Vortex states inside the superconducting phase of a thin microtriangle

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Abstract. – By combining the non-linear Ginzburg-Landau equations with a gauge transformation of the vector potential that accounts for the superconducting/vacuum boundary condition, the superconducting phase of a thin microtriangle under a perpendicular magnetic field is investigated. We determine the symmetry-breaking and symmetry-switching transitions that the nucleated order parameter may undergo when decreasing the temperature well below the phase boundary. It is shown that symmetry consistent vortex-antivortex patterns are stable in a broad range of temperatures and magnetic fields. The geometry of the sample also induces crossovers between vortex states unexpected for other regular polygons.

The confinement of the superconducting condensate in microstructures with sizes of the order of the superconducting coherence length, $\xi(T)$, provides a promising path for the development of new applications of superconductors. The reason is that the superconducting properties, due to their quantum origin, not only depend on the specific characteristics of each material, but also on sample's topology [1]. This opens the possibility of a quantum design of the critical parameters of these so-called mesoscopic superconductors which, also stimulated by recent advances in micro- and nanofabrication techniques, has attracted an intense research activity in the field [2]. As a consequence, it is now well established that in these materials the normal-superconducting phase boundary, $T_c(H)$, shows a cusp-like behaviour superimposed onto a linear background with periodicity and slope depending on the geometry of the sample [1,2]. Each of these cusps corresponds to an additional flux quantum, ϕ_0 , penetrating the superconductor and a subsequent increment of the fluxoid quantum number of the sample (the so-called vorticity), L , by 1. Besides the critical parameters, the symmetry constraints also affect the vortex matter in mesoscopic superconductors. For instance, as a consequence

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of their C_∞ -symmetry, in mesoscopic disks the superconductivity nucleates in a rotational invariant manner as a giant vortex in the center [3,4]. In contrast to that, antivortices can be spontaneously created in mesoscopic regular polygons to comply with the symmetry of the sample [5,6]. These vortex-antivortex patterns can be expanded by the inhomogeneous field of a magnetic dot placed on top of the samples which, in addition, may also induce multiquanta vortex entries along $T_c(H)$ [7].

However, to perform experimental studies and to develop potential applications of these microstructures it is of fundamental importance to determine the range of stability of the symmetric vortex-patterns nucleated at $T_c(H)$. Symmetry-switching and symmetry-breaking transitions are expected when the temperature is lowered, since the decrease of $\xi(T)$ will induce a progressive loss of the mesoscopic regime till, eventually, a recovery of the conventional Abrikosov lattice [8–11]. The physical mechanisms of these transitions have been studied in mesoscopic squares by using an effective two-state model that has revealed different types of switches between vortex states [8]. This method has also allowed to identify the structure of the symmetry-breaking transitions, that is equivalent to the pseudo Jahn-Teller instability of high-symmetry nuclear configurations in molecules. Detailed calculations performed by using the non-linear Ginzburg-Landau (GL) theory with the help of a Monte Carlo simulation have confirmed these results [8,9]. In this letter, we investigate the phase transitions between vortex-patterns inside the superconducting state of an equilateral microtriangle in the framework of the GL theory. We will show that the symmetry consistent vortex patterns at the boundary, including those involving vortex-antivortex pairs, are stable in a broad range of temperatures and fields. However, we have also found a new type of crossover between vortex-patterns that is not expected in other regular polygonal structures (as squares), and that can be attributed to the coincidence between the geometries of the sample and of the conventional Abrikosov lattice. The effects of these switchings between vortex-patterns on the magnetization will be finally discussed.

The loss of the symmetry when the temperature is lowered well below $T_c(H)$ can be accounted for within the GL functional by including the fourth power term of the amplitude of the order parameter in the free-energy expansion. This leads to the following full (non-linear) GL functional that has to be minimized [12]:

$$G = \int \left[\alpha |\Psi|^2 + \Psi^* \hat{L} \Psi + \beta |\Psi|^4 \right] d\vec{r}, \quad (1)$$

where $\hat{L} = \left(-i\hbar \vec{\nabla} - \frac{2e}{c} \vec{A} \right)^2 / 2m^*$, α and β are the GL parameters, $\vec{A}$ is the vector potential and Ψ is the order parameter. Our model is restricted to the case of a very thin superconductor, *i.e.*, a superconductor with thickness, d , much smaller than $\xi(T)$ and the magnetic penetration depth, $\lambda(T)$. As a consequence, the solutions of the GL functional will not depend on the coordinate along the direction perpendicular to the triangle, they can be applied to both type-I and type-II superconductors, and we have assumed in eq. (1) that the applied magnetic field coincides with the field inside the superconductor [13].

When applied to mesoscopic superconductors, eq. (1) has to be solved by simultaneously taking into account the appropriate boundary condition, which for a superconducting/vacuum interface can be written as $\left(\frac{\hbar \vec{\nabla}}{i} - \frac{2e}{c} \vec{A} \right) \Psi|_n = 0$ [12] (here n holds for the normal to the boundary line). However, the presence of $\vec{A}$ in this expression complicates the minimization of eq. (1) unless it is transformed into $\nabla \psi|_n = 0$ through a gauge transformation of the vector potential [14]. We have already used this procedure to study the nucleation of the superconductivity in an equilateral triangle in terms of the linearized Ginzburg-Landau (LGL) equation,

Fig. 1 – Calculated temperature *vs.* field (in units of, respectively, $S/\xi^2(T)$ and ϕ/ϕ_0 , where S is the sample's surface and $\phi = H \cdot S$ is the applied magnetic flux) phase diagram of a superconducting triangle together with schematic plots of the obtained vortex configurations. Vortices and antivortices are represented by, respectively, bold and open circles. The black lines separate the stability regions of these vortex patterns, that have been colored and labeled with letters according to the irreducible representations that determine the calculated solution. The numbers indicate the type of transition that the vortex states undergo when the corresponding line is crossed: 1 stands for the first-order symmetry switching transitions with change in L , 2 for the second-order symmetry breaking transitions with L -conservation, 3 for the first-order symmetry breaking transitions with change in L , and 4 for a novel switch between two symmetric states with the same L .

$\hat{L}\phi_i = \epsilon_i\phi_i$ [5,9]. Accordingly with the symmetry group of the problem (C_3), the eigenfunctions of this last equation, ϕ_i , can be classified in three different irreducible representations (irreps) denoted as A , $E+$ and $E-$, which are characterized by having, respectively, no vortex ($L = 0$), one vortex ($L = +1$) and one antivortex ($L = -1$) at the center of the sample.

A straightforward approach to apply the superconducting/vacuum boundary condition to eq. (1) is, therefore, to use the eigenfunctions of the LGL equation as basis set for an expansion of the superconducting order parameter, $\Psi = \sum_i c_i \phi_i$ (c_i being the complex coefficients of the expansion). The free energy functional may be then rewritten in a simplified form [13],

$$G\beta = \sum_i (\alpha + \epsilon_i) c_i'^* c_i' + \frac{1}{2} \sum_{ijkl} A_{ij}^{kl} c_i'^* c_j'^* c_k' c_l', \quad (2)$$

where $A_{ij}^{kl} = \int \phi_i^* \phi_j^* \phi_k \phi_l d\vec{r}$ and $c_i' = \sqrt{\beta} c_i$. The problem of solving the non-linear GL functional under the superconducting/vacuum boundary condition is now reduced to a search for the c_i' -values that minimize eq. (2). For that we have used a standard Monte Carlo method with a basis set of 24 functions (8 for each irrep). We have also checked that the results remain unchanged by using larger basis sets.

The normal-superconducting phase diagram of an equilateral triangle resulting from our calculations is shown in fig. 1 for a broad range of temperatures and fields. To obtain a sample-independent figure, temperature and field are given in units of, respectively, $S/\xi^2(T)$ and ϕ/ϕ_0 (here S is the sample's area and $\phi = H \cdot S$ is the applied magnetic flux). In the particular case of aluminium, by considering the typical parameters of already experimentally studied samples (*i.e.*, $S \simeq 2 \mu\text{m}^2$, $\xi(0) \simeq 150 \text{ nm}$ and $T_c \simeq 1.32 \text{ K}$) [9], the axes of fig. 1 will cover magnetic fields up to 120 Oe and almost the whole temperature region below T_c . The

Fig. 2 – Sequence of the symmetry-breaking transition from a vortex-antivortex molecule to a two-vortices state that can be observed at $\phi = 5.25\phi_0$. For clarity, the center of the sample has been zoomed sixteen times in (a) and eight times in (b), (c) and (d). The nucleated order parameter consists of one antivortex in the center of the triangle surrounded by three vortices at the bisectors. When the temperature decreases, this symmetric pattern starts to deform till the antivortex annihilates together with one vortex and the two remaining vortices migrate from the center of the sample.

solid lines represent the transitions that can take place between vortex patterns and they have been numbered according to their type (see below), while the different colors and letters identify the irreps that determine the obtained solution in each region of the phase diagram. As illustrated by fig. 1, the symmetric vortex configurations at the boundary are stable in broad temperature-field regions that, in real samples, can be enlarged by just decreasing S . Note also that the stability of some of the vortex patterns, in particular at low fields, may extend down to the lowest studied temperature, $S/\xi^2(T) = 100$. Indeed, this is the case of the solutions corresponding to $L = 0$ and $L = 1$, but it also applies for $L = 3$. For this last vorticity value the energy is minimized by placing three vortices at the bisectors of the triangle, thus complying with both the symmetry of the triangle and of the conventional Abrikosov lattice. A similar configuration with an extra vortex at the center of the sample is also stable for $L = 4$, since this spatial distribution also follows the geometry of the sample and has the structure of a deformed triangular lattice. However, fig. 1 also shows that the boundary lines that separate regions with different vorticity have positive slopes in broad magnetic field intervals. Therefore, even the stable vortex patterns described above may undergo a transition to another symmetric state with different L if the temperature is decreased at a magnetic field slightly above the lower cusp that limits the corresponding vorticity region at $T_c(H)$. These symmetry switching transitions are of the first order [8] and they are labeled in fig. 1 as 1.

Symmetry switching transitions are not the only ones that the vortex patterns may undergo in a triangle. As shown in fig. 1, for $L = 2$ the superconductivity nucleates by placing one antivortex at the center of the triangle surrounded by three vortices at the bisectors. However, deep inside the superconducting phase, this symmetric vortex-antivortex pattern undergoes the transition to a two-vortices state schematically shown in fig. 2. In this figure, we present the evolution with decreasing temperature of the order parameter distribution at the center of the sample (zoomed $16\times$ for panel (a) and $8\times$ for panels (b), (c) and (d)) for $\phi = 5.25\phi_0$. The nucleated order parameter changes substantially when the superconducting state is penetrated till the antivortex annihilates together with one vortex. Thereafter, the two remaining vortices migrate from the center of the triangle forming a non-symmetric pattern that results from the admixture of the original irrep $E-$ with irrep A . This switch corresponds to a second-order symmetry-breaking transition that conserves L [8], and it is labeled in fig. 1 as 2. The nucleated vortex-antivortex pattern for $L = 5$ undergoes an analogous transition.

Fig. 3 – Panels (a) to (d): Evolution of the vortex pattern at $\phi = 10.5\phi_0$ with decreasing temperature. Superconductivity nucleates by placing two vortices at each of the bisectors of the triangle, but deep inside the superconducting state the three inner vortices rotate by 60° and expand towards the edges till a perfect triangular lattice is formed. (e) Temperature dependence of the coefficients of the excited levels that admix with the nucleated order parameter (all of them belonging to irrep A) when the superconducting phase is penetrated at this field. The admixture is smooth and continuous, so that this shift cannot be considered as a true phase transition between vortex patterns.

Symmetry-breaking transitions are not exclusive for only the vortex-antivortex patterns. In fact, the tendency to lose symmetry effects and progressively recover the Abrikosov lattice at low temperatures is expected to be stronger at high magnetic fields, since under these conditions the sample will be more similar to a bulk material [8,9]. This aspect is confirmed by the results presented in fig. 1. Just as an example, let us consider the case of $L = 7$ at the phase boundary. Close to $T_c(H)$ the vortices are placed symmetrically by pairs along the bisectors of the triangle with an extra vortex at the center. When the temperature decreases irrep $E+$ mixes with irrep A and the four inner vortices rearrange themselves forming a rectangle, leading to a vortex pattern where some traces of a distorted triangular lattice can be observed. Note also that the presence at low temperatures of both symmetric and non-symmetric vortex patterns may induce, by changing the field at a constant T , first-order symmetry-breaking transitions with change in L that are labeled as 3 in fig. 1.

Types 1 to 3 transitions have been already found for a superconducting microsquare [8,9], but in the case of a triangle we have also identified a novel crossover (labeled as 4 in fig. 1) between two symmetric states with conservation of the vorticity. This happens between the sixth and the seventh cusp of the $T_c(H)$ -boundary, where close to the transition $L = 6$ is achieved by placing two vortices in each of the bisectors of the triangle. However, when the superconducting state is penetrated the three inner vortices experience a 60° rotation followed by an expansion towards the edges till a perfect triangular lattice is formed. The sequence corresponding to this transformation can be seen in fig. 3, where in panels (a) to (d) the order parameter distribution in the sample at $\phi = 10.5\phi_0$ is presented for four different temperatures. We have found that the origin of this rotation is the admixture of the lowest Landau level (LLL) corresponding to irrep A (that determines the solution at the boundary) with the three first excited levels of the same irrep. To illustrate this point, we present

Fig. 4 – Temperature dependence of the increment of the magnetization (multiplied by S), ΔM , under three different applied magnetic fields. For $\phi = 5.25\phi_0$ and $\phi = 10.5\phi_0$, the magnetic fields at which the switches between vortex patterns schematically shown in, respectively, figs. 2 and 3 take place, these ΔM curves present traces of a critical behaviour (for instance, inflexion points and step-like anomalies) around the transition points. In contrast to that, for $\phi = 6.5\phi_0$, a magnetic field that corresponds to the stable vortex configuration observed when $L = 3$, ΔM remains almost constant.

in fig. 3(e) the temperature dependence of the coefficients of these levels in the Ψ -expansion (normalized by the LLL coefficient, c_0), which clearly indicates that this transformation cannot be considered as a true phase transition between vortex patterns (this change between vortex states is then represented in fig. 1 by a dotted line), since no jump in the c_i coefficients is observed. Instead, the admixture, in particular with the first excited level, is smooth and starts at temperatures very close to the transition. This situation is in clear contrast with the transitions predicted for a microsquare, where the nucleated order parameter changes to another vortex state by admixing with (or by switching to) levels of a different irrep, thus always modifying its symmetry. We shall, therefore, attribute the origin of this novel modification of the vortex patterns to the coincidence between the geometries of the sample and of the Abrikosov lattice, so that the tendency to recover a triangular vortex lattice at low temperatures can be also accomplished by mixing different states within the same irrep.

The best way to study the vortex states and their stability in mesoscopic superconductors is by using local vortex imaging techniques as, for instance, Hall probe or scanning tunneling microscopies. However, the transitions between vortex patterns may also leave traces on the behaviour of different observables, thus opening new possibilities for their study. To illustrate this point, in fig. 4 we present the temperature dependence of the increment of the magnetization (multiplied by S), ΔM , under three different magnetic fields. These curves were obtained by first using $M = \frac{1}{2c_S} \int_S \vec{r} \times \vec{j} dx dy$ for the averaged magnetization, and then calculating ΔM with steps of 5 in $S/\xi^2(T)$ -units. It is clearly seen that the ΔM curve at $\phi = 5.25\phi_0$ (circles), corresponding to the vortex-antivortex annihilation at $L = 2$, has a step-like anomaly when the two remaining vortices depart from the center of the sample [$S/\xi^2(T) = 65$]. The curve for $\phi = 10.5\phi_0$ (squares), where $L = 6$ and the vortex rotation shown in fig. 3 takes place, also has some peculiarities although it does not correspond to a true phase transition between vortex patterns. Subsequently, no jump in ΔM is observed, but the magnetization progressively increases while the three inner vortices shrink, and then shows an inflexion point when the 60° rotation occurs ($S/\xi^2(T) = 70$) that leads to constant ΔM during the vortices expansion towards the edges. In contrast, for the stable configuration that can be observed when $L = 3$ ($\phi = 6.5\phi_0$ (triangles)) ΔM remains almost constant.

Figure 4 also shows that ΔM is of the order of $10^{-7} \text{ A} \cdot \text{m}$, what in a typical sample with thickness in the nanometric scale will lead to a magnetic moment in the range of $10^{-15} \text{ A} \cdot \text{m}^2$. Since conventional SQUID magnetometers have a resolution in between $10^{-10} \text{ A} \cdot \text{m}^2$ and $10^{-11} \text{ A} \cdot \text{m}^2$, these variations in the magnetization could be accurately measured in arrays of 10^6 samples, what is well within the current experimental capabilities.

To sum up, by using the full Ginzburg-Landau functional we have investigated the evolution of the vortex patterns inside the superconducting phase of a microtriangle. A tendency to recover a triangular lattice at low temperatures and high magnetic fields has been detected, and different types of transitions between vortex states have been identified. This includes a crossover between two symmetric states with the same L unexpected for other regular polygons [8] that can be attributed to the coincidence between the symmetries of the sample and of the Abrikosov lattice. The vortex patterns at the $T_c(H)$ -boundary, in particular those involving vortex-antivortex pairs, are found to be stable in broad temperature and fields regions, thus making their observation possible by using vortex imaging techniques. Our calculations also indicate that, alternatively, the transitions between vortex states could be studied through the specific features they leave in the magnetization *vs.* field curves. The extension of these results to the vortex-patterns stability in other mesoscopic superconducting systems deserves further studies. The case of hybrid superconductor/ferromagnet symmetric nanostructures could be of particular interest.

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