Reduced-Order-Model Interpolation for use in Global Modal Parameterization

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Abstract
Interpolation of reduced order models (ROM) is an important topic in many areas of model reduction. Recently, a system-level model reduction technique for flexible multibody systems, Global Modal Parameterization (GMP), has been proposed. This method is based on an interpolation of ROMs for different undeformed configurations in order to reduce model equation assembly time. As is shown in this paper, the way this interpolation is performed significantly impacts the accuracy of the simulation. This work compares 3 different interpolation strategies. Firstly two strategies which directly interpolate the reduced mass and stiffness matrix, through linear and quadratic interpolation, are considered. A third novel interpolation strategy, which operates in the eigenspace of the ROMs, is proposed in order to obtain better accuracy for the eigenfrequencies of the interpolated system. Finally the three approaches are compared through a numerical validation on a planar slider-crank mechanism.

1 Introduction

Interpolation of reduced order models (ROMs) is an issue in model order reduction for nonlinear and parameterized systems [12, 5, 9, 2, 1, 7]. The main issue is that there is not one single definition of what is expected of the interpolated model. In general it is imperative that the interpolated model is close to the exact model, but how this *closeness* is defined will be mostly case-dependent.

In this work, interpolation of ROMs for the simulation of reduced flexible multibody models is examined. Flexible multibody simulation is an application which typically requires large computational effort due to nonlinearity, a large number of degrees-of-freedom (DOFs) and differential-algebraic equations of motion. Recently, Brüls introduced the Global Modal Parameterization (GMP) as a reduction technique for flexible multibody simulations [5]. In this method, the system dynamics are parameterized by a minimal set of rigid DOFs. The method is split up into two parts: a preprocessing in which the reduced system properties are determined for a discretized set of possible configurations, and an actual processing phase in which the reduced system is simulated and system properties are obtained through interpolation from the previously calculated configurations.

The way this interpolation is performed, strongly affects the performance of the reduced simulation, and this work aims at examining which interpolation approach delivers the best results. For this purpose three different interpolation approaches are compared. The first two approaches are the linear and quadratic interpolation of the projection vectors and reduced mass- and stiffness matrices. This is a classical approach

which keeps both the stiffness and mass matrices close to those of the uninterpolated model at a given configuration. However, this approach generally does not assure a correspondence of the reduced system dynamics, since there is no explicit control on the eigenfrequencies of the interpolated system. Sec. 4 shows that these approaches can lead to large deviations on the eigenvalues, even though both mass and stiffness matrices show good correspondence. The third approach is a novel method in which the reduced mass and stiffness matrices are projected onto the eigenspace of the system and are interpolated in this eigenspace before transforming back to the original reduced space. This method allows direct interpolation of the eigenvalues of the reduced system and is called *frequency interpolation* in the remainder of this work. The numerical experiment in Sec. 4 demonstrates that this approach leads to more accurate results, even though the deviation on the mass and stiffness matrices is larger than in the case of direct interpolation.

Firstly a short overview of the GMP-method with its main characteristics is given. Section 3 describes the three proposed interpolation methods in detail. Finally a numerical validation of the GMP-method with the different interpolation strategies is performed in order to verify the properties of each method.

2 Global Modal Parameterization.

Many different formulations are used for the description of flexible multibody models and an exhaustive overview is given by Wasfy & Noor [14]. The equations of motion for a planar system can generally be written as a set of differential-algebraic equations, in the generalized coordinates $\boldsymbol{q}$:

$$\boldsymbol{M^{qq}(q)\,\ddot{q} + \mathcal{V}_{,q} + \Phi_{,q}^{T}\,\lambda} = \boldsymbol{g_{ext}^{q}} \quad (1)$$

$$\boldsymbol{\Phi(q)} = \boldsymbol{0} \quad (2)$$

In these equations:

- q is a vector of n generalized coordinates, which define the *configuration* of the system.
- $M^{qq}(q)$ is the configuration-dependent mass matrix.
- $\mathcal{V}_{,q}$ is the gradient of the internal potential energy. Only potential energy due to structural deformation is considered in this term. The generalized forces due to other potential energy sources (e.g. gravity) are taken into account through the source term g_{ext}^{q}.
- g_{ext}^{q} denotes the generalized forces due to external loads.
- $\Phi(q) = 0$ expresses m kinematic holonomic constraints[1]. Its gradient $\Phi_{,q}^{T}$ is assumed to be of full rank for all configurations q.[2]
- λ is a vector of m Lagrange multipliers to take the constraints into account.
- $\Phi_{,q}^{T}\,\lambda$ represent the reaction forces and moments enforcing the constraints.

The set of equations (1)-(2) is a differential-algebraic (DAE) set of equations of index 3. Solving this system in its original form requires implicit solution methods or specialized stabilized explicit solvers [13]. Model reduction techniques are employed to obtain a more suitable ODE description of the model for efficient simulation with standard explicit solvers.

Global Modal Parameterization (GMP), as proposed by Brüls [5], is a reduction technique which was first introduced for reduction of rigid systems [6] and afterwards adapted for the reduction of flexible mechanisms. Due to the assumptions made in deriving this method, it allows to turn the set of DAEs into a set of ODEs with far fewer DOFs.

In this work, the derivation of the model equations is given for a planar system with one rigid DOF. This allows for a more intuitive physical interpretation of the model equations.

[1] Non-holonomic constraints are not considered in this work.

[2] A gradient/Jacobian of an entity A with respect to a vector b will be denoted as $A_{,b}$.

2.1 Model reduction

The GMP-model reduction technique is based on dividing the motion into a rigid and a deformed part. In this approach, the flexible deformation is assumed to be a small linearized deviation from the rigid position. This is expressed by:

$$\boldsymbol{q} = \boldsymbol{\rho}(\boldsymbol{\theta}) + \boldsymbol{\Psi}^{\boldsymbol{q\delta}}(\boldsymbol{\theta})\boldsymbol{\delta} \tag{3}$$

The two terms in this equation are:

- $\boldsymbol{\rho}(\boldsymbol{\theta})$: this represents a nonlinear function which determines the generalized coordinates q for an undeformed configuration in function of the minimal amount of rigid DOFs θ.
- $\boldsymbol{\Psi}^{\boldsymbol{q\delta}}(\boldsymbol{\theta})\boldsymbol{\delta}$: this term expresses the flexible deformation. The deformation is determined as the product of a configuration-dependent modal matrix $\boldsymbol{\Psi}^{\boldsymbol{q\delta}}(\boldsymbol{\theta})$, which contains the deformation modes, and the respective participation factors δ, or flexible DOFs, of the deformation modes.

The reduced degrees-of-freedom can be grouped in the vector $\boldsymbol{\eta}$:

$$\boldsymbol{\eta} = \begin{bmatrix} \boldsymbol{\theta} \\ \boldsymbol{\delta} \end{bmatrix} \tag{4}$$

This specific parameterization allows a very strong reduction of DOFs in comparison to the original model. This reduction is twofold:

- Due to the system point-of-view, all redundant DOFs are eliminated.
- The description of the flexible deformation allows to only consider the most relevant deformation modes.

As can be seen from equation (3), the flexible deformation modes are also dependent on the rigid position. Eigenmodes are used as deformation modes, and these are strongly dependent on the actual configuration of the system. This description assumes that the flexible deformations remain small, such that the effect of the elastic deformation on the dynamic properties of the system remains negligible.

The projection from q to the reduced coordinates η happens through the use of rigid body $\boldsymbol{\Psi}^{\boldsymbol{q\theta}}$ and flexible deformation modes $\boldsymbol{\Psi}^{\boldsymbol{q\delta}}$ [5], which are both function of the rigid configuration θ. Different choices could be made for these modes, especially for the deformation modes [10], which might highly impact the obtained accuracy for a given number of reduced DOFs. In this work, the rigid motion vector is the null-vector of the stiffness matrix and the deformation modes are the most dominant flexible eigenmodes when the rigid DOF θ is fixed, as in [5]. This choice leads to a clear distinction between rigid and flexible motion, but also leads to a mode set which is not mass orthogonal.

2.2 Model equation derivation

As in [3], the equations of motion for the reduced model are derived from the augmented Lagrangian for the system. For an undamped multibody system, the Lagrangian $\mathcal{L}$ consists of kinetic energy $\mathcal{K}$, potential energy $\mathcal{V}$ and external work $\mathcal{W}$:

$$\mathcal{L} = \mathcal{K} - \mathcal{V} - \mathcal{W} \tag{5}$$

In order to take the constraints into account, the augmented Lagrangian $\mathcal{L}^*$ is employed:

$$\mathcal{L}^* = \mathcal{L} + \boldsymbol{\Phi}(\boldsymbol{q})^T \boldsymbol{\lambda} \tag{6}$$

In this formalism, auxiliary variables λ are used to take the constraints $\boldsymbol{\Phi}(\boldsymbol{q})$ into account. For a system with m constraints, there are m auxiliary variables λ.

From the expression of the augmented Lagrangian, the equations of motion can be derived. The reduced equations of motion are obtained by computing the derivative of $\mathcal{L}^*$, which is a function of the generalized coordinates q, with respect to the reduced degrees-of-freedom η and λ:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}^*}{\partial \dot{\boldsymbol{\eta}}}\right) - \frac{\partial \mathcal{L}^*}{\partial \boldsymbol{\eta}} = \frac{d}{dt}\left(\frac{\partial \mathcal{K}}{\partial \dot{\boldsymbol{\eta}}}\right) - \frac{\partial \mathcal{K}}{\partial \boldsymbol{\eta}} + \frac{\partial \mathcal{V}}{\partial \boldsymbol{\eta}} - \frac{\partial \mathcal{W}}{\partial \boldsymbol{\eta}} + \frac{\partial \Phi^T}{\partial \boldsymbol{\eta}}\lambda = 0 \tag{7}$$

$$\Phi(q) = 0 \tag{8}$$

This equation forms the basis of the model equations for the GMP-method.

In order to compute the different derivatives for the reduced DOFs, the derivative of the generalized coordinates q with respect to the reduced DOFs η is required. For a system with only one rigid DOF:

$$\boldsymbol{q}_{,\eta} = \begin{bmatrix} \boldsymbol{\Psi}^{q\theta} + \frac{\partial \boldsymbol{\Psi}^{q\delta}}{\partial \boldsymbol{\theta}}\boldsymbol{\delta} & \boldsymbol{\Psi}^{q\delta} \end{bmatrix} \tag{9}$$

The reduced equations of motion can be summarized as the sum of projected inertial forces $\boldsymbol{g}^{\boldsymbol{\eta}}_{\boldsymbol{iner}}$, projected internal elastic forces $\boldsymbol{g}^{\boldsymbol{\eta}}_{\boldsymbol{int}}$ and projected external forces $\boldsymbol{g}^{\boldsymbol{\eta}}_{\boldsymbol{ext}}$:

$$\boldsymbol{g}^{\boldsymbol{\eta}}_{\boldsymbol{iner}} + \boldsymbol{g}^{\boldsymbol{\eta}}_{\boldsymbol{int}} = \boldsymbol{g}^{\boldsymbol{\eta}}_{\boldsymbol{ext}} \tag{10}$$

This equation is an ODE and does not contain any terms related to the constraints due to the choice of the projection space.

In the following paragraphs, the different force terms resulting from the GMP formalism are discussed.

2.2.1 Inertial Forces

The inertial forces are obtained by deriving the kinetic energy $\mathcal{K}$ to the reduced DOFs. Details of this derivation are discussed by Brüls [3]. As derived by Naets [11], for a planar system with one rigid DOF and an unreduced description with a constant mass matrix, the reduced inertial forces are:

$$\begin{aligned} \boldsymbol{g}^{\boldsymbol{\eta}}_{\boldsymbol{iner}} &= \boldsymbol{q}_{,\boldsymbol{\eta}}{}^T \boldsymbol{M}^{\boldsymbol{qq}} \boldsymbol{q}_{,\boldsymbol{\eta}} \ddot{\boldsymbol{\eta}} + \boldsymbol{q}_{,\boldsymbol{\eta}}{}^T \boldsymbol{M}^{\boldsymbol{qq}} \left(\frac{\partial \boldsymbol{\Psi}^{\boldsymbol{q\theta}}}{\partial \boldsymbol{\theta}} \dot{\boldsymbol{\theta}}^2 + 2 \frac{\partial \boldsymbol{\Psi}^{\boldsymbol{q\delta}}}{\partial \boldsymbol{\theta}} \dot{\boldsymbol{\theta}} \dot{\boldsymbol{\delta}} + \frac{\partial^2 \boldsymbol{\Psi}^{\boldsymbol{q\delta}}}{\partial \boldsymbol{\theta}^2} \dot{\boldsymbol{\theta}}^2 \boldsymbol{\delta} \right) \\ &= \boldsymbol{M}^{\boldsymbol{\eta\eta}} \ddot{\boldsymbol{\eta}} + \boldsymbol{h}^{\boldsymbol{\eta}} \left(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}} \right) \end{aligned} \tag{11}$$

In Equation (11), the first term is the obvious projection of the inertial force onto the reduced space and the second term, denoted as h^η, arises due to the variable projection vectors.

2.2.2 Elastic Forces

The elastic forces are obtained by deriving the potential internal energy $\mathcal{V}$ of the system with respect to the reduced DOFs. Since this function is highly nonlinear, a Taylor-approximation around an undeformed configuration is used in order to derive the reduced internal forces [5]. This approach leads to:

$$\boldsymbol{g}^{\boldsymbol{\eta}}_{\boldsymbol{int}} = \boldsymbol{K}^{\boldsymbol{\eta\eta}} \boldsymbol{\eta} \tag{12}$$

Where $\boldsymbol{K}^{\boldsymbol{\eta\eta}}$ is the reduced stiffness matrix:

$$\boldsymbol{K}^{\boldsymbol{\eta\eta}} = \begin{bmatrix} 0 & 0 \\ 0 & (\Psi^{q\delta})^T \left(\frac{\partial^2 \mathcal{V}}{\partial \boldsymbol{q}^2} \right)_0 \Psi^{q\delta} \end{bmatrix} \tag{13}$$

2.2.3 External forces

The reduced external forces are obtained by projecting the unreduced forces directly onto the reduced projection space:

$$\boldsymbol{g}^{\boldsymbol{\eta}}_{\boldsymbol{ext}} = \boldsymbol{q}_{,\boldsymbol{\eta}}{}^T \boldsymbol{g}^{\boldsymbol{q}}_{\boldsymbol{ext}} \tag{14}$$

2.3 Practical implementation

During the simulation, the unreduced model will not be evaluated during each timestep and then reduced, since this would be much too costly due to the eigenvalue problem which has to be solved for model reduction. Instead, the reduction procedure is performed for a grid of possible undeformed configurations during a preprocessing step. These reduced models are stored such that they could be used during the actual simulation. Because it is highly unlikely that the mechanism will pass exactly through the predetermined states during simulation, interpolation between the precomputed configurations has to be performed to determine the ROM for the simulation configurations. It is of paramount importance that the interpolated model exhibits dynamical behaviour which is very similar to the exact ROM for a given configuration, in order to obtain reliable simulation results. Sec. 3 describes three different approaches to this interpolation and their effect on the accuracy of the simulation is validated numerically in Sec. 4.

3 Interpolation methods

The used interpolation method will highly impact the amount of discretization points which is necessary in order to obtain a certain accuracy. There will however be a strong trade-off between the accuracy of a method and the computational complexity. It is therefore important to correctly define the requirements for the interpolated model. It will be very application-dependent which properties the interpolated model should keep and which are less strict.

In the case of GMP, the requirement of the interpolated reduced model is simply that it is as close as possible to the exact reduced model for a given configuration. However, there is no strict demand on whether this means the mass and stiffness matrices should be as close as possible, or the eigenfrequencies should be matched as good as possible. No matter what method is used, a compromise will always be necessary.

In this section, three possible interpolation methods are discussed: linear interpolation of the system matrices, quadratic interpolation of the system matrices and a novel approach which linearly interpolates the eigenfrequencies of the reduced system.

3.1 Linear interpolation

The first method is to perform a linear interpolation of the reduced system matrices. The reduced mass, stiffness and projection matrix are all interpolated linearly in order to become an approximate model for a given configuration. In the discretization points this approach will provide exact results, but in between these points the results might deviate considerably. The used formulas are:

$$\theta = a\theta_i + (1-a)\theta_{i+1} \quad \text{with} \quad |a| < 1 \tag{15}$$

$$\boldsymbol{M}^{\eta\eta} = a\boldsymbol{M}_{\boldsymbol{i}}^{\eta\eta} + (1-a)\boldsymbol{M}_{\boldsymbol{i+1}}^{\eta\eta} \tag{16}$$

$$\boldsymbol{K}^{\eta\eta} = a\boldsymbol{K}_{\boldsymbol{i}}^{\eta\eta} + (1-a)\boldsymbol{K}_{\boldsymbol{i+1}}^{\eta\eta} \tag{17}$$

$$\boldsymbol{\Psi}^{q\eta} = a\boldsymbol{\Psi}_{\boldsymbol{i}}^{q\eta} + (1-a)\boldsymbol{\Psi}_{\boldsymbol{i+1}}^{q\eta} \tag{18}$$

In case the projection modes were mass orthonormal, this approach would also lead to an interpolation of the eigenvalues. In this case the mass matrix is the unity matrix and the stiffness matrix is a diagonal matrix with the eigenvalues on the diagonal. The interpolation maintains the diagonal structure and the eigenvalues are interpolated correspondingly. However, often, as in this work, a non-mass-orthogonal set of modes is used, in which case there is no direct interpolation of the eigenvalues and deviations on the eigenfrequencies of the interpolated model might differ considerably from those of the original model.

3.2 Quadratic interpolation

A second popular method is to perform quadratic interpolation. The accuracy of the results obtained through this approach will be similar to those obtained through other quadratic approximations such as piecewise quadratic functions or Lagrangian polynomials [5]. This approach will generally lead to much more accurate results than the linear approach, but comes at an added cost of determining the coefficients for the interpolation, which is very straight forward in the case of the linear interpolation. All matrices are again interpolated directly as is the case for the linear interpolation.

In order to determine the coefficients, the knowledge of the analytical solution for a 2×2-system is used. Interpolation is performed between 3 points θ_{i-1},θ_i and θ_{i+1}:

$$Y = A(\theta - \theta^{i-1})^2 + B(\theta - \theta^{i-1}) + C \tag{19}$$

$$A = \frac{(\theta_{i+1}-\theta_{i-1})(Y_i-Y_{i-1})-(\theta_i-\theta_{i-1})(Y_{i+1}-Y_{i-1})}{(\theta_i-\theta_{i-1})^2(\theta_{i+1}-\theta_{i-1})-(\theta_i-\theta_{i-1})(\theta_{i+1}-\theta_{i-1})^2} \tag{20}$$

$$B = \frac{-(\theta_{i+1}-\theta_{i-1})^2(Y_i-Y_{i-1})+(\theta_i-\theta_{i-1})^2(Y_{i+1}-Y_{i-1})}{(\theta_i-\theta_{i-1})^2(\theta_{i+1}-\theta_{i-1})-(\theta_i-\theta_{i-1})(\theta_{i+1}-\theta_{i-1})^2} \tag{21}$$

$$C = Y_{i-1} \tag{22}$$

All elements of the reduced system-matrices and projection modes are interpolated following this process. As is the case for the linear interpolation, this approach only interpolates the matrices and doesn't guarantee a certain course of the reduced eigenvalues. Again here the eigenvalues are interpolated in the case of mass orthogonal modes, but not in the general case.

3.3 Frequency interpolation

This new interpolation method is derived in order to maintain the best correspondence between the eigenvalues of the interpolated reduced system and the exact reduced system. As is shown in Sec. 4, regular interpolation of the system matrices might lead to large deviations on the eigenvalues. These eigenvalues have a large impact on the flexible response of a system. In case of e.g. active vibrational damping, a poor approximation of the eigenfrequencies might lead to poor corrective actions.

In order to achieve an interpolation of the eigenfrequencies, a intermediate transformation onto the eigenspace of the reduced model, through the mass-orthogonal eigenmodes Φ_i and Φ_{i+1}, is used. The projected matrices are now diagonal and are interpolated linearly, as described earlier, which preserves the structure and interpolates the eigenfrequencies as well.

$$\hat{\boldsymbol{M}}^{\eta\eta} = a{\boldsymbol{\Phi_i}}^T \boldsymbol{M_i^{\eta\eta}} \boldsymbol{\Phi_i} + (1-a){\boldsymbol{\Phi_{i+1}}}^T \boldsymbol{M_{i+1}^{\eta\eta}} \boldsymbol{\Phi_{i+1}} \tag{23}$$

$$\hat{\boldsymbol{K}}^{\eta\eta} = a{\boldsymbol{\Phi_i}}^T \boldsymbol{K_i^{\eta\eta}} \boldsymbol{\Phi_i} + (1-a){\boldsymbol{\Phi_{i+1}}}^T \boldsymbol{K_{i+1}^{\eta\eta}} \boldsymbol{\Phi_{i+1}} \tag{24}$$

The new diagonal matrices $\hat{\boldsymbol{M}}^{\eta\eta}$ and $\hat{\boldsymbol{K}}^{\eta\eta}$ are now available and have to be converted back to the original reduction space. A linear interpolation of the eigenmodes is used to perform the back-transformation:

$$\boldsymbol{\Phi} = a\boldsymbol{\Phi_i} + (1-a)\boldsymbol{\Phi_{i+1}} \tag{25}$$

$$\boldsymbol{M^{\eta\eta}} = (\boldsymbol{\Phi}^T)^{-1} \hat{\boldsymbol{M}}^{\eta\eta} \boldsymbol{\Phi}^{-1} \tag{26}$$

$$\boldsymbol{K^{\eta\eta}} = (\boldsymbol{\Phi}^T)^{-1} \hat{\boldsymbol{K}}^{\eta\eta} \boldsymbol{\Phi}^{-1} \tag{27}$$

The projection modes are interpolated correspondingly:

$$\boldsymbol{\Psi^{q\eta}} = (a\boldsymbol{\Psi_i^{q\eta}}\boldsymbol{\Phi_i} + (1-a)\boldsymbol{\Psi_{i+1}^{q\eta}}\boldsymbol{\Phi_{i+1}})\boldsymbol{\Phi}^{-1} \tag{28}$$

This approach delivers an interpolated model with a reliable approximation of the eigenfrequencies. With this method however, the course of the mass matrix, stiffness matrix and projection modes will not be a

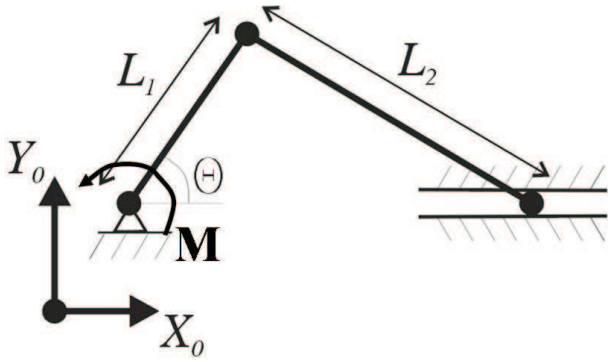


Figure 1: Planar slider-crank mechanism

direct interpolation anymore, which might lead to a larger deviation in these areas. Moreover this method will also be more expensive than the previous 2 methods since 2 eigenvalue problems have to be solved for each interpolation. Due to the limited dimension of the reduced system, the cost of this eigenvalue problem is expect to remain limited. Alternatively this eigenvalue problem could also be solved during the preprocessing phase in order to save online calculation time.

4 Numerical validation

The numerical validation of the aforementioned interpolation methods is performed on a planar slider-crank mechanism as depicted in Fig. 1. The properties of the system are summarized in Table 1. The original model of this system is constructed using a planar version of the large-rotation finite element (FE) model as proposed by Géradin & Cardona [8]. This FE model serves as a basis to construct the GMP model and is used as a benchmark during the simulation. The GMP-model is constructed with 7 deformation modes (8

properties	Crank	Connector
$E[GPa]$	210	210
$G[GPa]$	81	81
$L[m]$	0.3	0.7
$A[cm^2]$	1	1
$\rho[kg/m]$	7800	7800
$\#elem.$	5	5

Table 1: Properties of FE slider-crank model

modes in total, including the rigid mode) and a discretization step $\Delta\theta = 0.1rad$.

Firstly the the accuracy of the different interpolation methods for the mass and stiffness matrices and for the eigenvalues are considered between two discretization points. In a second subsection, a simulation of the mechanism is performed and the accuracy with respect to the unreduced model is compared.

4.1 Interpolation accuracy

Firstly the accuracy of the three interpolation strategies is compared for the mass matrix, the stiffness matrix and the eigenvalues for the reduced system. The original reduction space is constructed with a discretization step of $\Delta\theta = 0.1rad$ and an interpolation is performed with an interval of $\Delta\theta = 0.01rad$.

The average relative difference on the mass and stiffness matrix in function of the rigid configuration θ is depicted in Fig. 2. Fig. 2 shows that the best results are obtained with the quadratic interpolation. This is

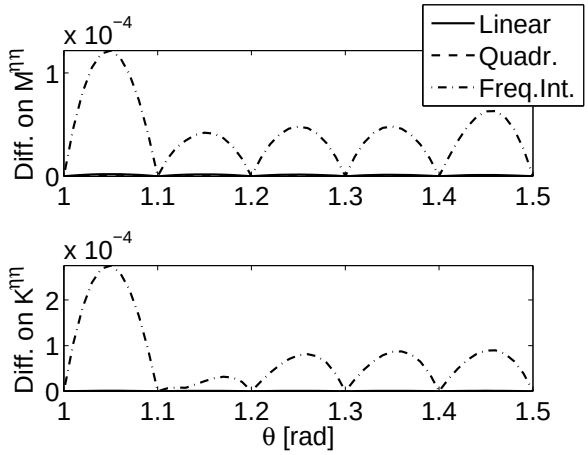


Figure 2: Average accuracy for interpolated reduced mass and stiffness matrix in between discretization points

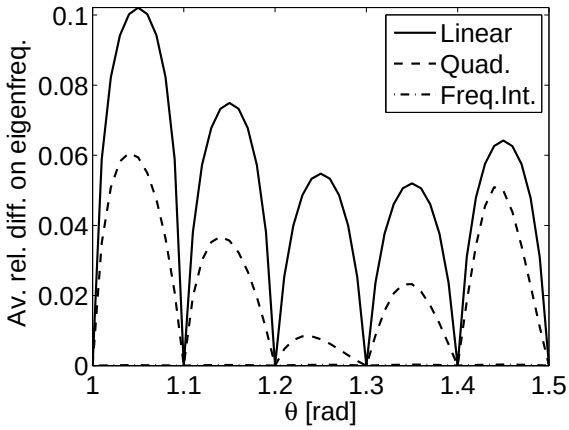


Figure 3: Comparison of eigenvalues in between discretization points

to be expected since this method uses the most information about the reduction space and interpolates both matrices directly. The quadratic interpolation is followed by the linear interpolation, because this method still works directly on both matrices. The frequency interpolation however, clearly produces less accurate results. This is due to the fact that this method actually interpolates the eigenfrequencies, which might lead to deviations for the mass and stiffness matrix.

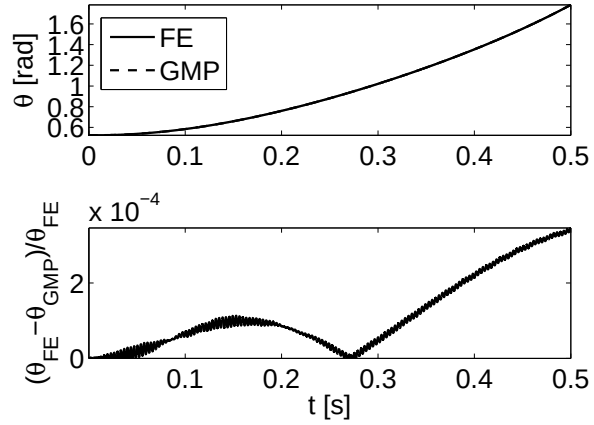
Fig.3 shows the average relative difference between the eigenvalues of the exactly reduced model and the interpolated reduced models. This figure clearly shows that the interpolation methods which work directly on the projection space, lead to important deviations in the eigenfrequencies of the reduced system. These deviations might lead to quite important changes in the dynamic behaviour of the interpolated system in comparison to the exact system. The frequency interpolation on the other hand delivers much better accuracy at the level of eigenfrequencies.

Based on both Fig. 2 and Fig.3,no clear answer is possible on which approach is best. The direct interpolation approaches will generally be the cheapest, while the frequency interpolation seems better capable of describing the dynamical phenomena of the reduced model. The effect on actual simulation results is presented in the following section.

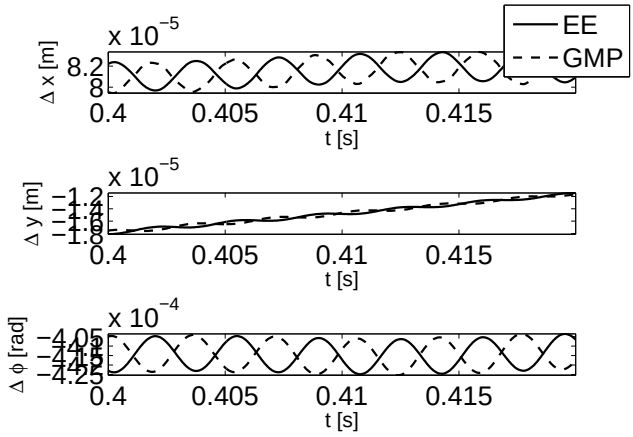
4.2 Simulation accuracy

This section shows the simulation results of the slider-crank mechanism when the rigid DOF is excited with a torque with amplitude $0.5Nm$ for the different interpolation schemes. At the beginning of the simulation the torque has sinusoidal ramp up with a frequency of $50Hz$. The simulation is performed using the generalized α-solver [4] with a constant timestep $h = 0.05ms$ and no numerical damping. The same GMP-model as in the previous section, with a discretization step of $\Delta\theta = 0.1rad$ and 7 flexible modes is used. For the inertial forces, the full inertial forces as described by Naets [11] are employed.

The results for the simulation of the GMP-model with linear interpolation are compared with those of the unreduced FE-model in Fig. 4. Fig. 4a shows the response of the rigid DOF θ and the relative difference



(a) θ and relative difference

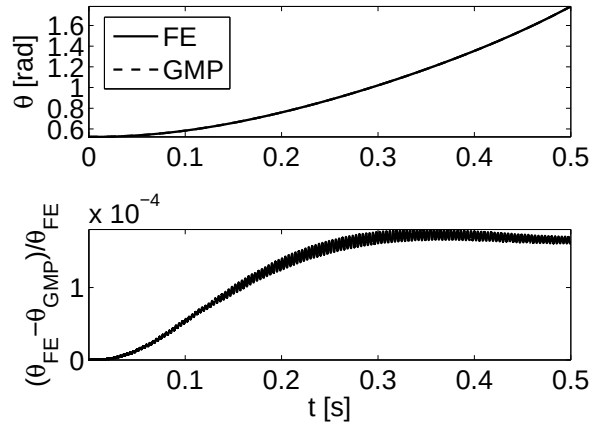


(b) Deformation of hinge-node

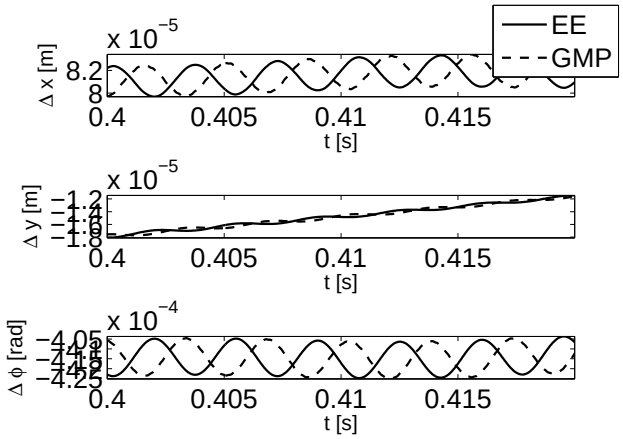
Figure 4: Simulation results with linear interpolation

with the unreduced model. Even though a very simple interpolation is employed and the discretization step is rather coarse, the rigid motion is quite accurately approximated. Fig. 4b shows the flexible deformation for the last node of the crank. This figure shows only a fragment of the total simulation to improve interpretability. The figure clearly shows that the flexible deformation is out of phase due to an accumulated effect of eigenfrequency estimation errors between the reduced model and the FE-model. This difference is due to the poor interpolation properties of the linear interpolation with respect to the frequency content.

For the GMP-model with quadratic interpolation, the simulation results are shown in Fig. 5. The accuracy for the rigid motion is improved in case of quadratic interpolation. Fig. 5b does however show that no noticeable improvement is present for the frequency of the flexible deformation. Even though the quadratic approach is



(a) θ and relative difference



(b) Deformation of hinge-node

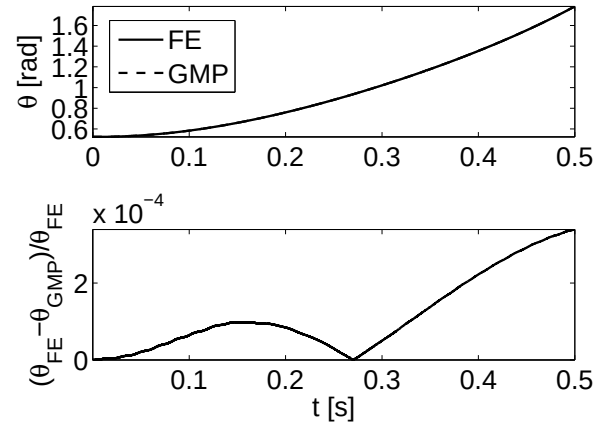
Figure 5: Simulation results with quadratic interpolation

capable in general to deliver better results than the linear approach, it also needs a rather fine discretization in order to reach good frequency accuracy in the case of non mass-orthogonal modes.

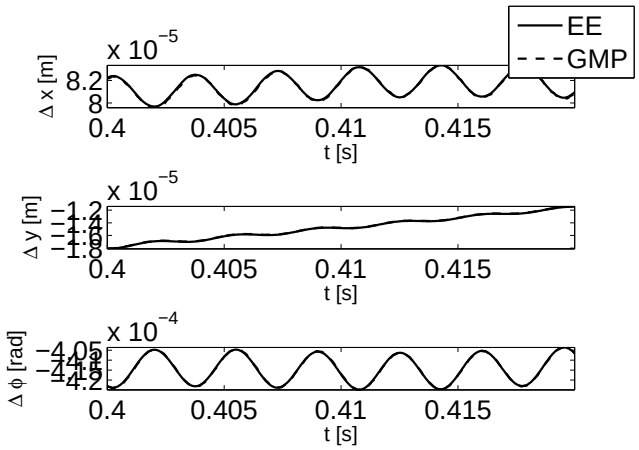
Finally, Fig. 6 shows the simulation results for the frequency interpolation. Fig. 6a shows that the accuracy for the rigid motion is similar to the linear interpolation. This can be expected, as after projection onto the eigenspace, a linear interpolation is performed. However the high frequency content of the relative difference is clearly reduced by employing this approach. When regarding the flexible deformation of the hinge-node in Fig. 6b, the advantage of the frequency interpolation becomes apparent. The flexible deformation of both the unreduced and reduced model are very similar and the difference is imperceivable on this scale.

5 Conclusion

Interpolation of reduced order models (ROMs) is an issue in many areas, such as nonlinear model reduction. One such method, in which ROM-interpolation is of paramount importance, is the Global Modal Parameterization (GMP). This method allows the reduction of flexible multibody models, which typically exhibit strong nonlinear behaviour. The GMP-method is based on a precomputed discretization of the possible configuration space of the system. During the actual simulation, the reduced model is evaluated by interpolating from this precomputed grid of possible configurations. This approach avoids the expensive evaluation of a multibody model during the simulation. It is however important that the interpolation procedure delivers



(a) θ and relative difference



(b) Deformation of hinge-node

Figure 6: Simulation results with frequency interpolation

physically valid approximations for the exact model.

In previous works on the GMP method, polynomial based interpolation schemes, such as linear or quadratic interpolation schemes, were used to directly interpolate the reduction modes, mass- and stiffness matrix. These are very straightforward and economical approaches which deliver good results in many cases. However, in general, these methods provide no certainty on the course of the eigenfrequency of the reduced model. This work demonstrates experimentally that the deviations of the eigenvalues for the interpolated system and the exact system might become quite large, even though the mass- and stiffness matrices are interpolated quite accurately. These deviations might lead to large differences in the flexible motion for the simulation of mechanisms which are excited with a high frequency content, as is shown through the numerical example of a slider-crank mechanism.

Due to the aforementioned problems, a novel interpolation technique is introduced in this work. The technique is based on a projection on the eigenspace of the reduced model, in which the eigenvalues of the model can be interpolated directly. Afterwards a back-transformation to the general reduced coordinates is performed. This approach allows to generate an interpolated ROM with interpolated eigenvalues, at the expense of a larger deviation in the reduced mass- and stiffness matrix, as is shown experimentally. The ability of this method to improve the flexible response of the GMP-model is demonstrated on a slider-crank mechanism. For this numerical example, the flexible deformation for the GMP-model is almost indistinguishable from the original FE-model, in contrast to the GMP-model with interpolation of the system matrices.

These results show the importance of adequate interpolation techniques for ROMs and allow the user of GMP to perform much more accurate highly dynamical simulations.

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