The Problem of Coronal Heating

J. Vranjes and S. Poedts

*Centre for Plasma Astrophysics, and Leuven Mathematical Modeling and Computational Science
Research Centre (LMCC), Celestijnenlaan 200B, 3001 Leuven, Belgium.*

Abstract. The inhomogeneous coronal plasma is a perfect environment for fast growing drift waves. The omnipresence of coronal magnetic loops implies gradients of the equilibrium plasma quantities like the density, magnetic field and temperature. These gradients are responsible for the excitation of drift waves that grow both within the two-component fluid description (both in the presence of collisions and without it) and within the two-component kinetic descriptions (due to purely kinetic effects). Some aspects of these phenomena are investigated here. In particular the analysis of the particle dynamics within the growing wave is compared with the corresponding fluid analysis. While both of them predict the stochastic heating, the threshold for the heating obtained from the single particle analysis is higher. The explanation for this effect is given. Also, the effects of the density gradient in the direction perpendicular to the magnetic field vector are investigated within the kinetic theory, in both electrostatic and electromagnetic regimes. The electromagnetic regime implies the coupling of the gradient-driven drift wave with the Alfvén wave. The growth rates for the two cases are calculated and compared. It is found that, in general, the electrostatic regime is characterized by stronger growth rates, as compared with the electromagnetic perturbations. The released amount of energy density due to this heating should be more dependent on the magnitude of the background magnetic field than on the coupling of the drift and Alfvén waves. The stochastic heating is expected to be much higher in regions with a stronger magnetic field. On the whole, the energy release rate caused by the stochastic heating can be several orders of magnitude above the value presently accepted as necessary for a sustainable coronal heating. The vertical stratification and the very long wavelengths along the magnetic loops imply that a drift-Alfvén wave, propagating as a twisted structure along the loop, in fact occupies regions with different plasma- β and, therefore, may have different (electromagnetic-electrostatic) properties, resulting in different heating rates within just one or two wavelengths.

Keywords: Solar corona, heating, drift waves

PACS: 96.60.P-, 96.60.Q-, 52.35.Kt

INTRODUCTION

In our recent papers [1]-[4] a novel approach and a new paradigm for the coronal heating has been put forward. The model is based on the drift wave theory, a well known subject in the general plasma theory, in laboratory plasma physics, and even in terrestrial ionospheric research [5]. Yet, in the past the drift wave theory has completely been overlooked in the context of solar plasmas. It implies the abundance of free energy for the instability of the drift wave already in the corona. That energy is stored in the gradients of the density, temperature and magnetic field. It makes the waves growing and it results in heating due to the polarization drift effects. The heating is stochastic by nature; for short enough wave-lengths in the direction perpendicular to the magnetic field vector, gyrating plasma particles feel a space-time variation of the wave-electric field, and their motion becomes stochastic and equivalent to the increase of the temperature. The nature of the heating is such that it acts essentially in the perpendicular direction,

and, in addition to this, more massive particles are in fact more effectively heated by that mechanism.

For the purpose of the present study, in order to quantitatively describe some details of the heating and acceleration of plasma particles, any of the mentioned instabilities of the drift wave will do. Hence, in the first part we shall use the kinetic, electrostatic density gradient driven instability from [1, 2].

The analysis performed in [1]-[4] was focused on the electrostatic domain of the drift wave instability. This implies a small plasma- $\beta = 2\mu_0 n_0 \kappa T / B_0^2$, e.g. of the order or below m_e/m_i . For the plasma- β above m_e/m_i , the electromagnetic effects and the coupling with the Alfvén wave will effectively take place and, as a first manifestation of this, only the bending of the magnetic field vector may be taken into account. Such a coupled drift-Alfvén mode has in fact been studied in our earlier work [6] by using two-component fluid theory, with a complete and self-consistent contribution of hot ions effects, appropriate for the hot solar corona. Within such a two-fluid theory, the drift-Alfvén mode is destabilized in the presence of collisions and, for a large enough parallel wave-number, the mode has all the features of a growing Alfvén wave. This is because of an exchange of identities of the drift and Alfvén modes [6, 7] occurring in a certain parameter domain. On the other hand, the drift-Alfvén wave instability within the collision-less kinetic theory has a completely different nature, and this will be the subject of the present work. In particular, we shall investigate the difference in the growth-rates of the electrostatic (drift) and the electromagnetic (drift-Alfvén) modes. Such a difference is expected because of the following two opposite effects: i) a part of the energy that drives the instability is spent on the bending of the magnetic field vector and this should in principle reduce the growth rate, yet in the same time, ii) this bending should partly reduce the electron mobility in the parallel direction. The effects of such a reduction should be similar to electron collisions studied in [6] and, as a result, the growth-rate may become increased. Hence, the total outcome of the EM effects will then depend on the mutual ratio of these two opposite effects.

PHYSICS OF HEATING

The physics of the mentioned stochastic heating is described in detail in various sources. Here, we give some short sketches of it by following [8], and using the standard drift wave theory. Within the two-fluid theory, the perpendicular velocity of the ion unit volume is given by the recurrent formula [2, 6]

$$v_{i\perp} = \frac{1}{B_0} \vec{e}_z \times \nabla_{\perp} \phi + \frac{v_{Ti}^2}{\Omega_i} \vec{e}_z \times \frac{\nabla_{\perp} n_i}{n_i} + \vec{e}_z \times \frac{\nabla_{\perp} \cdot \pi_i}{m_i n_i \Omega_i} + \frac{1}{\Omega_i} \frac{d}{dt} \vec{e}_z \times \vec{v}_{i\perp}, \quad \frac{d}{dt} \equiv \frac{\partial}{\partial t} + \vec{v} \cdot \nabla. \quad (1)$$

The four terms here correspond to, respectively, the $\vec{E} \times \vec{B}$ -drift, the diamagnetic drift $\vec{v}_{*i}$, the stress tensor drift $\vec{v}_{\pi i}$, and the polarization drift $\vec{v}_{pi}$. The velocity can be calculated up to small terms of any order using the drift approximation $|\partial/\partial t| \ll \Omega_i$. In some situations, in plasmas with relatively cold ions, as a first approximation one may assume the $\vec{E} \times \vec{B}$ -drift as the leading order one, and set it into the velocity $\vec{v}_{i\perp}$ in the polarization drift. The diamagnetic drift does not contribute to the ion flux $\nabla \cdot (n \vec{v}_{*i}) \equiv 0$, and also its

contribution to the convective derivative of the polarization drift is canceled by the part of the stress tensor flux $n\vec{v}_{\pi i}$ [9]. Therefore, we shall focus now on the $\vec{E} \times \vec{B}$ -drift, and the polarization drift $\vec{v}_{pi} = (d\vec{E}_{\perp}/dt)/(B_0\Omega_i)$, where $\vec{E}_{\perp} \equiv -\vec{e}_y k_y \phi_1$. These two together describe an elliptical motion in the presence of the wave, described by:

$$\vec{v}_E = \vec{e}_x \frac{k_y \phi_1}{B_0} \sin \varphi, \quad \varphi = k_y y + k_z z - \omega_r t, \quad (2)$$

$$\vec{v}_{pi} = -\vec{e}_y \frac{\omega_r k_y \phi_1(t)}{B_0 \Omega_i} \left[\left(1 - \frac{k_z}{\omega_r} \frac{dz}{dt} \right) \cos \varphi - \frac{\gamma}{\omega_r} \sin \varphi \right] / \left(1 - \frac{k_y^2 \phi_1(t)}{B_0 \Omega_i} \cos \varphi \right). \quad (3)$$

In deriving Eq. (3) the wave amplitude is taken as time dependent due to the wave instability, $\phi_1 = \hat{\phi} \exp(\gamma t)$, $\hat{\phi} = \text{const.}$, and the same holds for the coordinates $y(t), z(t)$ appearing in the wave phase $\varphi(t)$.

Crucial for the heating is the term $[k_y^2 \phi_1(t)/(B_0 \Omega_i)] \cos \varphi$ in the denominator. Looking back into the derivations, it turns out that it appears from the derivative $d(\sin \varphi)/dt \rightarrow dy/dt = v_y = v_{pi}$. Keeping it here has sense only in case of a relatively large displacement in the direction of the perpendicular wave vector. From Eq. (3) it is seen that for

$$a(t) \equiv k_y^2 \phi_1(t)/(B_0 \Omega_i) = k_y^2 \rho_i^2 e \phi_1(t)/(\kappa T_i) \geq 1, \quad (4)$$

the denominator may vanish and, consequently, the polarization drift should go to infinity. However, Eq. (3) is obtained approximately starting from the guiding center approximation that is violated for such a large polarization drift, and the analytical expression for the polarization drift in this limit becomes inaccurate. Yet, Eq. (3) describes the general trend for the polarization drift; a fast and significant acceleration and heating of the plasma particles will definitely take place, as experimentally verified in [10]-[12].

Particle motion

As discussed in [8], a direct numerical integration for plasma particles is required for a full validation of the above described heating mechanism for the solar corona. The numerical simulation has to take into account the actual values of the electric field in the position of a specific gyrating plasma particle. For that purpose, taking again the drift-wave electric field with a time-varying amplitude, the appropriate set of equations for a single particle (ion) reads

$$y''(t) + y(t) - a(t) \sin \left[y(t) + \frac{k_z}{k_y} z(t) - bt \right] = 0, \quad (5)$$

$$z''(t) - a(t) \frac{k_z}{k_y} \sin \left[y(t) + \frac{k_z}{k_y} z(t) - bt \right] = 0, \quad (6)$$

$$x'(t) - y(t) = 0. \quad (7)$$

Here, the prime denotes a derivative in time, and the spatial and time coordinates are normalized as $x, y, z \rightarrow k_y x, k_y y, k_y z$, $t \rightarrow \Omega_i t$, $b = \omega_r / \Omega_i$. Equation (7) is obtained after one integration where an integration constant appears that has been neglected.

To provide some details of the particle acceleration and heating by the growing drift wave in the context of the solar corona, we use a set of representative parameters from [1, 2] that are known to yield the instability: $B_0 = 10^{-2}$ T, $n_0 = 10^{15}$ m $^{-3}$, $L_n = [(dn_0/dx)/n_0]^{-1} = s \cdot 100$ m, $\lambda_y = 0.5$ m, and $\lambda_z = s \cdot 10^4$ m. As discussed in [2], the instability will in fact develop even when the density is varied by several orders of magnitude around the given value, and the same holds for the magnetic field and the temperature. Here, L_n is the characteristic scale-length for the inhomogeneous equilibrium density. The parameter s is introduced in [1, 2] for convenience only, because it was realized that the ratio γ/ω_r remains exactly the same in case the ratio λ_z/L_n is fixed. This further implies that we may go to very different scales, e.g., by taking s in the range $0.1 - 10^4$. As long as λ_y is kept constant, the ratio γ/ω_r will remain the same, although the actual values for these two quantities will certainly change. Hence, for the given parameters one finds $\gamma/\omega_r = 0.26$. In the case $s = 1$, this in fact implies $\omega_r = 254$ Hz and $\gamma = 66$ Hz. While in the case $s = 10^3$, this would give a mode with $\omega_r = 0.254$ Hz and $\gamma = 0.66$ Hz. Going to such small values of L_n (tens of meters) in fact makes sense because the perpendicular diffusion, that naturally develops in such a plasma with density gradients, is indeed very small, with the diffusion coefficient $D_\perp$ of the order of 0.01 m 2 /s and the corresponding diffusion velocity $D_\perp \nabla_\perp n/n$ of just a few millimeters per second [13]. Note that the parameters λ_z and L_n can of course also be varied independently of each other.

FIGURE 1. Left: normalized particle positions in the x, y -plane after it starts from the point A with $(x, y, z) = (0, 0, 0)$, for a growing electric field potential $\phi_1(t) = \hat{\phi} \exp(\gamma t / \Omega_i)$, and $\hat{\phi} = 0.86$ V. Right: corresponding normalized velocities v_x, v_z .

The set of ordinary equations (5)-(7) is solved numerically. We do this first for a relatively small wave amplitude in order to demonstrate some basic features of the particle motion due to the wave electric field. The result is shown in Fig. 1, where we give the projection of the proton positions in the perpendicular plane, $x(t), y(t)$, in the wave field with the starting amplitude $e\phi_1/(\kappa T_i) = 0.01$, that corresponds to $\phi_1 = 0.86$ V. Here, $\lambda_y = 0.5$ m, $\lambda_z = 2 \cdot 10^4$ m, which consequently [1, 2] yields the wave frequency $\omega_r = 254$ Hz. This all yields $a = 0.014$, hence the stochastic heating is absent. The maximum time in physical units here is 0.04 s, and in this moment the potential has

FIGURE 2. Left: change in time of the normalized kinetic energy of a particle with a unit mass $E = (v_x^2 + v_y^2 + v_z^2)/2$. Right: normalized displacement in the direction of the polarization drift $y(t)$, and the normalized perpendicular velocity component $y(t) \equiv v_x(t)$.

reached the value of 17 V.

In order to see what happens for larger amplitudes of the electrostatic drift wave when the stochastic heating is supposed to be in action, Eqs. (5)-(7) are solved by allowing a slightly larger time range. One of the results is shown in Fig. 2 (left) for the kinetic energy of a particle with unit mass $E(t)/m = [v_x(t)^2 + v_y(t)^2 + v_z(t)^2]/2$, with normalized velocities as above. The other parameters are the same as in the previous text and figures. Here, the stochastic heating takes place after around 0.078 s, in the moment when the growing wave amplitude reaches the value of around 150 V. Note that this is by about a factor 2.5 larger than the value obtained from the condition (4), which follows from an approximative procedure as explained earlier. Thus, a higher necessary threshold for the stochastic heating is expected.

The corresponding plot of the displacement in the direction of the wave $y(t)$, which is in fact also equal to the velocity $v_x(t)$ [see Eq. (7)] is shown in Fig. 2 (right). The plot of the velocity $v_y(t)$ (that is not given here) reveals the amplitude that is completely negligible until $\Omega_i t$ becomes close to $8 \cdot 10^4$; after that it is very stochastic and with the amplitude equal to that of the velocity $v_x(t)$. Thus we are in the range of parameters in which the expansion procedure used previously is not valid any longer because the polarization and $\vec{E} \times \vec{B}$ drifts are of the same order, and the stochastic heating is fully in action. The total time interval in the graphs is only about 3-4 wave-periods.

COMPARISON BETWEEN ELECTROSTATIC AND ELECTROMAGNETIC DRIFT-WAVE INSTABILITY

In the case of a plasma beta in the range $m_e/m_i \leq \beta \ll 1$, it is appropriate to take into account only the bending of the magnetic field. The perturbed density for the species $j = e, i$ is, in that case, described by [7]:

$$n_{j1} = -\frac{q_j n_{j0}}{\kappa T_j} \left\{ \phi_1 + (\omega - \omega_1) \phi_1 \sum_m \frac{\Lambda_m(b_j)}{\omega_2 - m\Omega_j} \left[W \left(\frac{\omega_2 - m\Omega_j}{|k_z| v_{Tj}} \right) - 1 \right] \right\}$$

$$+(\psi_1 - \phi_1) \left(1 - \frac{\omega_1}{\omega}\right) \sum_m \Lambda_m(b_j) W\left(\frac{\omega_2 - m\Omega_j}{|k_z|v_{Tj}}\right) \Big\}. \quad (8)$$

Here, $\omega_1 = \omega_{*j} - k_y g / \Omega_j$, $\omega_2 = \omega + k_y g / \Omega_j$, $\omega_{*j} = v_{*j} k_y$, where $\vec{v}_{*j}$ is the diamagnetic velocity. In the terms $\omega_{1,2}$ we have the gravity that will be kept for ions only. The other notation is as follows $b_j = k_y \rho_j^2$, $\Lambda_m(b_j) = I_m(b_j) \exp(-b_j)$, and the gyro-radius, thermal velocity, and gyro-frequency of the species j are given by $\rho_j = v_{Tj} / \Omega_j$, $v_{Tj}^2 = \kappa T_j / m_j$, and $\Omega_j = q_j B_0 / m_j$, respectively. I_m is the modified Bessel function of the first kind and order m , and $W(\chi) = (2\pi)^{1/2} \int_{-\infty}^{+\infty} \eta \exp(-\eta^2/2) d\eta / (\eta - \chi)$. The equilibrium magnetic field and density gradient are $\vec{B}_0 = B_0 \vec{e}_z$ and $\nabla n_{j0} = -\vec{e}_x dn_{j0}/dx$, and we use a local approximation and Fourier analysis with small perturbations of the form $\sim \hat{f}(x) \exp(-i\omega t + ik_y y + ik_z z)$, where $\hat{f}(x)$ is the x -dependent amplitude and $|d/dx| \ll |k_y|$.

The terms ϕ and ψ describe the potential of the electric field [7], $\vec{E} = -\nabla_{\perp} \phi - \vec{e}_z \partial \psi / \partial z$, and we use the parallel current

$$j_{z1} = -\frac{q^2 n_{j0}}{\kappa T_j k_z} \left[(\omega - \omega_1) \phi_1 \sum_m \Lambda_m(b_j) W\left(\frac{\omega_2 - m\Omega_j}{|k_z|v_{Tj}}\right) + (\psi_1 - \phi_1) \left(1 - \frac{\omega_1}{\omega}\right) \sum_m \Lambda_m(b_j) (\omega_2 - m\Omega_j) W\left(\frac{\omega_2 - m\Omega_j}{|k_z|v_{Tj}}\right) \right]. \quad (9)$$

For electrons it is good enough to use a negligible mass limit so that $\Lambda_0(b_e) \simeq 1$, while the deviation from unity of the corresponding term for ions is a finite Larmor radius effect. In the limit of frequencies $|\omega_2|$ much below Ω_j , one keeps only the term $m = 0$ in the summation for both electrons and ions [7]. For similar reasons, in the case $|\chi| < 1$, we shall use the approximate expression (for electrons) $W(\chi) \simeq i(\pi/2)^{1/2} \chi \exp(-\chi^2/2) + 1 - \chi^2 + \chi^4/4 \dots$, and for ions $|\chi| > 1$, $W(\chi) \simeq i(\pi/2)^{1/2} \chi \exp(-\chi^2/2) - 1/\chi^2 - 3/\chi^4 + \dots$. For these two species Eq. (8) then becomes [7]

$$\frac{n_{e1}}{n_0} = \frac{e}{\kappa T_e} \left\{ \phi_1 + i\phi_1 \left(\frac{\pi}{2}\right)^{1/2} \frac{\omega - \omega_{*e}}{|k_z|v_{Te}} \exp\left(-\frac{\omega^2}{2k_z^2 v_{Te}^2}\right) + (\psi_1 - \phi_1) \left(1 - \frac{\omega_{*e}}{\omega}\right) \left[1 + i\left(\frac{\pi}{2}\right)^{1/2} \times \frac{\omega}{|k_z|v_{Te}} \exp\left(-\frac{\omega^2}{2k_z^2 v_{Te}^2}\right)\right] \right\}, \quad (10)$$

$$\begin{aligned} \frac{n_{i1}}{n_0} = & -\frac{e}{\kappa T_i} \left\{ \phi_1 - \phi_1 \frac{\omega - \omega_1}{\omega_2} \Lambda_0(b_i) \left[1 + \frac{k_z^2 v_{Ti}^2}{\omega_2^2} + \frac{3k_z^4 v_{Ti}^4}{\omega_2^4} \right. \right. \\ & \left. \left. - i\left(\frac{\pi}{2}\right)^{1/2} \frac{\omega_2}{|k_z|v_{Ti}} \exp\left(-\frac{\omega_2^2}{2k_z^2 v_{Ti}^2}\right)\right] + (\psi_1 - \phi_1) \times \right. \\ & \left. \times \left(1 - \frac{\omega_1}{\omega}\right) \Lambda_0(b_i) \left[i\left(\frac{\pi}{2}\right)^{1/2} \frac{\omega_2}{|k_z|v_{Ti}} \exp\left(-\frac{\omega_2^2}{2k_z^2 v_{Ti}^2}\right) - \frac{k_z^2 v_{Ti}^2}{\omega_2^2} - \frac{3k_z^4 v_{Ti}^4}{\omega_2^4}\right] \right\}. \quad (11) \end{aligned}$$

The quasi-neutrality then yields the dispersion equation that will be discussed below in two limits.

Electrostatic drift wave

In the electrostatic limit, one may set $\phi_1 = \psi_1$ in the above expressions. Using the quasi-neutrality condition $n_i = n_e$ and Eqs. (10,11), one directly obtains the dispersion equation in the form $Re\Delta(\omega, \vec{k}) + iIm\Delta(\omega, \vec{k}) = 0$. The frequency is assumed to be complex, in the form $\omega = \omega_r + i\gamma$. Setting $Re\Delta(\omega_r, \vec{k}) = 0$, one then obtains the following equation for the real part of the frequency:

$$\begin{aligned} \omega_{2r}^5 \left(1 + \frac{T_e}{T_i}\right) - \Lambda_0(b_i) \frac{T_e}{T_i} \omega_r \omega_{2r}^4 + \Lambda_0(b_i) \frac{T_e}{T_i} \omega_1 \omega_{2r}^4 - \Lambda_0(b_i) k_z^2 c_s^2 \omega_r \omega_{2r}^2 \\ + \Lambda_0(b_i) \omega_1 k_z^2 c_s^2 \omega_{2r}^2 - 3\Lambda_0(b_i) k_z^4 v_{Ti}^2 c_s^2 \omega_r + 3\Lambda_0(b_i) k_z^4 v_{Ti}^2 c_s^2 \omega_1 = 0. \end{aligned} \quad (12)$$

In the limit of a negligible ion response along the magnetic field vector, $|\omega_r/k_z| \ll c_s$ with $c_s^2 = \kappa T_e/m_i$, and for small gravity effects, this gives the electrostatic drift wave frequency used in [1, 2]:

$$\omega_r = -\frac{\omega_{*i}\Lambda_0(b_i)}{1 - \Lambda_0(b_i) + T_i/T_e} = \frac{\omega_{*e}\Lambda_0(b_i)}{1 + [1 - \Lambda_0(b_i)]T_e/T_i}. \quad (13)$$

Here, $\omega_{*i} = -\omega_{*e}T_i/T_e$. Observe that after setting $\Lambda_0(b_i) \simeq 1 - b_i$, the term in the denominator becomes $1 + k_y^2 \rho_s^2$, where $\rho_s = c_s/\Omega_i$. Using the two-fluid description for comparison and as a guideline [6, 8], it can be shown that this same expression (describing the finite ion inertia) directly follows from the ion polarization drift, the latter playing an essential role in the process of stochastic heating.

On the other hand, for a purely parallel propagation from Eq. (12) one obtains the ion-acoustic (IA) mode in plasmas with hot ions

$$\omega_r^4 - k_z^2 c_s^2 \omega_r^2 - 3k_z^4 v_{Ti}^2 c_s^2 = 0. \quad (14)$$

The solutions $\omega_r^2 = (k_z^2 c_s^2/2)[1 + (1 + 12T_i/T_e)^{1/2}]$ for $T_i = T_e$ yield the frequency $\omega_r \simeq \pm 1.5 k_z c_s$. Hence, the ion temperature plays no important role in the real part of the IA wave frequency, while the opposite is certainly true with its imaginary part.

The growth rate γ is obtained approximately from $\gamma \simeq -Im\Delta(\omega_r, \vec{k})/[\partial Re\Delta/\partial \omega]_{\omega=\omega_r}$. This yields

$$\begin{aligned} \gamma = -\left(\frac{\pi}{2}\right)^{1/2} \frac{\omega_{2r}^2}{\omega_{*e}\Lambda_0(b_i)} \left[\frac{\omega_r - \omega_{*e}}{|k_z|v_{Te}} \exp\left(-\frac{\omega_r^2}{2k_z^2 v_{Te}^2}\right) \right. \\ \left. + \Lambda_0(b_i) \frac{T_e}{T_i} \frac{\omega_r - \omega_1}{|k_z|v_{Ti}} \exp\left(-\frac{\omega_{2r}^2}{2k_z^2 v_{Ti}^2}\right) \right] \left[1 + \frac{k_z^2 v_{Ti}^2}{\omega_{2r}^2} \left(3 - \frac{2\omega_{2r}}{\omega_{*i}}\right) + \frac{3k_z^4 v_{Ti}^4}{\omega_{2r}^4} \left(5 - \frac{4\omega_{2r}}{\omega_{*i}}\right) \right]^{-1}. \end{aligned} \quad (15)$$

The frequency on the right-hand side in Eq. (15) is to be obtained from Eq. (12). Note that in the absence of gravity $\omega_{2r} \rightarrow \omega_r$, and in addition, for a negligible ion response along the magnetic field vector, Eq. (15) yields the growth rates from [1, 2] that are used in the previous section too. For the assumed geometry ω_{*i} is negative and the ion contribution will always tend to reduce the growth rate, while electrons will make the mode growing. Clearly this purely kinetic instability can only take place provided that the frequency ω_r is below ω_{*e} .

Electromagnetic domain

To proceed with the electromagnetic perturbations, a procedure similar to the derivation of Eqs. (10) and (11) yields the electron parallel current

$$j_{ez1} = -\frac{e^2 n_0}{k_z \kappa T_e} (\omega - \omega_{*e}) (1 + i\Upsilon_e) \psi_1, \quad \Upsilon_e = \left(\frac{\pi}{2}\right)^{1/2} \frac{\omega}{|k_z| v_{Te}} \exp\left(-\frac{\omega^2}{2k_z^2 v_{Te}^2}\right). \quad (16)$$

The corresponding expressions for the ions are:

$$j_{iz1} = \frac{e^2 n_0}{k_z \kappa T_i} (\omega - \omega_1) \Lambda_0(b_i) \left[\left(1 + \frac{\omega_g}{\omega}\right) \psi_1 - \frac{\omega_g}{\omega} \phi_1 \right] \left[\frac{k_z^2 v_{Ti}^2}{\omega_2^2} \left(1 + \frac{k_z^2 v_{Ti}^2}{\omega_2^2}\right) - i\Upsilon_i \right], \quad (17)$$

$$\Upsilon_i = \left(\frac{\pi}{2}\right)^{1/2} \frac{\omega_2}{|k_z| v_{Ti}} \exp\left(-\frac{\omega_2^2}{2k_z^2 v_{Ti}^2}\right), \quad \omega_g = \frac{k_y g}{\Omega_i}.$$

The first necessary equation is obtained as above, by using the quasi-neutrality condition $n_{i1} = n_{e1}$ and Eqs. (10, 11). The second equation follows from the Ampère law that, with the help of Eqs. (16) and (17), yields

$$\psi_1 = \phi_1 \frac{s_1 + f_1}{s_2 + f_2}, \quad s_1 = k_y^2 \rho_s^2 k_z^2 c_a^2, \quad s_2 = s_1 + \omega(\omega_{*e} - \omega)(1 + i\Upsilon_e), \quad (18)$$

$$f_1 = \frac{T_e}{T_i} \Lambda_0(b_i) \omega_g (\omega - \omega_1) (\alpha - i\Upsilon_i), \quad f_2 = f_1 \frac{\omega_2}{\omega_g}, \quad \alpha = \frac{k_z^2 v_{Ti}^2}{\omega_2^2} \left(1 + \frac{k_z^2 v_{Ti}^2}{\omega_2^2}\right).$$

Here, $c_a^2 = B_0/(\mu_0 n_0 m_i)$ denotes the square of the Alfvén velocity. The terms $f_{1,2}$ originate from the ion parallel current and in many situations can be neglected.

The potential (18) is used to eliminate ψ_1 in the quasi-neutrality condition and in the end one obtains the following dispersion equation:

$$\begin{aligned} & -\frac{T_e}{T_i} \left[1 - \frac{\omega - \omega_1}{\omega_2} \Lambda_0(b_i) (1 + \alpha - i\Upsilon_i) + p \left(\frac{s_1 + f_1}{s_2 + f_2} - 1 \right) \frac{\omega - \omega_1}{\omega} \Lambda_0(b_i) (i\Upsilon_i - \alpha) \right] \\ & = 1 + i\Upsilon_* + p \left(\frac{s_1 + f_1}{s_2 + f_2} - 1 \right) \left(1 - \frac{\omega_{*e}}{\omega} \right) (1 + i\Upsilon_e). \end{aligned} \quad (19)$$

$$\Upsilon_* = \left(\frac{\pi}{2}\right)^{1/2} \frac{\omega - \omega_{*e}}{|k_z| v_{Te}} \exp\left(-\frac{\omega^2}{2k_z^2 v_{Te}^2}\right).$$

The parameter p is set here by hand, and for convenience only; taking $p = 0$ is equivalent to the electrostatic limit discussed in the previous section, while $p = 1$ implies the electromagnetic perturbations that are of interest here.

The dispersion equation (19) describes the coupled Alfvén and drift waves as well as the ion parallel (acoustic) response, together with the gravity and finite Larmor radius

effects for ions. The wave spectra are obtained following the same procedure as before, i.e. by setting $\omega = \omega_r + i\gamma$, etc. The real part of Eq. (19) yields:

$$0 = 1 + \frac{T_e}{T_i} \left[1 - \frac{\omega_r - \omega_1}{\omega_{2r}} \Lambda_0(b_i)(1 + \alpha_r) \right] + \left(\frac{s_{1r} + f_{1r}}{s_{2r} + f_{2r}} - 1 \right) \left[1 - \frac{\omega_{*e}}{\omega_r} - \Lambda_0(b_i) \alpha_r \frac{T_e}{T_i} \left(1 - \frac{\omega_1}{\omega_r} \right) \right]. \quad (20)$$

Here, the index r denotes the real part of the corresponding expressions.

It is interesting to compare these derivations with the results from the two-component fluid theory. Omitting the ion parallel response and gravity, Eq. (20) yields

$$(\omega - \omega_{*e})(\omega^2 - \omega_{*i}\omega - k_z^2 c_a^2) = k_z^2 c_a^2 k_y^2 \rho_s^2 (\omega - \omega_{*i}). \quad (21)$$

This equation is exactly the same as the corresponding two-fluid equation from [6], and it is also obtained from the kinetic derivation in [7]. We stress that such an agreement between the fluid and kinetic descriptions is only possible if the two-fluid derivations self-consistently include the gyro-viscosity stress tensor contributions [6]. From Eq. (21) it is seen that the coupling between the drift mode $\omega = \omega_{*e}$ and the Alfvén mode is due to the right-hand side, which here appears due to the finite-ion-mass effect $k_y^2 \rho_s^2$. In the fluid description, this term originates from the ion polarization drift $\vec{v}_{pi}$.

Frequencies, growth rates and heating

In both instabilities discussed above, the perpendicular ion motion is essentially the same: the typical dominant velocity is due to the $\vec{E} \times \vec{B}$ -drift and this holds as long as $\lambda_y \gg \rho_i$. The latter condition may be relaxed, the relative contribution of the polarization drift in that case increases and the foreseen stochastic heating for the given electrostatic-drift and drift-Alfvén instabilities will have a similar nature. Note that experimental verification performed in [11] in fact involved the excitation of the drift-Alfvén waves. It may be of importance to check the growth rates of these instabilities, calculated for the same or a similar set of physical parameters. This may give the answer about their relative importance in the solar corona.

To check our model and the differences between the electrostatic (ES) and electromagnetic (EM) cases, we take the same set of parameters as in the second section, with the plasma- $\beta = 0.64m_e/m_i$. The values for L_n used there are below the resolution of presently available instruments, that are around 200 km at best. The drift wave instability can easily be demonstrated also for L_n values above this limit [2]. As example we refer to the graphs from [2] where the predicted drift wave instability develops for the density scale-length L_n above 200 km. It should be stressed, however, that the actual radius of a magnetic loop with a cylindric cross section can be much larger than the parameter L_n , implying the density that is in fact more or less constant in the main body of the structure, with only a relatively sharp decrease at its outer region. In the case of such magnetic structures, the predicted drift wave activity is expected mainly in their outer regions.

FIGURE 3. Left: the drift wave frequency for several different values of plasma β in terms of the perpendicular wave-length. Right: the corresponding normalized growth rates.

In Fig. 3 we plot the frequency ω_r (left) and the growth-rate (right) for the case $s = 1$, and in terms of the perpendicular wavelength λ_y . The four lines in the figure represent the real part of the frequency calculated from Eq. (19) in the following manner. The full line is obtained after setting $p = 0$, that is equivalent to simply neglecting the EM effects, corresponding to the purely electrostatic analysis given in [2]. The dotted line is obtained after setting $p = 1$, yielding consequently the drift wave frequency when the EM effects are taken into account. As expected, in view of the given small plasma- β , the frequency remains almost unchanged. The frequency is passing through a maximum, and this follows from the fact that $\omega_r \sim k_y / (1 + k_y^2 \rho_s^2)$ (note that $k_y^2 \rho_s^2 \simeq 9$ at $\lambda_y = 0.2$ and $k_y^2 \rho_s^2 \simeq 0.36$ at $\lambda_y = 1$). The plasma- β can be varied by changing several parameters. In the present case this is done by the variation of the number density. Hence, we take it to be $n_0 = 2 \cdot 10^{16} \text{ m}^{-3}$ and $n_0 = 5 \cdot 10^{16} \text{ m}^{-3}$, and calculate the frequency from Eq. (19). This is represented by the dashed and dash-dotted lines, respectively. It is seen that the drift-wave frequency becomes reduced and this can be attributed to its coupling with the Alfvén wave. Note that setting $s = 10^3$ (thus simultaneously changing L_n to 10^2 km , and λ_z to 10^4 km , implying larger coronal loops) reduces the frequency approximately to ω_r/s , while in the same time the dispersion lines keep exactly the same shape. Hence, similar to the electrostatic analysis in [2], the reduction of frequency due to variation of s remains more or less the same even in the presence of EM perturbations discussed here. The plot of the growth rate shows that the EM effects can make an important modification of the drift wave for larger values of the plasma- β . The reason for the reduced growth rate for short λ_y can be understood from Eq. (21): for larger k_y the coupling term on the right-hand side becomes more important, a greater amount of energy is spent on the Alfvén mode and, because of the fixed amount of the free energy stored in the background density gradient, the growth rate is therefore reduced. Hence, the physics of the stochastic heating predicted in [1]-[4] will remain nearly the same even in the case of the coupling with the Alfvén mode, provided a low plasma- β . On the other hand, for a larger plasma- β , the increasing electromagnetic effects will impose longer growth times.

The plasma- β can change also by varying the temperature and/or magnetic field. However, the drift wave frequency is proportional to the temperature and also strongly

FIGURE 4. Left: the drift wave frequency obtained from Eq. (19) in terms of the plasma temperature and for several values of the plasma number density (per cubic meter). Right: the corresponding drift wave growth rate.

depends on the magnetic field [see Eq. (13)], so that the effect of the perturbed magnetic field alone on the drift-wave in that case is not so transparent. In Fig. 4 we give the drift wave spectrum in terms of the plasma temperature for several values of the plasma density. Other parameter values are $\lambda_y = 0.5$ m, $\lambda_z = 200$ km, and $L_n = 1$ km. The electromagnetic effects are again most effectively seen by taking several values of the plasma density and varying the temperature. At $T = 1.4 \cdot 10^6$ K the frequency is reduced by factor 1.4 for the density increased from $n_0 = 10^{15}$ m $^{-3}$ to $n_0 = 10^{16}$ m $^{-3}$. At the same time the corresponding growth rate is reduced by a factor 3.2.

The graphs presented in this section are solely for the drift wave part of the spectrum from Eq. (19). The Alfvén mode, that is also described by Eq. (19), plays no important role in the present study dealing with the stochastic heating. The coupling of the two modes is in fact given in detail in [6] using the two-component fluid descriptions.

Considering a specific single coronal loop and in view of the given geometry that implies a very elongated wave front in the axial direction, $\lambda_z \gg \lambda_\perp$, the results presented above may imply the following. The waves propagate both axially and poloidally, with drastically different wavelength components in the two directions. In the cylindric geometry of a magnetic loop, the wave front is thus twisted along the loop. An extremely large axial component of the wavelength implies a wave that simultaneously takes place in areas with gradually varying (with altitude) plasma parameters and consequently different plasma- β . At higher altitudes with a lower density, the perturbations may be electrostatic and develop on a shorter time scale. The associated heating may rapidly develop at such places and it can then spread along the common wave-front towards lower regions where, due to the increased plasma- β , it is additionally accompanied by the EM effects that develop on longer characteristic times.

However, this scenario with an increased plasma- β due to higher plasma density can be partly counteracted by the lower temperature at lower altitudes, and as a result the energy release rate and the heating, together with the variation of magnetic topology, may not be so drastically different along the given magnetic loop. One possible example of this can be seen from Fig. 4: for the given normalized temperature $T = 1$ from the line 1 (point A in Fig. 4), the plasma- β is $0.64m_e/m_i$, while for example at $T = 0.4$

from the line 3 (point B), the plasma- β is $2.5m_e/m_i$. This is of course just a rough estimate because the points A and B belong to two separate dispersion lines. One particular plasma mode is determined by one particular dispersion line, yet in view of the parameters changing with the altitude such a transition may be expected. Hence, the drift wave spreading along the loop will have an electrostatic nature at the first point, and it will be EM at the second one, implying a difference in the heating rate and the magnetic variation.

FIGURE 5. The drift wave frequency and the growth rate for $B_0 = 3 \cdot 10^{-2}$ T; other parameters are the same as for line 3 from Fig. 4.

The magnetic field intensity too varies with the altitude, and this may additionally change both the plasma- β and also the wave properties. Hence, assuming the magnetic field is stronger by a factor 3 (i.e. taking $B_0 = 3 \cdot 10^{-2}$ T), and for other parameters as for the line 3 from Fig. 4, after solving Eq. (19) again, in Fig. 5 we present the wave frequency and the growth rate in terms of the temperature. Observe that, for example, at the normalized temperature $T = 0.4$ (the point C) the plasma- $\beta = 0.3m_e/m_i$, so due to the stronger magnetic field the mode is now in the electrostatic regime (compare with the point B from Fig. 4). At the same point we have the frequency and the growth rate $\omega = 13.3 + i0.22$ Hz. Hence, for such a stronger magnetic field the growth rate becomes about 50 times lower, as compared to the electrostatic case for the point A from Fig. 4.

The points A and C in the previous examples may belong to the same magnetic loop. However, different loops may have different plasma- β and these points may also represent such a situation. Therefore, the heating in different loops may be with or without a detectable variation of the magnetic topology. Observations of strong energy release events in the past (even in the range of flares) have shown that both scenarios are indeed possible; examples without magnetic variations can be seen in [14]-[16]. The qualitative analysis described above will additionally be supported below by some more numbers.

According to [10, 11] the stochastic heating by the drift wave is in action provided a strong enough wave potential amplitude (4), and the maximum achieved ion velocity due to this heating is given by

$$v_{max} \simeq [k_y^2 \rho_i^2 e \phi_1 / (\kappa T_i) + 1.9] \Omega_i / k_y. \quad (22)$$

The effective stochastic temperature is then $T_{max} = m_i v_{max}^2 / (3\kappa)$. From Eq. (4) it follows that the condition for the stochastic heating will sooner be satisfied in regions of a weaker

magnetic field, regardless of the starting temperature. We shall apply these expressions using the ES and EM growth rates given above in order to quantitatively verify the differences due to eventual magnetic nature of the perturbations.

For the parameters corresponding to the point A from Fig. 4, the expression (4) for $a = 1$ yields the required potential $\phi_1 = 61$ V. The maximum achieved stochastic velocity from Eq. (22) is 221 km/s, and the achieved stochastic temperature is $T_{max} = 1.97 \cdot 10^6$ K. Assuming some small accidental initial perturbations with the amplitude $e\hat{\phi}/(\kappa T_i) = 0.01$, i.e., $\hat{\phi} = 0.86$ V we can calculate the time t_g required to achieve the required value for the heating $\phi_1 = \hat{\phi} \exp(\gamma t_g)$. This yields $t_g = \ln(\phi_1/\hat{\phi})/\gamma = 0.4$ s. Note also that here $e\phi_1/(\kappa T_i) \simeq 0.7$. The total released energy density is $E_{max} = n_0 m_i v_{max}^2/2 = 0.04$ J/m³, and the energy release rate $\Gamma_{max} = E_{max}/t_g = 0.1$ J/(m³s). Hence, Γ_{max} is about 1700 times the required value for the coronal active regions, that amounts to $\simeq 6 \cdot 10^{-5}$ J/(m³s).

Taking as another example the point B in Fig. 4 (i.e., the same magnetic field 10^{-2} T but different density and starting temperature) yields $E_{max} = 0.4$ J/m³, $t_g = 1.3$ s, and $\Gamma_{max} = 0.3$ J/(m³s), $T_{max} = 1.97 \cdot 10^6$ K; $\beta = 2.5m_e/m_i$. So the present case is weakly electromagnetic and it is accompanied with the increase in the energy density and the energy release rate (because the density is higher), as compared to the electrostatic case from the point A, although it implies a longer growth time. The achieved stochastic temperature T_{max} and velocity v_{max} are the same as in the point A because a is kept fixed, and $\phi_1 = 61$ V, $\hat{\phi} = 0.34$ V as in the previous case.

On the other hand, taking as example the point C from Fig. 5, and the threshold (4), yields: $\hat{\phi} = 0.34$ V, $\phi_1 = 546$ V, $v_{max} = 663$ km/s, $T_{max} = 1.8 \cdot 10^7$ K, $E_{max} = 3.67$ J/m³, $t_g = 33$ s, and $\Gamma_{max} = 0.11$ J/(m³s). The stronger necessary potential here is due to the increased value of the magnetic field, see Eq. (4). Hence, the energy release rate Γ_{max} is almost the same as for the point A, yet the characteristic time t_g for the point A is more than 80 times shorter. In the same time, the maximum released energy density in the area with such a stronger magnetic field B_0 is for about a factor 90 larger in comparison with the point A, with the achieved stochastic temperature that goes to 18 million K. The reason for the larger energy density is clearly the larger maximum stochastic velocity in the area where both the magnetic field and density are larger. The fact that plasma- β for this stronger magnetic field is only $0.3m_e/m_i$ tells us that the increased stochastic energy density can be related to the EM effects and the coupling with the Alfvén wave. However, a much more pronounced effect on the heating should be attributed to the increased magnetization of the plasma species. Thus, the areas with stronger background magnetic fields are subject to stronger stochastic heating. The magnetic field used here is in agreement with observations of active regions showing the field strength of a few times 0.01 T, that in fact may easily go above 0.1 T, implying a possibly still stronger heating within the scenario presented above.

Note also that the two potentials for the points A and C, 61 V and 546 V, respectively, are obtained assuming $a = 1$ in Eq. (4). These two potentials yield the electric field $k_y \phi_1$ in the perpendicular direction 0.77 kV/m and 6.9 kV/m, respectively. Now, to have the threshold $a = 1$, the required potential $\phi_1 \sim \lambda_y B_0^2$, so a slight increase in these two parameters will yield even stronger electric fields. Taking as example $\lambda_y = 2$ m, $B_0 = 4 \cdot 10^{-2}$ (instead of $\lambda_y = 0.5$ m, $B_0 = 3 \cdot 10^{-2}$ as in the point C) yields the perpendicular electric field at which the stochastic heating takes place $E_y \simeq 27$ kV/m.

The three obtained values for the electric field, together with the corresponding magnetic field values, yield the $\vec{E} \times \vec{B}$ -drift ($= E/B_0$) of the plasma as a whole in the perpendicular direction 77, 230, and 675 km/s, respectively. In view of the meter-sized perpendicular wavelengths these plasma flows (drifts) could eventually be observed only by spectral analysis. Hence, we conclude that i) exceptionally strong perpendicular electric fields are expected during the proposed stochastic heating, and this particularly within stronger magnetic structures, and, ii) the perpendicular stochastic heating, as single particle interaction with the wave, is accompanied with collective plasma drifts.

SUMMARY

In this work, the trajectories of individual plasma particles have been investigated within the framework of the drift wave instability and the associated stochastic heating, with the aim to describe some fine details of the particle dynamics during the heating process. It follows that the general trend of the ion behavior in the wave field with a growing amplitude can be described analytically from the fluid point of view. However, a more detailed picture is obtained by following a single particle which is moving in the wave field.

The kinetic theory of the drift wave shows that the mode is almost always unstable due to purely kinetic effects, and it couples naturally to the Alfvén wave. The higher the plasma- β is, the better the coupling. The electrostatic drift wave in the solar corona is expected to be more unstable as compared to the regime in which the two modes are coupled. Essential for plasma heating is the electrostatic part of such an electromagnetic drift-Alfvén wave. The heating is stochastic by nature and it possesses such properties that it is able to satisfy numerous heating requirements in the solar corona. From the analysis it also follows that the regions with stronger magnetic fields will be subject to much stronger heating. Note that such a relation between the temperature and the magnetic field has been established long ago [17]; in the present work we give a new, alternative explanation for this phenomenon.

The electric field associated with the drift wave implies the possibility of the acceleration of plasma particles (primarily electrons) in the direction parallel to the magnetic field vector, and the development of drifts (in the perpendicular direction) of the plasma as a whole due to the $\vec{E} \times \vec{B}$ -drift that is the same for both electrons and protons. This issue is discussed in [2, 3]. The mean free path of the plasma species j is proportional to v_{Tj}^4 and therefore the parallel acceleration by the electric field is always more effective on particles that are already faster, i.e., those from the tail in the starting (Maxwellian) distribution, because those have more time/space to interact with the field. This will consequently result in a very different distribution function with a much longer high-velocity tail, and resembling the κ -distribution observed in the outer solar atmosphere. The proposed particle acceleration within the present model appears as a natural development of the drift wave instability for which the source is clearly identified, thus removing the standard problem of various acceleration schemes that typically suffer from a common problem, the lack of a proper source.

The energy for driving the instability is stored in the gradients of the background plasma. The predicted drift wave instability will inevitably lead to the flattening of the

background plasma gradients and the instability at this particular region will cease. However, the coronal magnetic structures are ever-changing and evolving, and the plasma inhomogeneity is the consequence of that. If it vanishes at one place it reappears elsewhere and the instability will reestablish itself again. The same evolving magnetic structure within which the instability and heating is developing may also simply move through the space. Such motions are observed and this all seems to be dictated by phenomena that are in much lower layers in the Sun.

ACKNOWLEDGMENTS

The results presented here are obtained in the framework of the projects G.0304.07 (FWO-Vlaanderen), C 90347 (Prodex), GOA/2009-009 (K.U.Leuven). Financial support by the European Commission through the SOLAIRE Network (MTRN-CT-2006-035484) is gratefully acknowledged.

REFERENCES

1. J. Vranjes and S. Poedts, *EPL* **86**, 39001 (2009).
2. J. Vranjes and S. Poedts, *MNRAS* **398**, 918 (2009).
3. J. Vranjes and S. Poedts, *Phys. Plasmas* **16**, 092902 (2009).
4. J. Vranjes and S. Poedts, *MNRAS* **400**, 2147 (2009).
5. M. C. Kelley, *The Earth's Ionosphere* (Academic Press, San Diego, 1989).
6. J. Vranjes and S. Poedts, *Astron. Astrophys.* **458**, 635 (2006).
7. J. Weiland, *Collective Modes in Inhomogeneous Plasmas* (Institute of Physics Pub., Bristol, 2000).
8. P. M. Bellan, *Fundamentals of Plasma Physics* (Cambridge Univ. Press, Cambridge, 2006).
9. J. Vranjes and S. Poedts, *Astron. Astrophys.* **503**, 591 (2009).
10. J. M. McChesney, R. A. Stern, and P. M. Bellan, *Phys. Rev. Lett.* **59**, 1436 (1987).
11. J. M. McChesney, P. M. Bellan, and R. A. Stern, *Phys. Fluids B* **3**, 3363 (1991).
12. S. J. Sanders, P. M. Bellan, and R. A. Stern, *Phys. Plasmas* **5**, 716 (1998).
13. J. Vranjes and S. Poedts, *Astron. Astrophys.* **482**, 653 (2008).
14. T. J. Janssens, *Solar Phys.* **27**, 149 (1972).
15. E. B. Mayfield and G. A. Chapman, *Solar Phys.* **70**, 351 (1981).
16. M. I. Pudovkin, S. A. Zaitseva, N. O. Shumilov, and C. V. Meister, *Solar Phys.* **178**, 125 (1998).
17. L. P. van Speybroeck, A. S. Krieger, and G. S. Vaiana, *Nature* **227**, 818 (1970).