. cons. 2* the associated electricity consumption with the convex OCP, *Rel. red.* is the electricity consumption reduction achieved by the nonlinear OCP compared to the convex OCP.

Figure 8: Variable electricity price.

of 30 minutes and (iv) a desired zone temperature $T_{z,ref}$ of 20°C. The electricity price is variable, as shown in Figure 8, and the ambient air temperature varies sinusoidally around 10°C with an amplitude of 2°C, as depicted in Figure 9. The choice of a relatively high mean temperature and a small temperature amplitude compared to the first scenario, is made to increase the incentive for the nonlinear, electricity price dependent formulation to shift the heat production from day to night. First, in the case of a cold night, the heat pump would have to operate at almost full capacity during the night to maintain the desired room temperature, thereby leaving little room for shifting an additional fraction of the heat production from on-peak to off-peak electricity price period. Second, the higher the day-night temperature variation, the larger the COP drop at night, which counteracts the advantage of the low electricity price.

4.2 Optimal control profiles

Figure 10 shows the optimal profiles of the control variables for three values of the weighting factor K . As for the first scenario, the profiles of the nonlinear and convex OCP are equal for $K = 1.0$ since the zone temperature must remain as close as possible to the desired temperature $T_{z,ref}$. At $K = 0.5$ the profiles differ significantly. With the nonlinear OCP formulation,

Figure 9: Ambient air temperature profile for the second scenario.

significantly more heat is produced during the night at low electricity price. This will cause the zone temperature to rise during the night and to drop during the day. The temperature variation remains however relatively small thanks to the large thermal inertia of the heating system. Important to notice is that the heat production is not entirely concentrated during the night. First, the lower ambient air temperature at night negatively affects the COP. Second, if the heat pump operation is concentrated in one time interval, the supply temperature significantly rises, which also negatively affects the COP. Both effects are taken into account by the nonlinear OCP as the COP dependency on both the ambient air temperature and the supply water temperature are incorporated in the formulation. In general, the OCP formulation must explicitly take the temperature dependency of the COP into account if the objective is electricity cost minimization given a time varying electricity price. Neglecting the T_{ws} dependency of the COP while taking the variation in electricity price into account, results in an over-concentration of the heat production during off-peak period which in turn results in a lower COP and thus a higher electricity consumption than predicted. For $K = 0.005$, thermal discomfort is hardly penalized. To minimize electricity cost, first, a lower amount of heat is delivered and, second, almost no heat is produced during on-peak period.

4.3 Pareto fronts

Figure 11 shows the Pareto fronts for a weighting factor between $K = 0.075$ and $K = 1.0$. The results obtained for smaller K -values are not depicted as they give rise to a quadratic mean difference between the zone temperature and the reference temperature, ΔT_{av} , of more than 1°C which is considered as non acceptable (see Table 4). Comparison of the Pareto-fronts reveals that the nonlinear OCP formulation results in a decreased electricity cost compared to the convex formulation, even for very small levels of thermal discomfort. With the convex OCP formulation, on the contrary, a reduction of the electricity cost yields a significant increase in the thermal discomfort level.

4.4 Quantification of the difference between nonlinear and convex OCP

Table 4 tabulates the electricity cost reduction realized by the nonlinear optimal control formulation relative to the convex formulation as a function of the thermal discomfort. Analogously to the previous section, the relative reduction increases with smaller weighting factors. The results show that the nonlinear controller already offers a 6% electricity cost reduction at a ΔT_{av} smaller than 0.5°C . This illustrates the importance of minimizing the electricity cost instead of the heating energy for the case of an electricity price with a different day and night tariff, and thus the added value of a nonlinear versus a convex OCP formulation.

Figure 10: The optimal heat pump power profiles resulting from the nonlinear OCP formulation (left) and the convex OCP formulation (right), for three different weighting factors ($K = 1, 0.5, 0.005$) between energy cost and thermal discomfort for the case with variable electricity price.

Figure 11: The Pareto fronts of the nonlinear and convex OCP for the case with variable electricity price. Values for weighting factors: $K = 0.075, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$ and $K = 1.0$.

K [-]	Therm. disc. [K ² h]	ΔT_{av} [°C]	Elec. cost 1 [€]	Elec. cost 2 [€]	Rel. red. [%]
1.0	0.00	0.0	1.45	1.45	0.00
0.9	0.01	0.020	1.39	1.44	3.39
0.8	0.03	0.035	1.38	1.44	4.26
0.6	0.07	0.054	1.36	1.44	5.02
0.4	0.14	0.076	1.35	1.43	5.38
0.2	0.45	0.14	1.34	1.42	5.74
0.1	1.77	0.27	1.31	1.39	5.99
0.05	7.10	0.54	1.25	1.34	6.29
0.01	141	2.42	0.91	0.99	8.35
0.005	423	4.20	0.62	0.71	12.5

Table 4: Results as a function of the weighting factor K for the case with variable electricity price: *Therm. disc.* is the thermal discomfort with the nonlinear OCP for the given weighting factor K , ΔT_{av} is the corresponding quadratic mean difference between the zone temperature and the reference temperature, *Elec. cost 1* is the associated electricity cost with the nonlinear OCP, *Elec. cost 2* the associated electricity cost with the convex OCP, *Rel. red.* is the electricity cost reduction achieved by the nonlinear OCP compared to the convex OCP.

5 Conclusion

The contribution of this paper to the research of control of air-source heat pumps connected to floor heating systems is twofold. First, a control methodology is presented which allows the user to easily define a trade-off between thermal comfort on the one hand and energy cost on the other hand. For each choice of weighting factor between these two conflicting objectives, the heat pump operation is defined which reaches the required thermal comfort level at the lowest cost. To this end, the control task is formulated as an optimal control problem, taking into account the building dynamics, the heat pump characteristics, the diurnal variation of the ambient air temperature and the electricity price tariff-structure. The integration of both the building dynamics and the heat pump characteristics in the optimization allows to exploit the storage capacity of the floor heating system to minimize the energy cost of the heat production. The results show that, for a scenario with a constant electricity price, heat production is shifted towards the afternoon to benefit from the higher ambient air temperature - and thus higher heat pump efficiency. With a day-night electricity price tariff, on the contrary, heat pump operation is shifted towards the night to benefit from off-peak electricity price. The multi-objective approach clearly shows that the flexibility to shift the heat production is inversely correlated with the required thermal comfort level. The second contribution of this paper is the answer to the question how much can be gained from addressing the control task from a systems' perspective, i.e. including both building dynamics and heat pump characteristics in the optimization, compared to optimizing the building loads without consideration of the heat production efficiency. The motivation for this comparison lies in the fact that the former requires solving a nonlinear optimization problem, while the latter results in a convex optimization problem which is significantly easier to solve. The results indicate that, for a comfort level allowing a deviation of 0.5 °C from the zone reference temperature and a constant electricity price, optimization at system level allows energy savings up to 4% compared to optimization at the building level only. In case of a day-night electricity price tariff, the energy cost savings amount up to 6%. Future research will focus on the exploitation of more advanced multi-objective optimization techniques (e.g., Logist et al. 2010) for the efficient analysis and optimization of heat pump systems. The constructed Pareto optimal sets or trade-off charts will be a valuable tool for practitioners afterwards.

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