OInduced: An Efficient Algorithm for Mining Induced Patterns from Rooted Ordered Trees

Mostafa Haghir Chehreghani, Morteza Haghir Chehreghani, Caro Lucas, and Masoud Rahgozar

***Abstract*—Frequent tree patterns have many practical applications in different domains such as XML mining, web usage analysis, etc. In this paper, we present *OInduced*, a novel and efficient algorithm for finding frequent ordered induced tree patterns. *OInduced* uses a breadth-first candidate generation method and improves it by means of an indexing scheme. We also introduce frequency counting using tree encoding. For this purpose, we present two novel tree encodings, *m-coding* and *cm-coding*, and show how they can restrict nodes of input trees and compute frequencies of generated candidates. We perform extensive experiments on both real and synthetic datasets to show efficiency and scalability of *OInduced*.**

***Index Terms*—Rooted ordered labeled tree, frequent tree pattern, induced subtree, breadth first candidate generation, frequency counting, tree encoding.**

I. Introduction

MINING frequent tree patterns is very useful in domains such as user web log analysis, XML document mining, web mining, bioinformatics and network routing. For example, in [35], tree patterns are used as a powerful tool to distinguish users according to their behavior on the web. In this work, first, log data are converted into rooted ordered trees and a set of frequent patterns is extracted from them. Then, based on these patterns, a structural classifier is built to classify different users. Structural classifiers show higher performance compared to traditional classifiers which treat each tree as a bag of words [35].

In this paper, we focus on the problem of extracting induced patterns from a database of rooted ordered trees. Several algorithms have been proposed to find induced patterns from a collection of rooted ordered tree. The well-known algorithm in this context is *FREQT* [2]. FREQT uses an occurrence-list based approach for frequency counting. For each subtree, all the nodes in the database are stored in a list in which the rightmost node of the subtree can be mapped. The size of the occurrence list kept for each frequent pattern can be large ($O(|V|)$, where $|V|$ is the number of nodes of the database). This makes the algorithm inefficient, especially for dense datasets in which the correlation among trees is very high.

Recently, *iMB3Miner* [22] tries to restrict invalid candidates using a tree model guided approach. For frequency counting, iMB3Miner uses the information gathered for guided candidate generation. However, the amount of this information is high. Each occurrence of a candidate C is encoded as an occurrence coordinator whose size is $|C|$.

In this paper, we develop more efficient data structures for storing the information used in frequency counting. To do so, we initiate frequency counting based on tree encoding. The key contributions of our work are as follows:

1) We develop a new equivalence class extension to extend each candidate by only frequent trees. We use breadth first search and take advantage of an indexing scheme to perform the class extension, effectively.
2) We present two new tree encodings and accordingly, develop a novel and efficient approach for frequency counting. We show that successful occurrences of a candidate must satisfy a number of conditions and the presented tree encodings can check the conditions, efficiently. The size of each occurrence in the proposed method is $O(1)$.
3) We introduce a new and efficient algorithm, called *OInduced*, for the problem of finding all the frequent induced ordered tree patterns from a single tree or from a forest of trees. We compare *OInduced* with most efficient previous works, and by performing extensive experiments, we show that *OInduced* provides significant improvements for both real data and synthetic data.

The rest of this paper is organized as follows. In section 2, some preliminaries and definitions related to tree mining and tree patterns are given. In section 3, we have a brief overview on the related works. Section 4 describes our proposed candidate generation method. In section 5 we present two new tree encodings as well as the method used for frequency counting. We experimentally evaluate the effectiveness of *OInduced* in section 6. Finally, the paper is concluded in section 7.

II. Preliminaries and problem statement

To explain the problem of mining frequent tree patterns in a collection of trees we provide the following definitions:

a) Rooted labeled tree: A rooted labeled tree $T = (V, E, L)$ is a connected directed acyclic graph (DAG) with V as the set of nodes and $E = \{(x,y)|x, y \in V\}$ as the set of edges. $L : V \to \mathbb{N}$ is a labeling function that assigns an integer to each node of the tree. A distinguished node r is

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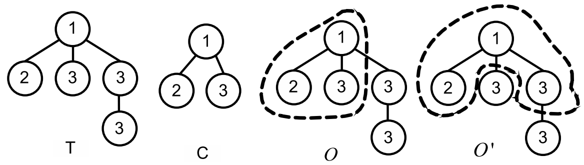


Fig. 1: An example of occurrences. O and O' are two occurrences of C in T.

considered as the root, and for any other node x, there is a unique path from r to x. A *rooted labeled ordered tree* has a left-to-right ordering among each set of siblings.

b) Zaki's string representation: Zaki's string representation S for a tree T is defined as follow: labels of the nodes are added to S in the preorder traversal of T, and when a backtracking from a child to its direct parent occurs, a unique symbol (e.g. -1) is added to S [32]. For convenience, through the paper, we present each tree by its string representation. For example, tree T of Figure 1 is presented as "1 2 -1 3 -1 3 3".

c) Induced subtree: For a rooted labeled tree $T = (V, E, L)$, a rooted labeled tree $T' = (V', E', L')$ is an *induced subtree* of T (or T' is *isomorphic* to a subtree of T), if and only if: (1) $V' \subseteq V$, (2) $E' \subseteq E$, (3) $L' \subseteq L$ and the labeling of V' in T is preserved in T' and (4) if defined for rooted ordered trees, the left-to-right ordering among the siblings in T is preserved among the corresponding nodes in T'.

If a k-candidate (a candidate tree with k nodes) C_k is an induced subtree of an input tree T, an *occurrence* O_k of C_k in T is the subtree of T which is isomorphic to C_k. Two distinct occurrences can share some nodes in common, but they cannot consist of entirely the same nodes. For example, in Figure 1, T is an input tree, C is a candidate, and O and O' are two occurrences of C in T. O and O' share two nodes in common: the nodes with lables 1 and 2.

d) Embedded subtree: For a rooted labeled tree $T = (V, E, L)$, a rooted labeled tree $T' = (V', E', L')$ is an embedded subtree of T if and only if: (1) $V' \subseteq V$, (2) v_1 is the parent of v_2 in T' if v_1 is an ancestor of v_2 in T, (3) $L' \subseteq L$ and the labeling of V' in T is preserved in T' and (4) if defined for rooted ordered trees, the left-to-right ordering among the siblings in T is preserved among the corresponding nodes in T'.

e) Per-tree support (per-tree frequency, per-transaction frequency) and occurrence-match support (occurrence-match frequency): Given a database D consisting of rooted ordered labeled trees and a subtree S the per-tree support (or per-tree frequency) of S is the number of trees in D for which S is an induced subtree. The occurrence-match support (or occurrence-match frequency) of S is defined as the number of occurrences of S in D. Per-tree support can be expressed more formally as follows:

$$support_T(S, D) = \sum_{T \in D} IsInd(S, T)$$

where $IsInd(S, T)$ is 1 if S is the induced subtree of T and 0 otherwise. Occurrence-match support can be represented as follows:

$$support_O(S, D) = \sum_{T \in D} NumInd(S, T)$$

where $NumInd(S, T)$ is the number of occurrences of S in T.

f) Frequent tree: Tree C is frequent if its per-tree support (occurrence-match support) is more than or equal to a user-specified per-tree (occurrence-match) $minsup$ value. The problem of mining frequent tree patterns in a database of tree-structured data is concerned with finding all frequent trees. The desired type of patterns in the mining process can differ based on the type of the application. In this paper, our concern is mining frequent induced patterns from rooted ordered labeled trees. Both of per-tree frequency and occurrence-match frequency are allowed in this work. There is no overall agreement on the definition of support for different applications. It seems that occurrence-match frequency is more applicable for structured data [22]. For simplicity, through the paper, we use the term frequency (support) to refer to occurrence-match frequency (occurrence-match support); unless we explicitly say that frequency (support) refers to per-tree frequency (per-tree support).

III. Related works

Recently, many algorithms have been proposed in the literature for finding frequent tree patterns from a collection of trees. Wang et al. [26] motivate the schema discovery in the general setting. They also investigate discovering typical structures from web documents and propose algorithms for discovering similar structures and structural association rules among a collection of tree-structured data [27] and [28].

Feng et al. [9] introduce an XML-enabled association rule template which is flexible to represent both simple and complex rules. They continue the work by presenting template models to help users to specify the interesting XML associations to be mined and propose techniques for template-guided mining of association rules [8].

Zaki introduces *TreeMiner* [32] to mine embedded ordered frequent tree patterns. For frequency counting, he uses a new data structure called *scope-list* and defines join operations for vertical frequency counting. TreeMiner stores each occurrence in $O(k)$ space, where k is the size of the tree. He also introduces the rightmost path extension to generate non-redundant candidates. Later, he proposes *SLEUTH* for mining embedded unordered tree patterns [33]. Asai et al. [2] independently propose the rightmost candidate generation. They developed *FREQT* for mining frequent induced ordered tree patterns. In FREQT, for each occurrence, a list stores all nodes in the database for which the rightmost node of the occurrence can be mapped.

Independently, Asai et al. and Nijssen et al. extend FREQT to discover induced unordered tree patterns and present *Unot* [3] and *uFreqt* [17] algorithms. For frequency counting, Unot uses an occurrence list based approach in which each occurrence is stored in $O(k)$ space, where k is the size of the tree. uFreqt uses a different occurrence list based approach for frequency counting that its size is bounded by the product of the size of the database and the size of the pattern.

HybridTreeMiner [6] discovers induced unordered tree patterns and uses a breadth-first candidate generation method. However, occurrence lists in HybridTreeMiner must record occurrences of a candidate in all possible orders. *PathJoin* [29] assumes that labels for the children of each node are unique and finds induced unordered maximal patterns. The number of maximal patterns is much less than the number of all the frequent tree patterns.

Chi et al. [5] propose *FreeTreeMiner* for mining induced unordered free trees. To compute the frequency of a candidate C, FreeTreeMiner uses a tree isomorphism algorithm based on bipartite graph matching. Its time complexity is $O(|T| \times |C| \times \sqrt{|C|})$, where $|T|$ and $|C|$ are the sizes of T and C, respectively.

TreeFinder [24] uses an Inductive Logic Programming approach to mine unordered, embedded subtrees, but it is not a complete method and may loose many frequent trees. *SingleTreeMining* [20] is an algorithm proposed for mining rooted unordered trees with application to phylogenetic. Chi et al. propose *CMTreeMiner* [7] for mining both closed and maximal frequent trees. This algorithm traverses an enumeration tree that systematically enumerates all subtrees, and uses an enumeration DAG to prune the branches of the enumeration tree that do not correspond to closed or maximal frequent subtrees.

Xiao et al. [30] propose *TreeGrow* for mining unordered maximal embedded tree patterns. However, TreeGrow assumes that the labels for the children of each node are unique. Their candidate generation method is localized so as to avoid unnecessary computational overhead.

The methods of [15], [16] and [21] discover frequent tree patterns in web documents by using *tag tree patterns* as hypotheses. A tag tree pattern is an edge labeled tree which has structured variables and a variable can match to an arbitrary subtree.

XSpanner [25] is a pattern growth-based method and can mine embedded ordered trees. The pseudo-projection step in XSpanner is expensive that reduces its performance. Tatikonda et al. [23] propose a generic approach for mining tree patterns. They develope *TRIPS* and *TIDES* algorithms using two sequential encodings of trees to systematically generate and evaluate the candidate patterns. However, TRIPS and TIDES can only work with per-tree support. Tan et al. [22] present a unique embedding list representation of the tree structure, which enables efficient implementation of their *Tree Model Guided* (*TMG*) candidate generation.

To find frequent unordered tree patterns, most of the proposed algorithms use a *canonical form* and extend only candidates that are in the canonical form. A canonical form is a unique way to represent a labeled tree. Luccio et al. [13], [14] define sorted pre-order string method. This method for a rooted unordered tree is defined as the lexicographically smallest one among those preorder strings of the ordered trees that can be obtained from the unordered tree. They show that for a rooted unordered tree, its canonical representation based on the pre-order traversal can be obtained in linear time, using the tree isomorphism algorithm of *Aho* [1]. Later, Asai et al. [3], Nijssen et al. [17], and Chi et al. [5] independently define similar canonical representations.

Efficient algorithms for mining frequent graph patterns which are the general form of frequent tree patterns can be found in [10], [12] and [31]. In [10] a graph transaction is represented by an adjacency matrix and frequent patterns appearing in the matrices are mined using the basket analysis algorithms. Kuramochi et al. [12] propose *FSG* to find all connected subgraphs that appear frequently in a large graph database. FSG incorporates some optimizations for candidate generation and counting to scale to large graph databases. Yan et al. [31] present *CloseGraph* for mining closed graph patterns and develop pruning techniques based on early termination.

The tree matching problem, i.e. finding occurrences of a pattern tree in a target tree is studied in [11], and several dynamic programming methods are presented. Shasha et al. [18] survey the algorithms proposed for processing queries on trees and describe algorithms for search in graphs. In [19] the authors present an algorithm to the nearest neighbor search problem for unordered labeled trees. Their algorithm is based on storing the paths of the trees in a suffix array and then counting the number of mismatching paths between a query tree and a data tree.

In general, finding frequent patterns includes two main steps: candidate generation and frequency counting. The well-known method for candidate generation in trees is the *right-most path extension* method, and *equivalence class extension* has widely been used in embedded pattern mining algorithms to improve rightmost path extension. Initial frequency counting methods, in fact, are tree matching algorithms which compute frequencies of patterns, independently. Later, vertical frequency counting methods are introduced that are highly data structure dependent. They usually define join operations on the used data structure and compute frequencies of larger candidates by joining occurrences of smaller ones.

IV. Candidate Generation

Our candidate generation method, which is in fact an extension of the well-known rightmost path extension method, generates candidates in a breadth-first way. The rightmost path extension is shown to be complete and non-redundant for generating embedded and induced candidates [2], [32] and [34]. In this method a node is added anywhere in the rightmost path of a k-candidate C and generates a $k+1$-candidate C'. In its simple form, it extends each candidate by connecting all frequent nodes to all nodes of the rightmost path.

Algorithms such as [32] try to improve candidate generation using equivalence class extension. The main observation behind equivalence class extension is that only known frequent elements are used to extend a candidate [32]. An equivalence class is defined as follows: two trees C and C' are in the same equivalence class if they differ only in the rightmost node. Equivalence class extension has been vastly used to improve embedded candidate generation. In the following, a new equivalence class based extension method is presented for induced candidate generation. Our method extends a candidate

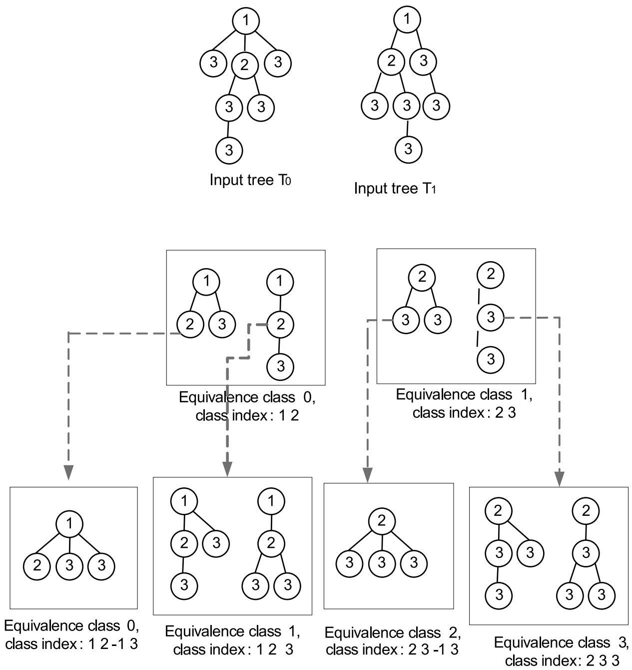


Fig. 2: An example of rp_extension. $minsup$ is equal to 2.

C through two different classes. One class is the class to which C belongs. To find the other class an efficient indexing scheme is presented.

In equivalence class extension, two k-candidates C and C, join together and the rightmost node of C, (the second tree) is added to a position in the rightmost path of C (the first tree). In the extended $k+1$-candidate, the parent of the rightmost node of C' is either the rightmost node of C or another node in the rightmost path of C. The first case, denoted by *rn_extension*, generates deeper candidates and in the second case, denoted by *rp_extension*, the number of children of the rightmost path increases (wider candidates are generated).

A. *rp_extension*

Definition 1: Position of $x \in V(T)$, denoted by $pos_T(x)$, is defined as its depth in T. The position can uniquely distinguish a node in a path. Let X be the node in the position $p-1$ of the rightmost path of T. When we say node N is added to the position p of the rightmost path of T, we mean that N becomes the rightmost child of X.

For rp_extension, our method acts as [34] proposed for embedded candidate generation: for every two candidates C and C' belonging to a same equivalence class, the rightmost node of C' is added to the position $pos_{C'}(rightmost\ node\ of\ C')$ of the rightmost path of C. If $pos_{C'}(rightmost\ node\ of\ C')$ refers to the rightmost node of C, the extension is invalid. So, we will have the following restriction for the rp_extension: $pos_C(rightmost\ node\ of\ C) \geq pos_{C'}(rightmost\ node\ of\ C')$.

Figure 2 shows an example of rp_extension in which T_0 and T_1 are two input trees, $minsup$ is equal to 2, and level 3 contains all the frequent candidates with 3 nodes. Since only frequent candidates are used for future extension, non-frequent candidates are deleted after applying a direct frequency counting operation. For example consider the tree ”1 2 3” belonging to equivalence class 0. Since the $position$ of the rightmost node of ”1 2 3”, (i.e. 2) is greater than the $position$ of the rightmost node of ”1 2 -1 3” (i.e. 1), ”1 2 3” can join with ”1 2 -1 3”. The resultant candidate, ”1 2 3 -1 -1 3”, is generated by adding the rightmost node of ”1 2 -1 3”, (i.e. ”3”) to the $position$ 1 of ”1 2 3”. ”1 2 3” can also join with itself and generate candidate ”1 2 3 -1 3”. Extension of each candidate generates a new equivalence class. Figure 2 contains all 4-candidates (frequent and non-frequent) generated via rp_extension.

B. *rn_extension*

Definition 2: Index of an equivalence class, denoted by ***E***, is defined as the tree consisting of the first $k-1$ nodes which are shared among all members of the class.

Definition 3: First $k-1$ subtree of tree T, denoted by $first_{k-1}(T)$, is the subtree generated by removing the rightmost node of T.

Definition 4: If tree T has more than one leaf, its *second rightmost leaf*, denoted by srl, is defined as the leaf which has the greatest preorder number among all the leaves except the rightmost node.

Definition 5: The last $k-1$ subtree of tree T, denoted by $last_{k-1}(T)$, is the subtree generated by removing either: 1) the root of T (if T has only one leaf), or 2) the srl of T (if T has more than one leaf).

For example, in Figure 2, $first_{k-1}$ of ”1 2 3 -1 -1 3” is ”1 2 3”, its srl is the node ”3” in *position* 2 and its $last_{k-1}$ is ”1 2 -1 3”. The $last_{k-1}$ of ”1 2 3” is ”2 3”, since it has only one leaf.

Theorem 6 helps us to find the equivalence class that rn_extends a candidate.

Theorem 6: k-candidate C_k can be rn_extended if there exists a k-candidate C'_k such that $last_{k-1}(C_k)$ and $first_{k-1}(C'_k)$ are identical.

Proof: Consider candidate C_{k+1} generated by adding a child N to the rightmost node of C_k. If another node M, $M \neq N$, is deleted from C_{k+1}, candidate C'_k is generated. Then, C_{k+1} can be generated by joining C_k and C'_k.

- M can not be an intermediate node (intermediate node is neither root nor leaf); because in this situation, removing M converts a parent-child relation into an ancestor-descendant relation and for induced patterns these relations are not equivalent.
- M can be the root of C_{k+1}. If the root of C_{k+1} has only one child, no problem arises. However, if the root of C_{k+1} has more than one child, removing the root generates a forest in which the size of each tree is smaller than k, instead of generating a single k-candidate.
- M can be an arbitrary leaf node, e.g. the srl. If C_{k+1} has more than one leaf, no problem arises. However, if C_{k+1} has only one leaf, nodes M and N will be equivalent and therefore, in this state M can not be removed.

If C_{k+1} has one leaf, its root will have only one child. So, in this case the root can be deleted. Now we can claim that

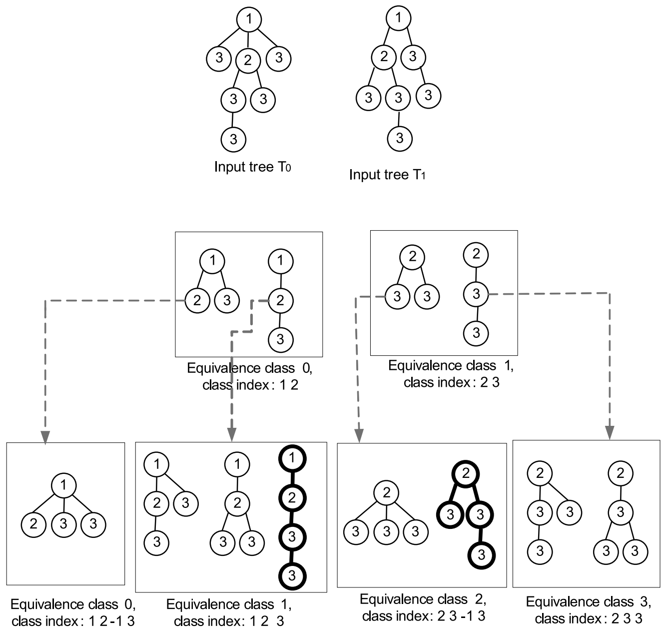


Fig. 3: An example of rn_extension. Trees with strong lines are generated via rn_extension. $minsup$ is equal to 2.

C'_k which rn_extends C_k and generates C_{k+1}, looses either the root of C_{k+1} if (C_{k+1} has only one leaf), or the srl of C_{k+1} (if C_{k+1} has more than one leaf). On the other hand, C_k looses the rightmost node of C_{k+1} and keeps its other nodes. Therefore, C_k must join with a tree that has N as the rightmost node, instead of either the srl or the root. This means that $last_{k-1}(C_k)$ and $first_{k-1}(C'_k)$ must be identical. ■

Lemma 7: All trees that can rn_extend C_k belong to a same equivalence class.

Proof: According to Theorem 6, the first $k-1$ nodes of all trees rn_extending C_k must generate the tree $last_{k-1}(C_k)$. Therefore, they belong to a same equivalence class. ■

For a C_k, since members of a single equivalence class rn_extends C_k, we can use the following notation: *an equivalence class E' rn_extends C_k*.

Lemma 8: Tree C_k can be rn_extended by an equivalence class E' if the index of E' and $last_{k-1}(C_k)$ are identical.

Proof: Directly from Theorem 6. ■

The equivalence class rn_extending a tree C_k and the equivalence containing C_k (that rp_extends C_k) can be either the same or not.

Figure 3 shows examples of rn_extension. In this figure, trees with strong lines are 4-candidates generated via rn_extension. First consider "1 2 3". This tree has only one leaf node, therefore its $last_{k-1}$ misses the root. The resultant subtree, i.e. "2 3", is the index of class 1 of level 3. Therefore, as Theorem 6 says, members of this class can rn_extend "2 3". Some of the extensions have been depicted in the figure. Now, consider "2 3 -1 3" which has two leaves, so its $last_{k-1}$ misses the srl. The resultant subtree, i.e. "2 3", is the index of class 1. So, "2 3 -1 3" can be rn_extended e.g. by "2 3 3" to generate candidate "2 3 -1 3 3".

Assume that the equivalence class E' satisfies the condition presented in Theorem 6 for rn_extension of C_k which C_k itself belongs to the equivalence class E. E and E' are at the same level (their indices have the same size), therefore our proposed method for candidate generation must construct the state space in a breadth first manner. First, C_k is rp_extended by all members of E. Then, we look for an equivalence class E' whose index is $last_{k-1}(C_k)$. If there exists such a class, the elements of E' rn_extend C_k. Figure 4 shows the high level pseudo code of our candidate generation method. Lines 4-9 demonstrate how C_k can be rp_extended and lines 11-14 show how C_k can be rn_extended. We will explain line 10 in details in the next subsection.

C. Finding the equivalence class that rn_extends a tree

An important issue is finding the equivalence class E' that rn_extends C_k. An inefficient solution is to compare $last_{k-1}(C_k)$ with all class indices, until the satisfying one is found. The class indices of a specific level and as well as the trees of a single equivalence class can be generated in an ordered way. This can improve the search process. However, still there exists a problem: although members of an equivalence class are ordered and they share k-1 prefix, their $last_{k-1}$ are not ordered. The reason is that for each tree the node which is deleted and generates $last_{k-1}$ can be either the root or the srl.

Here, we propose a simple and efficient indexing scheme to find the equivalence class rn_extending a tree. Lemma 9 and Theorem 10 provide the rationale behind the indexing scheme.

Lemma 9: Assume that tree C_{k-1} is **rp_extended** by tree C'_{k-1} and generates tree C_k. Then, C'_{k-1} will be $last_{k-1}(C_k)$.

Proof: Since the rightmost node of C'_{k-1} is added to a non-leaf node of C_{k-1} and generates a new leaf, C_k has more than one leaf. On the other hand, when the rightmost leaf of C'_{k-1} is added to C_{k-1}, the rightmost leaf of C_{k-1} will be the second rightmost leaf of the resultant tree C_k. Since C_{k-1} and C'_{k-1} belong to the same equivalence class, they share the first k-1 nodes. So removing the node corresponding to the rightmost node of C_{k-1} from C_k (which is the srl of C_k), will generate C'_{k-1}. This means that C'_{k-1} is $last_{k-1}(C_k)$. ■

For example, in Figure 3, "1 2 3" is rp_extended by "1 2 -1 3" and generates "1 2 3 -1 -1 3". On the other hand, "1 2 3 -1 -1 3" has more than one leaf and its $last_{k-1}$ is generated by removing its srl. Therefore, $last_{k-1}$ of "1 2 3 -1 -1 3" is "1 2 -1 3".

Theorem 10: Suppose that tree C_{k-1} is extended (via either rp_extension or rn_extension) by tree C'_{k-1} and generates tree C_k. C_k can be rn_extended by the class whose index is C'_{k-1}.

Proof:

1) First, assume that C_{k-1} is rp_extended by C'_{k-1}. According to Lemma 9, C'_{k-1} becomes $last_{k-1}(C_k)$. Therefore, C'_{k-1} will be the index of the class which rn_extends C_k and generates candidates with k+1 nodes.
2) Then, assume that C_{k-1} is rn_extended by C'_{k-1}. There are two possible situations:

Extend

Require: candidate C_k;
Ensure: all (k+1)-extensions of C_k;
$Output \leftarrow \emptyset$;
for all candidates C'_k in the equivalence class of C_k **do**
  if $pos_{C_k}(rightmost\ node\ of\ C_k) \geq pos_{C'_k}(rightmost\ node\ of\ C'_k)$ **then**
    Generate candidate C_{k+1} by adding the rightmost node of C'_k to $pos_{C'_k}(rightmost\ node\ of\ C'_k)$ of C_k;
    $Output \leftarrow Output \cup C_{k+1}$;
  end if
end for
Find the equivalence class E' that its index satisfies the condition of Theorem 6.
for all candidates $C'_k \in E'$ **do**
  Generate candidate C_{k+1} by adding the rightmost node of C'_k to C_k as the child of the rightmost node of C_k;
  $Output \leftarrow Output \cup C_{k+1}$;
end for
return $Output$;

Fig. 4: High level pseudo code of the candidate generation method.

a) C_k might have more than one leaf. As a result, C_{k-1} will have more than one leaf, and C_{k-1} and C_k will have the same slr. On the other hand, $last_{k-2}(C_{k-1})$ and $first_{k-2}(C'_{k-1})$ are identical and since $V(C_k) \setminus V(C_{k-1})$ is the rightmost node of C'_{k-1}, $last_{k-1}(C_k)$ will be generated by adding the rightmost node of C'_{k-1} to $last_{k-2}(C_{k-1})$, and this tree is C'_{k-1}.

b) C_k might have one leaf. Then C_{k-1} will have one leaf and the roots of C_{k-1} and C_k will be the same. On the other hand, $last_{k-2}(C_{k-1})$ and $first_{k-2}(C'_{k-1})$ are the same and since $V(C_k) \setminus V(C_{k-1})$ is the rightmost node of C'_{k-1}, $last_{k-1}(C_k)$ will be generated by adding the rightmost node of C'_{k-1} to $last_{k-2}(C_{k-1})$, and this tree is C'_{k-1}.

■

For example, in Figure 3, ”1 2” is rn_extended by ”2 3” and generates ”1 2 3”. The class rn_extending ”1 2 3” is the class whose index is ”2 3”. ”2 3” is rp_extended by ”2 3” and generates ”2 3 -1 3”. The class rn_extending ”2 3 -1 3” is the class whose index is ”2 3”.

To find the class which rn_extends a candidate C_k, two new integers are assigned to C_k: Id1 and Id2. Id1 determines C_k is which tree of level k, and Id2 determines $last_{k-1}(C_k)$ is which class of level k-1. $last_{k-1}(C_k)$ is the index of the class which rn_extends C_k. Assume that C_{k+1} is a new tree generated by joining C_k (as the first subtree) with C'_k (as the second subtree). The Id2 of C_{k+1} is set to the Id1 of C'_k. Theorem 10 provides the rationale behind this assignment. The Id1 of C_{k+1} can be easily determined by means of a counter that increases by one for each generated tree at level k+1.

To correctly refer to the equivalence class rn_extending C_k, we need to generate all the classes at level k-1, even those having no member. If do so, Id2 will refer directly to the equivalence class rn_extending the tree. In general, the number of classes at level k must be equal to the number of frequent trees at level k-1.

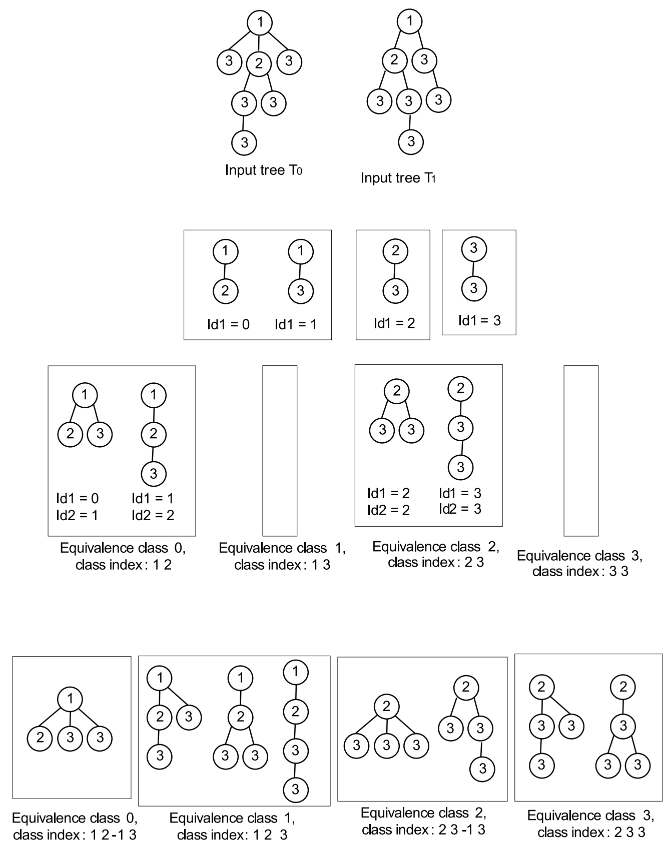


Fig. 5: An example of the indexing scheme.

Figure 5 shows how the indexing scheme can be applied to our running example. At level 3 all the classes are generated, even those without any member. ”1 2” is rn_extended by ”2 3” and generates ”1 2 3”. ”2 3” is the third tree of level 2 (so its Id1 would be 2), therefore ”1 2 3” will be rn_extended by the third class of level 3. ”2 3” is rp_extended by ”2 3”

Encoding

Require: an input tree T.
Ensure: *m-coding* and *cm-coding* of nodes of T.
$mid \leftarrow 0$.
m-coding(root(T))$\leftarrow 0$.
for all nodes x in preorder traversal of T **do**
  for all children r of x in right-to-left order **do**
    $mid \leftarrow mid + 1$.
    m-coding(r)$\leftarrow mid$.
  end for
  cm-coding(x)$\leftarrow mid$.
end for
return *m-coding* and *cm-coding*.

Fig. 6: High level pseudo code of *m-coding* and *cm-coding*

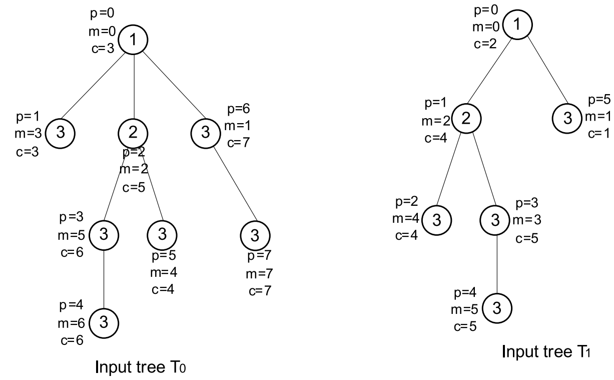


Fig. 7: *p-coding*, *m-coding* and *cm-coding* of the input trees. p refers to $p-coding$, m refers to $m-coding$, and c refers to $cm-coding$.

and generates ”2 3 -1 3”. ”2 3” is the third tree at level 2, therefore, ”2 3 -1 3” will be rn_extended by the third class at level 3.

V. Frequency counting

In this section, we develop a new method for frequency counting which is based on tree encodings. We first introduce two new tree encodings, and then explain how these encodings among with an already proposed encoding can be used to compute frequencies of candidates.

A. M-coding

In this encoding, an auxiliary integer, called mid, is used which is initiated by 0. *M-coding* of the root is set to 0. The tree is traversed in preorder and when a node x is met: the children of x are scanned from right to left and for each child r: mid is increased by one and the *m-coding* of r is set to the new value of mid. Since the nodes of the tree are traversed in preorder, when determining the *m-coding* of the children of a node, its *m-coding* has already been determined.

B. Cm-coding

Cm-coding of node x in input tree T is *m-coding* of its leftmost child, i.e. the greatest *m-coding* among its children. When a node is met in preorder traversal of the tree, *m-coding* of its children are assigned, therefore *cm-coding* of each node can be determined in $O(1)$ time complexity. Figure 6 presents the high level pseudo code of determining *m-coding* and the *cm-coding*. By one scan of T, *m-coding* and *cm-coding* of all nodes of T are determined.

As an example of the tree encodings, consider Figure 7 which presents the *p-coding*, *m-coding* and *cm-coding* of the input trees of our running example. *P-coding* refers to the preorder number of a node in an input tree. While *p-coding* is a depth-first traversal, *m-coding* and *cm-coding* are combined depth-first/breadth-first traversals.

C. Frequency counting

As mentioned above, tree C_k can be extended in two different ways: rp_extension and rn_extension. Each extension requires its particular method for frequency counting. In the rest of this section, we use the following assumptions and notations. We assume that occurrence O_k of k-candidate C_k, occurrence O_N of node N and occurrence O_{k+1} of k+1-candidate C_{k+1} occur in the input tree T. RN refers to the rightmost node of C_k and RP refers to the rightmost path of C_k excluding its rightmost node, i.e. $V(RP) \cup V(RN)$ forms the nodes of the rightmost path of O_k. O_{RN} refers to the rightmost node of O_k and O_{RP} refers to the rightmost path of O_k excluding its rightmost node, i.e. in O_k, O_{RN} and O_{RP} are the occurrences of RN and RP, respectively. We use the notation $par_T(x)$ to refer to the parent of node x in tree T.

1) Frequency counting for rp_extended candidates: Suppose that C_{k+1} is generated by adding node N to C_k via rp_extension. We want to know if adding O_N to the rightmost node of O_k generates occurrence O_{k+1}. The input tree T can be divided into the partitions depicted in Figure 8. $B1$ is the path between the root of T and the root of O_k. RC includes the *right children* of the nodes of $B1$ and the *right children* of the nodes of O_{RP}. Let a be a node on $B1$ and assume that its child b belongs to $B1$, too. *Right children* of a are the children whose preorder numbers are greater than the preorder number of b. Now, let a be a node in O_{RP} and assume that its child b belongs to O_{RP}, too. *Right children* of a are the children of a whose preorder numbers are greater than the preorder number of b. $B2$ is the path between O_{RN} and z, where z is the last node met before O_{RP} in the preorder traversal of T.

To generate an occurrence O_{k+1} of C_{k+1}, O_N must belong to the dotted region. For this purpose, O_N must satisfy Properties 11, 12 and 13.

Property 11: *p-coding*(O_N)>*p-coding*(O_{RN}).

Proof: Assume that O_N is added to node x in O_k. x is an ancestor of O_{RN}, and O_N is a right child of x, therefore, the preorder number of O_N is greater than the preorder number of O_{RN}. ■

Property 12: *m-coding*(O_N)<*m-coding*(O_{RN}).

Proof: There exist two possible situations: 1) O_N is not

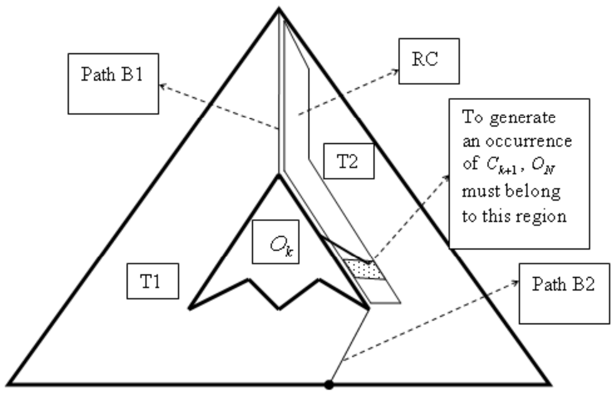
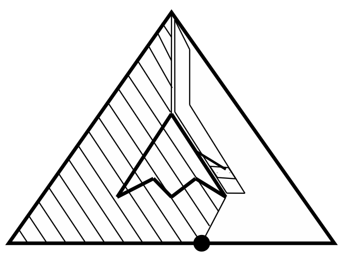
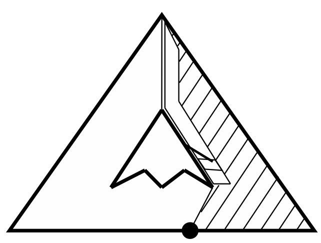


Fig. 8: Partitioning an input tree T.



(a) Hachured parts are eliminated by applying Property 11.



(b) Hachured parts are eliminated by applying Property 12.

Fig. 9: The restrictions of properties 1 and 2 on the input tree T.

added to the parent of O_{RN}: since the *p-coding* of the parent of O_N is smaller than the *p-coding* of the parent of O_{RN}, therefore, the *m-coding* of O_N will be smaller than the *m-coding* of O_{RN}. 2) O_N is added to the parent of O_{RN}: since O_N is the right sibling of O_{RN}, the *m-coding* of O_N will be smaller than the *m-coding* of O_{RN}. ■

O_N can be anywhere in T. It can be seen easily that if Property 11 is applied to O_N, it can not be selected from the hachured parts of Figure 9a. Property 12 limits O_N to the non-hachured parts of Figure 9b. Intersection of non-hachured parts of Figures 9a and 9b is the RC area, i.e. applying Properties 11 and 12 to O_N restricts it to the RC area. It is necessary to apply another restriction on O_N to limit it to the dotted region.

Property 13: $pos_T(O_N) - pos_T(O_{RN}) = pos_{C_k}(N) - pos_{C_k}(RN)$

Proof: The length of the path between every pair of nodes in O_k is equal to the length of the path between the corresponding nodes in T. Since O_k is an induced subtree of T and it preserves the parent-child relation, the length of the

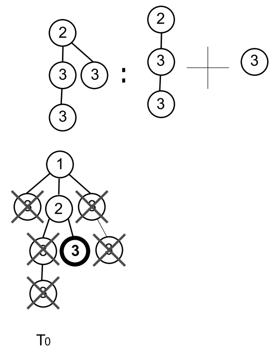


Fig. 10: An example of frequency counting for rp_extended candidates.

path between every pair of nodes in O_k is equal to the length of the path between the corresponding nodes in T. Since O_k is an occurrence of C_k in T, the length of the path between every pair of nodes in C_k is equal to the length of the path between the corresponding nodes in T. Therefore:

$$pos_T(O_N) - pos_T(par_T(O_{RN})) =$$
$$pos_{C_k}(N) - pos_{C_k}(par_{C_k}(RN))$$

Furthermore:

$$pos_T(par_T(O_{RN})) = pos_T(O_{RN}) - 1$$
$$pos_T(par_T(RN)) = pos_T(RN) - 1$$

Therefore:

$$pos_T(O_N) - pos_T(O_{RN}) = pos_{C_k}(N) - pos_{C_k}(RN)$$

■

If O_N satisfies Properties 11, 12 and 13, it can generate an occurrence of C_{k+1} by appending to O_k.

As an example, consider tree ”2 3 3 -1 -1 3” of Figure 5 generated through rp_extension. Encodings of the rightmost node of ”2 3 3” in T_0 are: *p-coding*=4, *m-coding*=6, *cm-coding*=6 and *position* of the rightmost node of ”2 3 3” in T_0 is 3. Encodings of the rightmost node of ”2 3 3” in $T1$ are: *p-coding*=4, *m-coding*=5, *cm-coding*=5 and *position* of the rightmost node of ”2 3 3” in T_1 is 3. Figure 10 shows different occurrences of ”3”. Only one occurrence satisfies all the conditions mentioned in Properties 11-13. Therefore, ”2 3 3 -1 -1 3” will have one occurrence in the input trees of our running example.

2) Frequency counting for rn_extended candidates: Assume that C_{k+1} is generated by adding node N to C_k through rn_extension. We want to see if adding O_N to the rightmost node of O_k generates the occurrence O_{k+1}. Tree T can be divided into the partitions depicted in Figure 11. This partitioning is slightly different from the partitioning of Figure 8, especially RC contains the right children of the nodes of $B1$ and the right children of the nodes of O_{RP} and all children of O_{RN}. $B1$, $B2$ and z are defined similar to Figure 8.

O_N can be anywhere in T. In order to generate an occurrence O_{k+1} of C_{k+1}, it must belong to the doted region of

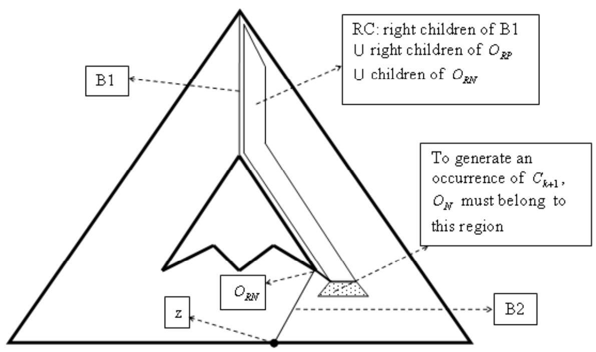
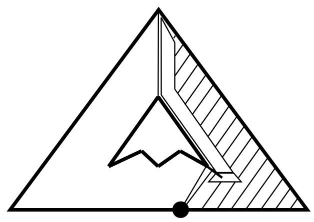
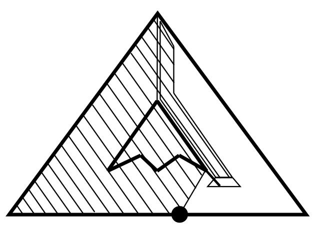


Fig. 11: Another partitioning of an input tree T.



(a) Hachured parts are eliminated by applying Property 14.



(b) Hachured parts are eliminated by applying Property 15

Fig. 12: How properties 4 and 5 can restrict partitions of an input tree T.

Figure 11. For this purpose, O_N must satisfy Properties 14 and 15.

Property 14: *cm-coding*(O_N)≤*m-coding*(O_{RN}).

Proof: Directly from the definition of *cm-coding*. ■

Property 15: *m-coding*(O_N)>*m-coding*(O_{RN}).

Proof: When O_N is a child of O_{RN}, the parent of O_N is met after the parent of O_{RN} in the preorder traversal, therefore, *m-coding* of O_N will be greater than *m-coding* of O_{RN}. ■

It can be seen easily that if Property 14 is applied to O_N, it can not be selected from the hachured parts of Figure 12a. Property 15 limits O_N to non-hachured parts of Figure 12b. Intersection of non-hachured parts of Figures 12a and 12b is the doted region. This means that O_N can generate an occurrence of C_{k+1} by appending to O_k iff it satisfies Properties 14 and 15.

Figure 13 shows how Properties 14 and 15 can be used to determine frequencies of rn_extended candidates. Consider tree ”1 2 3 3” which is generated via rn_extension of ”1 2 3”. ”1 2 3” has 4 occurrences in the input trees, 2 occurrences in T_0 and 2 occurrences in T_1. For each occurrence of ”1 2 3” in T_i ($i \in \{0,1\}$) all occurrences of ”3” occurring in T_i

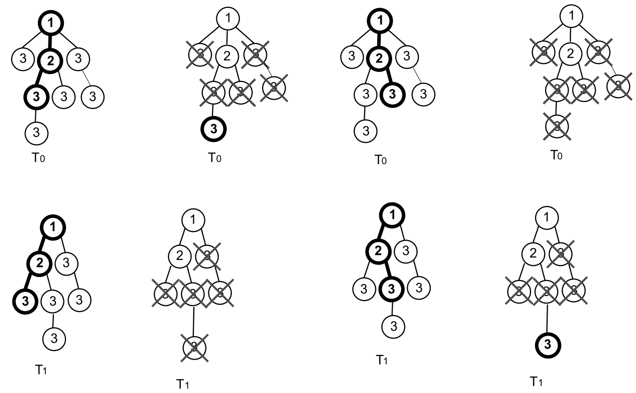


Fig. 13: An example of frequency counting for rn_extensded candidates.

are tested to determine which one satisfies Properties 14 and 15. Figure 13 presents these 4 different cases. For each case, the occurrences of ”3” with strong lines satisfy the conditions. As depicted in the figure, two occurrences of ”3” satisfy the conditions, therefore, ”1 2 3 3” would have 2 occurrences in the input trees.

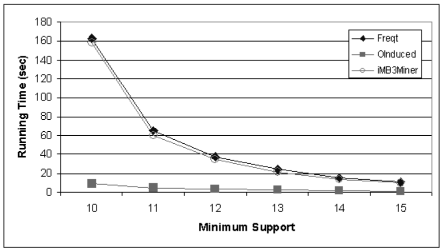
Exertion of the conditions presented in Properties 11-15 requires storing *p-coding*, *m-coding*, *cm-coding* and *position* of the rightmost node of each occurrence. After extending an occurrence O_k by O_N, O_N will be the rightmost node of the resultant occurrence O_{k+1}, therefore the encodings and the position of O_N will be assigned to O_{k+1}. Our algorithm for frequency counting works very efficient: it can compute frequency of a candidate by storing only 4 integers per each occurrence.

The *OInduced* algorithm takes as input an integer value $minsup$ defined by the user and a forest of rooted ordered labeled trees in Zaki's string representation format. The $minsup$ value can be selected to be either per-tree or occurrence-match. *OInduced* performs a breadth-first search in the state space of candidates and determines frequency of each candidates according to the before mentioned encodings. Figure 17 shows the high level pseudo code of *OInduced*.

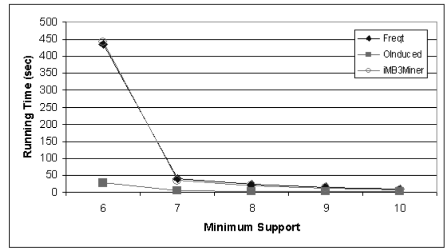
VI. Experimental results

We perform extensive experiments to evaluate the efficiency of the proposed algorithm using data from real applications as well as synthetic datasets. We do our experiments on a 1.8GHz Intel Pentium IV PC with a 2GB main memory, running UNIX operating system. All the algorithms are implemented in C++ using standard template libraries. For our comparison, we select *iMB3Miner* [22] and *FREQT* [2] which are the well-known algorithms developed to find induced patterns from rooted ordered trees. *OInduced*, *FREQT*, and *iMB3Miner* can work with both per-tree frequency and occurrence-match frequency. Here, due to lack of space, we only report results on occurrence-match frequency. Similar results can be obtained for per-tree frequency.

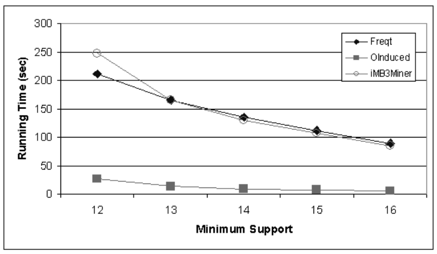
The widely used real dataset is CSLOGS [34]. This dataset contains the web access trees of the CS department of the



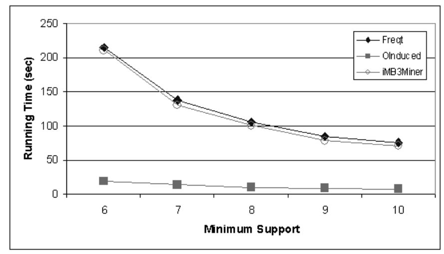
(a) Minimum support vs. running time over CSLOG1.



(b) Minimum support vs. running time over CSLOG2.



(c) Minimum support vs. running time over CSLOG12.



(d) Minimum support vs. running time over CSLOG3.

Fig. 15: Comparisons over user web log data.

OInduced

Require: a database D consisting of rooted ordered labeled trees, a user defined $minsup$ (either *per-tree* or *occurrence-match*).
Ensure: All frequent induced tree patterns.
$Output \leftarrow \emptyset$.
$F1\_SET \leftarrow$ the set of all frequent nodes and their encodings.
$F2\_SET \leftarrow \emptyset$.
while $F1\_SET \neq \emptyset$ **do**
  for all $P_k \in F1\_SET$ **do**
    $Ext \leftarrow$ **Extend**(P_k).
    for all $P_{k+1} \in Ext$ **do**
      if $support(P_{k+1}) \geq minsup$ **then**
        $F2\_SET \leftarrow F2\_SET \cup P_{k+1}$.
      end if
    end for
  end for
  $Output \leftarrow Output \cup F1\_SET$.
  $F1\_SET \leftarrow F2\_SET$.
  $F2\_SET \leftarrow \emptyset$.
end while
return $Output$.

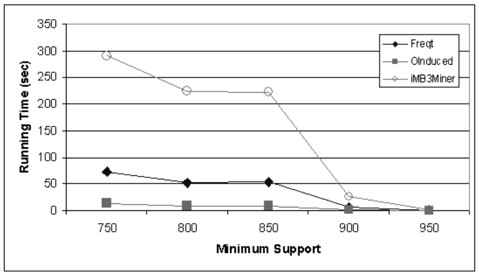
Fig. 14: High level pseudo code of OInduced.

Rensselaer Polytechnic Institute during one month and contains 59,691 transactions, 716,263 nodes and 13,209 unique vertex labels. Each distinct label corresponds to the URLs of a web page. The average string encoding length for the dataset is 23.3 [34]. This dataset is used for embedded pattern mining with pre-tree frequency. When used for occurrence-match frequency, all the algorithms have problems in finding frequent tree patterns. The problem arises from the fact that the dataset is a quite large dataset and during the occurrence-match frequency, the algorithms are overwhelmed by many occurrences.

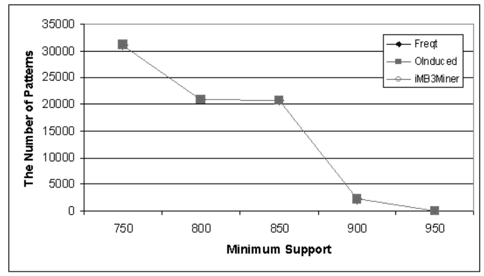
In [35], log file of each week is separated into a different dataset and three different datasets are generated: CSLOG1 for the first week, CSLOG2 for the second week and CSLOG3 for the third week. Furthermore, they generated a new dataset called CSLOG12 by combining CSLOG1 and CSLOg2. CSLOG1 contains 8,074 trees, CSLOG2 contains 7,404 trees, CSLOG3 contains 7,628 trees, and CSLOG12 contains 13,934 trees. Here, we use these datasets to evaluate our proposed algorithm. Figure 15 compares *OInduced* against *iMB3Miner* and *FREQT* over CSLOG1, CSLOG2, CSLOG3, and CSLOG12, respectively. Over all the datasets, *OInduced* significantly outperforms *iMB3Miner* and *FREQT*, especially for the lower values of $minsup$. For example, on CSLOG1 and at $minsup = 10$, *OInduced* works more than 18 times faster than *FREQT* and *iMB3Miner*.

The second real dataset used in this paper is the Multicast dataset which consists of MBONE multicast data measured during the NASA shuttle launch between the 14th and 21st of February, 1999 [4]. It has 333 distinct vertices where each vertex takes the IP address as its label. The Multicast dataset was sampled from this NASA dataset with 10 minutes sampling interval and has 1,000 transactions. In this dataset, there exist strong correlations among transactions and very large frequent patterns occur even at a high $minsup$. Figure 16 compares performance of the algorithms over the Multicast dataset. On this dataset, *OInduced* outperforms the other algorithms, especially; it significantly outperforms the *iMB3Miner* algorithm. For example, at $minsup = 750$, *OInduced* works more than 5 times faster than *FREQT* and more than 20 times faster than *iMB3Miner*.

We also evaluate the efficiency of *OInduced* using synthetic

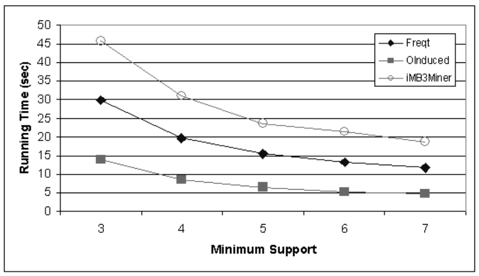


(a) Minimum support vs. running time.

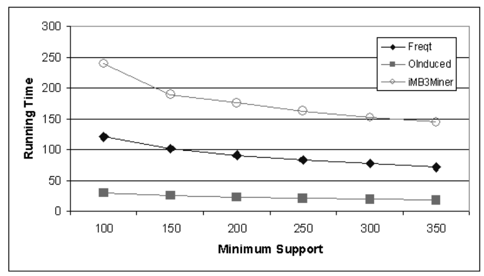


(b) The number of extracted patterns

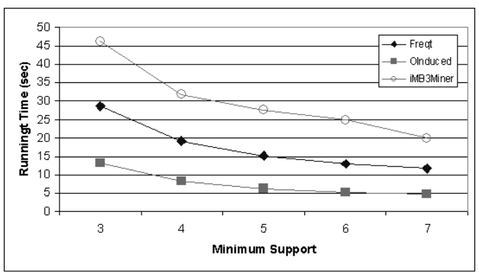
Fig. 16: Comparison over the Multicast dataset.



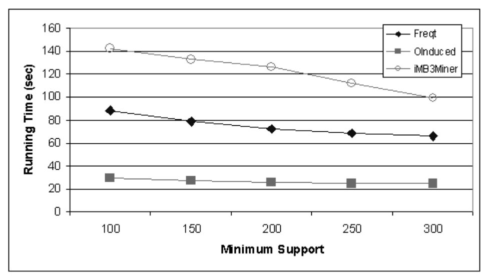
(a) Minimum support vs. running time over D10.



(b) Minimum support vs. running time over F3.



(c) Minimum support vs. running time over N1M.



(d) Minimum support vs. running time over TM.

Fig. 17: Comparisons over synthetic datasets.

datasets which are generated by the method described in [34]. The synthetic data generation program mimics the web site browsing behavior of the user. First a master web site browsing tree is built and then the subtrees of the master tree are generated. The synthetic tree generation program is adjusted by 5 parameters: 1) the number of labels (N), 2) the number of nodes in the master tree (M), 3) the maximum fan-out of a node in the master tree (F), 4) the maximum depth of the master tree (D), and 5) the total number of trees in the dataset (T).

The first synthetic dataset is D10 and uses the following default values for the parameters: $N = 100$, $M = 10,000$, $D = 10$, $F = 10$, $T = 100,000$. Figure 17a compares the running time of the algorithms on D10. As depicted in the figure, *OInduced* always outperforms *iMB3Miner* and *FREQT*.

We generate F3 as a narrow dataset and set all values to the default expect for $F = 3$. As depicted in Figure 17b, over this dataset *OInduced* works faster than *iMB3Miner* and *FREQT*, and *FREQT* outperforms *iMB3Miner*. For example at $minsup = 100$, *OInduced* outperforms *FREQT* by a factor of 4 and outperforms *iMB3Miner* by a factor of 8.

In N1M, N is set to $1,000,000$, so the average frequency of distinct labels becomes very low (i.e. $M \div N = 10,0000 \div 1,000,000 = 0.01$). Figure 17c presents the efficiency of *OInduced* against *iMB3Miner* and *FREQT* over N1M. Similar to the previous comparisons, *OInduced* outperforms the other algorithms.

To study how the algorithms behave on very large datasets, we compare them on T1M. For T1M, the parameters are set as follows: $N = 100$, $M = 10,000$, $D = 10$, $F = 10$, $T = 1,000,000$. Figure 17d compares *OInduced* against *iMB3Miner* and *FREQT* over T1M. As depicted in the figure, *OInduced* always outperforms *iMB3Miner* and *FREQT*.

Finally, to show how the algorithms scale, we generate three datasets with different sizes (different values for T), while the other parameters are set to the default values. At a fixed $minsup$ (i.e. 2), as depicted in Figure 18, we can see a linear increase in both running time and the number of patterns with increasing the number of trees for *OInduced*, *iMB3Miner* and *FREQT*. *OInduced* is more efficient than *iMB3Miner* and

FREQT. Both of horizontal and vertical axes in Figure 18 are depicted in logarithmic scale.

VII. Conclusion

In this paper, we introduced *OInduced* used to discover all frequent induced patterns from a collection of rooted, ordered and labeled trees. *OInduced* uses breadth-first search to generate candidates and takes advantage of equivalence classes to extend each candidate by only known frequent candidates. Then, an indexing scheme is used to improve the breadth-first equivalence class extension. We also presented two new tree encodings, *m-coding* and *cm-coding*, which are based on combined depth-first/breadth-first traversals of input trees. *OInduced* benefits from these encodings to restrict the nodes of input trees and quickly compute frequencies of candidates. We compared *OInduced* with the well-known algorithms, *iMB3Miner* and *FREQT*. Experiments on both real and synthetic data show that *OInduced* significantly reduces the running time and scales linearly with respect to the size of input trees.

Acknowledgment

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References

[1] A. V. Aho, J. E. Hopcroft and J. E. Ullman, The Design and Analysis of Computer Algorithm, Addison-Wesley, 1974.

[2] T. Asai, K. Abe, S. Kawasoe, H. Arimura, H. Satamoto and S. Arikawa, Efficient Substructure Discovery from Large Semi-Structured Data, Proc. Second SIAM Intl Conf. Data Mining, Apr. 2002.

[3] T. Asai, H. Arimura, T. Uno and S. Nakano, Discovering Frequent Substructures in Large Unordered Trees, Proc. Sixth Intl Conf. Discovery Science, Oct. 2003.

[4] R. C. Chalmers and K. C. Almeroth, Modeling the Branching Characteristics and Efficiency Gains of Global Multicast Trees, INFOCOM 2001.

[5] Y. Chi, Y. Yang and R.R. Muntz, Indexing and Mining Free Trees, Proc. Third IEEE Intl Conf. Data Mining, 2003.

[6] Y. Chi, Y. Yang, and R.R. Muntz, HybridTreeMiner: An Efficient Algorihtm for Mining Frequent Rooted Trees and Free Trees Using Canonical Forms, Proc. 16th Intl Conf. Scientific and Statistical Database Management, 2004.

[7] Y. Chi, Y. Yang, Y. Xia, R. R. Muntz, CMTreeMiner: Mining Both Closed and Maximal Frequent Subtrees. PAKDD, 63-73, 2004.

[8] L. Feng and T. S. Dillon, Mining XML-Enabled Association Rule with Templates, In Proceedings of KDID 04, 2004.

[9] L. Feng, T. S. Dillon, H. Weigand and E. Chang, An XML-Enabled Association Rule Framework. In Proceedings of DEXA 03, pp 88-97, 2003.

[10] A. Inokuchi, T. Washio and H. Motoda, An Apriori-based Algorithm for Mining Frequent Substructures from Graph Data, 4th European Conference on Principles of Knowledge Discovery and Data Mining, September 2000.

[11] P. Kilpelainen and H. Mannila, Ordered and Unordered Tree Inclusion, SIAM Journal of Computing, 24(2), 340356, 1995.

[12] M. Kuramochi and G. Karypis, Frequent Subgraph Discovery, Proceedings of the IEEE International Conference on Data Mining (ICDM01), November 2001.

[13] F. Luccio, A. M. Enriquez, P. O. Rieumont and L. Pagli, Exact Rooted Subtree Matching in Sublinear Time, Technical Report TR-01-14, Universita Di Pisa, 2001.

[14] F. Luccio, A. M. Enriquez, P. O. Rieumont and L. Pagli, Bottom-up Subtree Isomorphism for Unordered Labeled Trees, Technical Report TR-04-13, Universita Di Pisa, 2004.

[15] T. Miyahara, T. Shoudai, T. Uchida, K. Takahashi, H. Ueda, D. Cheung, J. G. Williams, L. Qing, Discovery of Frequent Tree Structured Patterns in Semistructured Web Documents, PAKDD, Hong Kong, China, 2001.

[16] T. Miyahara, Y. Suzuki, T. Shoudai, T. Uchida, K Takahashi, H. Ueda, Discovery of Maximally Frequent Tag Tree Patterns with Contractible Variables from Semistructured Documents, PAKDD, 133-144, 2004.

[17] S. Nijssen and J. N. Kok, Efficient Discovery of Frequent Unordered Trees, Proc. First Intl Workshop Mining Graphs, Trees, and Sequences, 2003.

[18] D. Shasha, J. Wang, and R. Giugno, Algorithms and Applications of Tree and Graph Searching, In Proceedings of the ACM International Symposium on Principles of Database Systems (PODS), 2002.

[19] D. Shasha, J. Wang, H. Shan, and K. Zhang, ATreeGrep: Approximate Searching in Unordered Trees, Proceedings of the International Conference on Scientific and Statistical Database Management (SSDBM), 2002.

[20] D. Shasha, J. Wang and S. Zhang, Unordered Tree Mining with Applications to Phylogeny, Proc. Intl Conf. Data Eng., 2004.

[21] Y. Suzuki, T. Miyahara, T. Shoudai, T. Uchida, Y. Nakamura, Discovery of Maximally Frequent Tag Tree Patterns with Height-Constrained Variables from Semistructured Web Documents, WIRI, 104-112, 2005.

[22] H. Tan, F. Hadzic, T. S. Dillon, E. Chang, L. Feng, Tree Model Guided Candidate Generation for Mining Frequent Subtrees from XML Documents, ACM Transaction on Knowledge Discovery from Data (TKDD), 2(2), 2008.

[23] S. Tatikonda, S. Parthasarathy, T. M. Kur, "TRIPS and TIDES: New Algorithms for Tree mining", CIKM 06, 455-464, 2006.

[24] A. Termier, M. C. Rousset and M. Sebag, TreeFinder: a First Step towards XML Data Mining, Second IEEE International Conference on Data Mining (ICDM'02), p. 450, 2002.

[25] C. Wang, M. Hong, J. Pei, H. Zhou, W. Wang and B. Shi, Efficient Pattern-Growth Methods for Frequent Tree Pattern Mining, Proc. Pacific-Asia Conf. Knowledge Discovery and Data Mining, 2004.

[26] K. Wang and H. Liu, Schema Discovery for Semistructured Data, In Proc. of the 3rd. International Conference on Knowledge Discovery and Data Mining (SIGKDD03), pages 271-274, California, USA, August 1997.

[27] K. Wang and H. Liu, Discovering Typical Structures of Documents: A Road Map Approach. In Proc. of the ACM SIGIR International Conference on Research and Development in Information Retrieval, pages 146-154, Melbourne, Australia, August 1998.

[28] K. Wang and H. Liu, Discovering Structural Association of Semistructured Data, IEEE Transactions on Knowledge and Data Engineering, 12(2):353-371, 2000.

[29] Y. Xiao, J. F. Yao, Z. Li and M. H. Dunham, Efficient Data Mining for Maximal Frequent Subtrees, Proc. Intl Conf. Data Mining (ICDM), 2003.

[30] Y. Xiao, J. F. Yao, G. Yang, Discovering Frequent Embedded Subtree Patterns from Large Databases of Unordered Labeled Trees, International Journal of Data Warehousing and Mining, 1(2), 70-92, 2005.

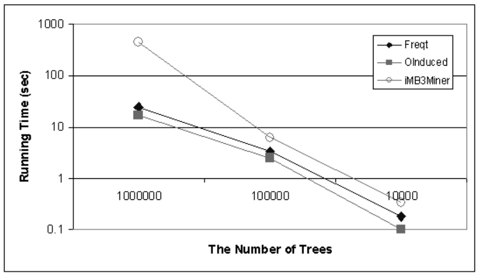
[31] X. Yan and J. Han, CloseGraph: Mining Closed Frequent Graph Patterns, ACM SIGKDD Int. Conf. on Knowledge Discovery and Data Mining, August 2003.

[32] M. J. Zaki, Efficiently Mining Frequent Trees in a Forest, Proc of the Int. Conf. Knowledge Discovery and Data Mining (SIGKDD02), July 2002.

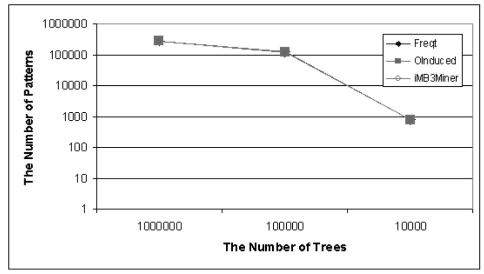
[33] M. J. Zaki, Efficiently Mining Frequent Embedded Unordered Trees, Fundam. Inform. 66(1-2): 33-52, 2005.

[34] M. J. Zaki, Efficiently Mining Frequent Trees in a Forest: Algorithms and Applications, IEEE Transaction on Knowledge and Data Engineering, 17 (8), pp. 1021-1035, 2005.

[35] M. J. Zaki and C. C. Aggarwal, XRules: An Effective Structural Classifier for XML Data, Machine Learning, 62(1-2), pp. 137-170, 2006.



(a) Minimum support vs. running time.



(b) The number of extracted patterns.

Fig. 18: Scale up comparison. Minimum-support is equal to 2. Both of horizontal and vertical axes are in logarithmic scale.

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