Addendum to “Interface tension of Bose-Einstein condensates”

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In this addendum to our paper [B. Van Schaeybroeck, Phys. Rev. A **78**, 023624 (2008)] we present an exact solution of the interface tension of Bose-Einstein condensates in a particular parameter regime.

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I. EXACT SOLUTION

In Ref. [1], we argued that the tension of an interface between two weakly interacting Bose-Einstein condensates (phases 1 and 2) can be written in the form

$$\gamma_{12} = 4P\xi_1 \mathcal{F}(\xi_2/\xi_1, K). \quad (1)$$

Also, when the condensates are trapped by the potentials U_2 and U_1 , the total excess energy $\Omega_{\mathcal{A}}$ can be reduced to a simple integral of the trapping potential along the interface area $\mathcal{A}$:

$$\Omega_{\mathcal{A}} = \frac{\sqrt{2m_1}}{4\pi\hbar a_{11}} \mathcal{F}(\xi_2/\xi_1, K) \int_{\mathcal{A}} d\mathbf{r} [\mu_1 - U_1(\mathbf{r})]^{3/2}, \quad (2)$$

where a_{11} , m_1 , μ_1 and U_1 are the intraspecies scattering length, the particle mass, the chemical potential, and the applied trapping potential of phase 1. The parameters K and ξ_2/ξ_1 can be expressed in terms of the atomic masses and the interspecies and intraspecies scattering lengths:

$$K = \frac{m_1 + m_2}{2\sqrt{m_1 m_2}} \frac{a_{12}}{\sqrt{a_{11} a_{22}}} \text{ and } \xi_2/\xi_1 = \sqrt[4]{\frac{m_1 a_{11}}{m_2 a_{22}}}. \quad (3)$$

In Ref. [1], approximations are given for the function $\mathcal{F}(\xi_2/\xi_1, K)$ in large regions of the parameter space $(\xi_2/\xi_1, K)$. However, an exact solution exists when the parameter ξ_2/ξ_1 and K are related by

$$\sqrt{K-1} = \frac{\sqrt{2}}{3} \left(\frac{1}{\xi_2/\xi_1} - \xi_2/\xi_1 \right). \quad (4)$$

In that case, the interface tension is determined by the function

$$\mathcal{F}(\xi_2/\xi_1, K) = \frac{\sqrt{2}}{3} (1 - \xi_2/\xi_1). \quad (5)$$

This result is also in agreement with two approximate solutions found Ref. [1]. First of all, in the regime of weak segregation (treated in Sec. IV A), the approximation of Eq. (16) is, to lowest order in the small parameters $K-1 \ll 1$ and $1 - \xi_2/\xi_1 \ll 1$, equal to Eq. (5). Second, in the regime of strong segregation ($K \rightarrow \infty$) and strong healing length asymmetry ($\xi_2/\xi_1 \ll 1$), Eq. (4) reduces to $\kappa = [\sqrt{K} \xi_2/\xi_1]^{-1} = 3/\sqrt{2}$. In that case, our expansion, Eq. (27) of Ref. [1] which is valid for small values of κ and which yields $\mathcal{F} = \sqrt{2}/3 - 0.4757(\xi_2/\xi_1)$, is in good agreement with Eq. (5).

The exact relation (5) can be understood by the presence of an extraordinary first-order wetting transition on the wetting line, which is analytically given by Eq. (4). The wetting transition is explained in Refs. [2,3].

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[1] B. Van Schaeybroeck, Phys. Rev. A **78**, 023624 (2008).

[2] J. O. Indekeu and B. Van Schaeybroeck, Phys. Rev. Lett. **93**,

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[3] B. Van Schaeybroeck and J. O. Indekeu (unpublished).