Vortex–antivortex patterns in mesoscopic superconductors

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Abstract

We have studied the nucleation of superconductivity in mesoscopic structures of different shape (triangle, square and rectangle). This was made possible by using an analytical gauge transformation for the vector potential $\mathbf{A}$ which gives $\mathbf{A}_n = 0$ for the normal component along the boundary line of the rectangle. As a consequence the superconductor–vacuum boundary condition reduces to the Neumann boundary condition. By solving the linearized Ginzburg–Landau equation with this boundary condition we have determined the field–temperature superconducting phase boundary and the corresponding vortex patterns. The comparison of these patterns for different structures demonstrates that the critical parameters of a superconductor can be manipulated and fine-tuned through nanostructuring.

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1. Introduction

With the help of the modern microfabrication techniques, it is possible to make submicron superconducting structures [1,2]. In these mesoscopic structures where the penetration depth and coherence length are larger than the sample size the boundary dominates the nucleation of superconductivity. For example, a type I superconductor can exhibit type II behavior when prepared as a thin film [3,4] and in mesoscopic superconductors vortices do not have to order themselves in the Abrikosov lattice [4]. These new features of mesoscopic superconducting structures have initiated an interest to use them as components for supercomputers or even quantum computing [5]. In recent years we have been concentrating on the singly connected structures such as triangles, squares and rectangles. These geometries form the building blocks of more complicated structures.

The theoretical description of superconductivity in a mesoscopic structure with an applied magnetic field requires the solution of the Ginzburg–Landau equations with the following boundary condition imposed on the order parameter Ψ at the superconductor/vacuum interface [6]:

$$\left(-i\hbar\nabla - \frac{2e}{c}\mathbf{A}\right)\Psi|_n = 0, \quad (1)$$

where $\mathbf{A}$ is the vector potential corresponding to the applied magnetic field and Ψ is the complex superconducting order parameter.

We are especially interested in the influence of the boundary geometry on the superconductivity, which is the strongest at the field–temperature (H – T) phase boundary between the superconducting and normal state because the penetration depth and coherence length diverge at this phase boundary. However when studying the nucleation of superconductivity it is sufficient to use the linearized Ginzburg–Landau equation (LGL) [7]

$$\frac{1}{2m^*}\left(-i\hbar\nabla - \frac{2e}{c}\mathbf{A}\right)^2\Psi = \frac{\hbar^2}{2m^*\xi^2(T)}\Psi, \quad (2)$$

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where $\xi(T)$ is the temperature dependent coherence length. This already reduces the complexity of the problem since the LGL equation looks similar to the well-known Schrödinger equation for a free particle in an external magnetic field. However this still poses significant problems when solving this equation, certainly with the boundary condition (1) which complicates the problem enormously. By using a gauge for the vector potential which gives $A_n = 0$ for the normal component along the boundary line [8], the boundary condition can be reduced to the Neumann boundary condition

$$\nabla \Psi|_n = 0. \quad (3)$$

This makes the solution of the LGL equation much more feasible.

When solving the LGL equation for a triangle [9], square [10] and rectangle [11] it is necessary to choose a basis set. For example the solutions for the square can be classified after irreducible representations (irreps) A, B, E_+ and E_- of the symmetry group C_4 of the problem since the symmetry of the structure is preserved due to the fact that the LGL equation is a linear differential equation. A convenient basis set is then provided by the eigenfunctions used for the particle in a square box problem obeying Neumann boundary conditions. For each irrep a restricted basis set of these functions is then used to solve the eigenvalue problem in the LGL equation. The resulting lowest eigenvalue describes the H – T phase boundary for the nucleation of superconductivity, while the lowest eigenfunction Ψ corresponds to the order parameter at the phase boundary. In Fig. 1 it is clearly seen how the lower eigenvalues of the LGL equation, corresponding to the different irreps, generate the H – T phase boundary. Following the phase boundary (the lowest eigenvalue) we notice that it shows an oscillatory cusplike behavior as a function of flux and every oscillation cusp corresponds to a different irrep. The sequence of the oscillations is always A, E_+ , B and E_- in the case of a square.

2. Influence of symmetry

Since the LGL equation is a linear differential equation, it does not break the symmetry of the boundary condition. This means that different geometrical structures will have different order parameter patterns describing the nucleation of superconductivity. For example in a disk superconductivity will nucleate in a rotational invariant manner. The well known solution has a C_∞ symmetry [12,13]. The consequence is that at the phase boundary single quantum and giant vortices in the disk always sit in the center of the disk. So the flux can only enter the superconducting structure through a giant vortex in the center. But if L is the number of flux

Fig. 1. Lower eigenvalues of the LGL equation for the mesoscopic square, as a function of the magnetic flux ϕ/ϕ_0 , with superconductor-vacuum conditions. The different lines correspond to the four irreducible representations (irreps) A (full black line), B (dashed black line), E_+ (dotted black line) and E_- (full gray line). Because the problem has a discrete C_4 symmetry there is a ‘repulsion’ of the levels, giving a regular pattern of avoided crossing between levels belonging to the same irrep. The flux is defined as $\Phi = HS$, with S the surface of the square, H the applied magnetic field and $\Phi_0 = h/2ec$ the superconducting flux quantum. On the vertical axis, the critical temperature T_c is linearly decreasing with increasing $S/\xi^2(T)$. The dashed straight line is the upper critical field H_{c2} in a bulk Type-II superconductor, and $H_{c3} = 1.69H_{c2}$ (dotted straight line) is the surface critical field for a semi-infinite slab with a straight superconductor/vacuum border.

quanta ϕ_0 carried by the giant vortex then the kinetic energy of the giant vortex is proportional to L^2 , which is quite unfavorable since L distinct vortices have a kinetic energy proportional to L .

Looking at a superconducting structure with a lower symmetry than C_∞ , we can expect that the giant vortex is not the preferred nucleation form. We see for example for the square and triangle that the C_4 and C_3 symmetry is preserved by putting the vortices in the center and on the diagonals. In the center it is possible to have no vortex, one vortex (ϕ_0), one antivortex ($-\phi_0$) and in the case of the square a giant vortex with flux $2\phi_0$.

When, e.g. we want to put three vortices into a square, it turns out to be preferable to create an *extra vortex–antivortex pair* instead of having a giant vortex with flux $3\phi_0$. As can be seen in Fig. 2, the vortex pattern with an antivortex in the center, is quite small. Even more so this whole structure can be found in a region where $|\Psi|^2$ is already seven orders lower than in the corners.

The work by Bonca and Kabanov [14] and Baelus et al. [15] shows that this vortex–antivortex pattern becomes unstable when going deeper into the

Fig. 2. The figures show the density of the order parameter $|\Psi|^2$. The logarithm of $|\Psi|^2$ is proportional to the gray scale used in the pictures. The darker the color the lower the $|\Psi|^2$. Black indicates the position of vortices and anti-vortices. The triangle is shown for a field of $5\phi/\phi_0$. The vorticity at this applied field is two and is generated by three vortices on the lines connecting the center and corners and one antivortex in the center. The square is shown for a field of $6\phi/\phi_0$. The vorticity at this field is three and is generated by four vortices on the diagonal and one anti-vortex in the center. Both figures are a zoom of the central region of the structures. The triangle has a side that is $\frac{1}{16}$ of the full triangle and the sides of the square are $\frac{1}{5}$ of the original.

superconducting state for $\kappa \gg 1$. However more recent work by Misko et al. demonstrates that the vortex–antivortex pattern is stable when the mesoscopic superconductor is type I ($\kappa < 1/\sqrt{2}$) [16]. This is easily understood when considering that the vortex–antivortex interaction changes sign when changing from a type II to a type I superconductor. Since the vortex–antivortex interaction is *repulsive* in a *type I* superconductor the vortex–antivortex pattern remains stable even deep in the superconducting state.

In the rectangles with the C_2 symmetry, we see that the vortex patterns are different from these in the square (see Fig. 3). The vortices prefer to be located on a row along the longest axis except at higher fields where some vortices are on the “diagonal” [11]. The symmetry-induced antivortex we have found for the square (or rectangle with aspect ratio one) disappears already at the aspect ratio of 1.02. It is remarkable that superconductivity nucleates with a vortex pattern that is primarily one-dimensional even when the aspect ratios are still relatively small. We only see a two-dimensional pattern in the rectangles with smaller aspect ratios, because the field, at which the vortices move off the axis to the corners, increases dramatically with the aspect ratio.

3. Phase boundary

Analyzing the changes in the phase boundary for different superconducting mesoscopic structures, compared to bulk material (see Fig. 4), it is clearly seen that the disk has a nucleation field $H_{c3}(T)$ higher than that of a plane boundary. Moreover the square is in that respect

Fig. 3. The figures show the density of the order parameter $|\psi|^2$ with the same color scale as in Fig. 2. The rectangle is shown for aspect ratio two. The top figure is at a field of $6\phi/\phi_0$ and the bottom figure is at a field of $19.5\phi/\phi_0$. As shown in the figures the vortices are in a row along the longest axis, but at higher fields some vortices also position themselves off the axis, closer to the corners.

Fig. 4. Calculated superconducting T_c – H phase boundary for disk (light gray), square (gray), triangle (black) and rectangle (dashed black) with superconductor–vacuum boundary conditions. The flux is defined as $\phi = HS$, with S the surface of the mesoscopic structure and H the applied magnetic field. The rectangle is shown for different aspect ratios. From the top to bottom curve the aspect ratios for the corresponding rectangle are 2, 16 and 64. With increasing aspect ratio the phase boundary evolves from a predominantly linear behavior with oscillations to a parabolic behavior without any oscillations. The observed $T_c(H)$ oscillations are clearly the strongest with the disk.

better than the disk and the triangle is even better than the square. The rectangle, on the other hand, has some very different phase boundaries depending on the aspect ratio. When increasing the aspect ratio from one (square) to ∞ (line) the phase boundary moves down in the T – H graph below the $H_{c3}(T)$ for the square and the triangle. This behavior was expected since for a one-dimensional system the T_c shift with field is parabolic.

Looking closer at the phase boundary we see that the $T_c(H)$ line for the disk shows pronounced oscillations. Every cusp corresponds to the increase of the vorticity by one. This is also the case for the square and the triangle. However the square has $T_c(H)$ oscillations with a smaller amplitude than the disk. In the triangle the oscillations are further suppressed. The $H_c(T)$ curve for the rectangle starts from the square (aspect ratio one) and the amplitude of the oscillations drops with increasing aspect ratio. Already at an aspect ratio of four the phase boundary is almost predominantly linear except for the lowest fields. When the aspect ratio is further increased the phase boundary for the rectangle changes into the parabolic $T_c(H)$ dependence expected for a line.

4. Conclusions

We have demonstrated that nanostructuring has a profound impact on the nucleation of superconductivity which is described by very different $T_c(H)$ lines for triangles, squares and rectangles.

The symmetry of the boundary in a mesoscopic superconducting structure induces new vortex matter which are not present in bulk superconductors. For example it is quite surprising that the appearance of an antivortex is preferred instead of the generation of a giant vortex in a triangle and square mesoscopic superconducting structure.

To observe these novel “vortex molecules” at the phase boundary local vortex-imaging techniques sensitive to the $|\Psi|^2$ density, such as scanning tunneling microscopy, are needed. These vortex structures are quite small and imbedded in a region where the order parameter is already suppressed. So any technique used to observe the zeroes of $|\Psi|^2$ needs a very good spatial resolution and a broad range in magnitude. The

combination of these two requirements makes it a great challenge for experiments aimed at the observation of the $|\Psi|^2$ pattern in the superconducting structure.

The results presented here are also of interest for symmetrically confined vortex matter in superfluids and Bose–Einstein condensates.

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