Polyhedral Process Networks

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Outline

- 1 Introduction and Motivation
- 2 Polyhedral Process Networks
 - Polytope Model
 - Dataflow Analysis
 - Parametric Counting
 - Computing Parametric Upper Bounds
- 3 Conclusion

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Motivation

Goal: map sequential programs to parallel hardware (MPSoC)

```

1 for( i=0; i<2*Hw; i++ )
  for( j=0; j<2*Hcl; j++ )
    a[i][j] = RandRange();

6 for( i=0; i<Hw; i++ ) {
  // 1D DWT in vertical direction with subsampling
  for( j=0; j<2*Hcl; j++ ) {
    tmpLine = (i==Hw-1) ? a[2*i+1][j] : a[2*i+2][j];
    Hf[j] = high_filt_vert( a[2*i+1][j], a[2*i+1][j], tmpLine );
11    tmp = (i==0) ? Hf[j] : oldHf[j];
    low_filt_vert( tmp, a[2*i+1][j], Hf[j], oldHf[j], Lf[j] );
  }

16 // 1D DWT in horizontal direction with subsampling
  for( j=0; j<Hcl; j++ ) {
    tmp = (j==Hcl-1) ? Lf[2*j] : Lf[2*j+2];
    HL[i][j] = high_filt_hor( Lf[2*j], Lf[2*j+1], tmp );
21    tmp = (j==0) ? HL[i][j] : HL[i][j-1];
    LL[i][j] = low_filt_hor( tmp, Lf[2*j], HL[i][j] );
  }

26 // 1D DWT in horizontal direction with subsampling
  for( j=0; j<Hcl; j++ ) {
    tmp = (j==Hcl-1) ? Hf[2*j] : Hf[2*j+2];
    HH[i][j] = high_filt_hor( Hf[2*j], Hf[2*j+1], tmp );
31    tmp = (j==0) ? HH[i][j] : HH[i][j-1];
    LH[i][j] = low_filt_hor( tmp, Hf[2*j], HH[i][j] );
  }
}

// The Outputs
for( i=0; i<Hw; i++ )
  for( j=0; j<Hcl; j++ ) {
36    Sink( LL[i][j] );
    Sink( HL[i][j] );
    Sink( LH[i][j] );
    Sink( HH[i][j] );
  }
}

```

?

Motivation

Goal: map sequential programs to parallel hardware (MPSoC)

```

1 for( i=0; i<2*Hw; i+= 1 )
  for( j=0; j<2*Hcl; j+= 1 )
    a[i][j] = ReadImage();

6 for( i=0; i<Hw; i+= 1 ) {
  // ID DWT in vertical direction with subsampling
  for( j=0; j<2*Hcl; j+= 1 ) {
    tmpLine = (i==Hw-1) ? a[2*i+1][j] : a[2*i+2][j];
    Hf[j] = high_filt_vert( a[2*i+1][j], a[2*i+1][j], tmpLine );
11  tmp = (i==0) ? Hf[j] : oldHf[j];
    low_filt_vert( tmp, a[2*i+1][j], Hf[j], oldHf[j], Lf[j] );
  }

  // ID DWT in horizontal direction with subsampling -----
  for( j=0; j<Hcl; j+= 1 ) {
    tmp = (j==Hcl-1) ? Lf[2*j] : Lf[2*j+2];
    HL[i][j] = high_filt_hor( Lf[2*j], Lf[2*j+1], tmp );
21  tmp = (j==0) ? HL[i][j] : HL[i][j-1];
    Lf[i][j] = low_filt_hor( tmp, Lf[2*j], HL[i][j] );
  }

  // ID DWT in horizontal direction with subsampling -----
  for( j=0; j<Hcl; j+= 1 ) {
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// The Outputs -----
for( i=0; i<Hw; i+= 1 )
  for( j=0; j<Hcl; j+= 1 ) {
36  Sink( Lf[i][j] );
    Sink( HL[i][j] );
    Sink( Lf[i][j] );
    Sink( HH[i][j] );
  }
}

```

process network

Kahn Process Networks

- Parallel specification
⇒ easier to map onto parallel hardware
- Consists of
 - ▶ independent processes
 - ▶ communication channels
⇒ unbounded FIFOs
 - ▶ synchronization: blocking read

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 - ▶ communication channels
⇒ FIFOs / memories
 - ▶ synchronization: blocking read + blocking write

Polyhedral Process Networks

- Parallel specification
 - ⇒ easier to map onto parallel hardware
 - ⇒ automatically derived from sequential program
- Consists of
 - ▶ independent processes
 - ⇒ one process for each statement
 - ▶ communication channels
 - ⇒ FIFOs / memories
 - ⇒ obtained from dependence analysis
 - ▶ synchronization: blocking read + blocking write

MPSoCs Programming and Mapping

- Programming model: KPN
- Example of a MPSoC platform:
 - 2 PowerPC processors
 - 2 MicroBlaze processors
 - 4 Communication memories (CM)
 - 4 Communication controllers (CC)
 - Crossbar component

- Mapping:
 - The processes are mapped onto the processors using standard GCC compiler.
 - The FIFOs are mapped onto the CMs depending on the process mapping.
 - CCs take care about the inter processor communication and synchronization

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Main difficulty: what are the channels and how big are they ?

- dataflow analysis

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Given a communication channel, how many tokens are in the channel at any given time?

- computing upper bounds

Given a communication channel, what is the maximal number of tokens in the channel during the execution?

Polytope Model

```
for (i = 1; i <= N; ++i)
  for (j = 1; j <= i; ++j)
    a[i][j]=
```

Assumptions on sequential code:

- iterators are integers
- loops with affine bounds
- affine conditions
- affine index expressions

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for (i = 1; i <= N; ++i)
  for (j = 1; j <= i; ++j)
    a[i][j] =
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Iteration domain: $P = \{ [i, j] \mid i \geq 1$

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Assumptions on sequential code:

- iterators are integers
- loops with affine bounds
- affine conditions
- affine index expressions

Lexicographical Order

```
#define N 5  
for (i = 1; i <= N; ++i)  
    for (j = 1; j <= i; ++j)  
        a[i][j]=
```

$$S = \{(i, j) \mid i \geq 1 \wedge i \leq N \wedge j \geq 1 \wedge j \leq i\}$$

Execution order: (1,1), (2,1), (2,2), (3,1), (3,2), (3,3), (4,1), (4,2), (4,3), (4,4) (5,1), (5,2), (5,3), (5,4), (5,5)

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Lexicographical order:

$$\mathbf{a} \prec \mathbf{b} \equiv \bigvee_{i=1}^n \left(a_i < b_i \wedge \bigwedge_{j=1}^{i-1} a_j = b_j \right)$$

$\Rightarrow$ smaller in first position where vectors differ

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Given a read from an array element, what was the last write to that array element?

Toy Example

```
int N;

int main(void)
{
    int i, j;
    int a[N];

    for (i = 0; i < N; ++i)
        a[i] = Read();
    for (i = 0; i < N; ++i)
        for (j = 0; j < N - i; ++j)
            a[i+j] = f(a[i+j]);
    for (i = 0; i < N; ++i)
        Write(a[i]);
}
```

Toy Example

```
int N;

int main(void)
{
    int i, j;
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    for (i = 0; i < N; ++i)
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Dataflow Analysis

Given a read from an array element, what was the last write to the same array element before the read?

Step 1: last write through given write access

```
for (i = 0; i < N; ++i)
  for (j = 0; j < N - i; ++j)
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for (i = 0; i < N; ++i)
  Write(a[i]);
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*Given a read from an array element, what was the last write to the **same array element** before the read?*

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for (i = 0; i < N; ++i)
  Write(a[i]);
```

Access relations:

$$A_1 = \{(i, j) \rightarrow (i + j) \mid 0 \leq i < N \wedge 0 \leq j < N - i\}$$

$$A_2 = \{(i) \rightarrow (i) \mid 0 \leq i < N\}$$

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$$\text{Map to all writes: } R' = A_1^{-1} \circ A_2 = \{(i) \rightarrow (i', i - i') \mid 0 \leq i' \leq i < N\}$$

Dataflow Analysis

*Given a read from an array element, what was the **last** write to the same array element before the read?*

Step 1: last write through given write access

```
for (i = 0; i < N; ++i)
  for (j = 0; j < N - i; ++j)
    a[i+j] = f(a[i+j]);
for (i = 0; i < N; ++i)
  Write(a[i]);
```

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Last write: $R = \text{lexmax } R' = \{(i) \rightarrow (i, 0) \mid 0 \leq i < N\}$

Dataflow Analysis

*Given a read from an array element, what was the last write to the same array element **before the read**?*

Step 1: last write through given write access

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for (i = 0; i < N; ++i)
    for (j = 0; j < N - i; ++j)
        a[i+j] = f(a[i+j]);
for (i = 0; i < N; ++i)
    Write(a[i]);
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In general: impose lexicographical order on shared iterators

Toy Example

```
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int main(void)
{
    int i, j;
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    for (i = 0; i < N; ++i)
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}
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            a[i+j] = f(a[i+j]);
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```

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Step 1: last write through given write access

Step 2: combination of results over all write accesses

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Step 2: combination of results over all write accesses
+ read accesses from same statement/process

Dataflow Analysis

Step 1: last write through given write access

Step 2: combination of results over all write accesses
+ read accesses from same statement/process

Dataflow Analysis: Result on Toy Example

```

for (i = 0; i < N; ++i)
    a[i] = Read();
for (i = 0; i < N; ++i)
    for (j = 0; j < N - i; ++j)
        a[i+j] = f(a[i+j]);
for (i = 0; i < N; ++i)
    Write(a[i]);

```

FIFO or Reordering Channel?

FIFOs are more efficient to implement in hardware

⇒ check whether communication channel is a FIFO

FIFO: reads occur in same order as corresponding writes

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FIFO: reads occur in same order as corresponding writes

Communication channel with dependence relation R is **not** a FIFO iff there is a pair of reads $\mathbf{r}_1$ and $\mathbf{r}_2$ such that $\mathbf{r}_1$ occurs before $\mathbf{r}_2$, but the writes occur in the opposite order

$$T = \{(\mathbf{r}_1, \mathbf{r}_2) \in (\text{dom } R)^2 \mid \mathbf{r}_1 \prec \mathbf{r}_2 \wedge R(\mathbf{r}_1) \succ R(\mathbf{r}_2)\}$$

$T = \emptyset \Rightarrow$ communication channel is FIFO

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$T = \emptyset \Rightarrow$ communication channel is FIFO

From now on we assume all channels are FIFOs
(Reordering channels require extra reordering buffer)

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Lexicographic maximum of parametric polytope (isl, pipelib, PPL)

⇒ piecewise quasi-affine map

Commercial Break: isl

isl is an

- LGPL
- thread-safe
- C

library for manipulating **integer sets and relations**

- bounded by *affine constraints*
- involving *parameters* and
- *existentially quantified variables*

Supported operations include

- basic operations such as *intersection*, *union* and *set difference*
- *integer projection*
- *set coalescing*
- *convex hull*
- *integer affine hull*
- computing the *lexicographic minimum* of a relation,

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- counting

Given a communication channel, how many tokens are in the channel at any given time?

Parametric Counting

Given a communication channel, how many tokens are in the channel at any given time?

Consider **intra**-process communication channel (FIFO)

Iteration domain of process: S

Dependence relation of channel: $R \subset S \times S$

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Consider **intra**-process communication channel (FIFO)

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Dependence relation of channel: $R \subset S \times S$

Number of tokens in channel before iteration $\mathbf{i} \in S$: number of writes to channel before $\mathbf{i}$ minus number of reads from channel before $\mathbf{i}$

$$n(\mathbf{i}) = \#\{\mathbf{w} \in \text{ran } R \mid \mathbf{w} \prec \mathbf{i}\} - \#\{\mathbf{r} \in \text{dom } R \mid \mathbf{w} \prec \mathbf{i}\}$$

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```
for (i = 0; i < N; ++i)
  for (j = 0; j < N - i; ++j)
    a[i+j] = f(a[i+j]);
```

$$R = \{(i, j) \rightarrow (i - 1, j + 1) \mid 1 \leq i \wedge 0 \leq j < N - i\}$$

$$n(i, j) = \begin{cases} j - 1 & \text{if } i = 0 \wedge 1 \leq j < N \\ N - i & \text{if } j = 0 \wedge 1 \leq i < N \\ N - i - 1 & \text{if } 1 \leq i < N \wedge 1 \leq j < N - i \end{cases}$$

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Ordering of iterations

- fixed within the same process
- unknown at compile-time between different processes
 - $\Rightarrow$ combine all processes into a single process
 - $\Rightarrow$ fix a particular schedule
 - $\Rightarrow$ perform counting in the combined process

Fixed schedule only used during buffer size computation

Computed sizes will be valid for given schedule

$\Rightarrow$ sizes allow for deadlock-free execution (at least one valid schedule)

Process Merging

Given: iteration domains of dimension $d_i \leq D$

- 1 Place in a common iteration space of dimension D
(no change of order within an iteration domain)
Heuristic: identify dimensions related through dependence relation

Process Merging: Common Iteration Space

```
for (i = 0; i < N; ++i)
  for (j = 0; j < N - i; ++j)
    a[i+j] = f(a[i+j]);
for (i = 0; i < N; ++i)
  Write(a[i]);
```

$$R = \{(i) \rightarrow (i', j') \mid 0 \leq i < N \wedge i' = i \wedge j' = 0\}$$

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Process Merging

Given: iteration domains of dimension $d_i \leq D$

- ① Place in a common iteration space of dimension D
(no change of order within an iteration domain)
Heuristic: identify dimensions related through dependence relation
- ② Add constant offsets to all iteration domains
(without violating any dependence)
Heuristic: put dependent iterations “close” together

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Main difficulty: what are the channels and how big are they ?

- dataflow analysis
 - Lexicographic maximum of parametric polytope (isl, pipelib, PPL)
 - $\Rightarrow$ piecewise quasi-affine map
- counting

Given a communication channel, how many tokens are in the channel at any given time?

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Number of elements in parametric polytope (barvinok)

⇒ piecewise quasi-polynomial

Derivation of Polyhedral Process Networks

Main difficulty: what are the channels and how big are they ?

- dataflow analysis

Lexicographic maximum of parametric polytope (isl, pipelib, PPL)

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Given a communication channel, what is the maximal number of tokens in the channel during the execution?

Computing Parametric Upper Bounds

Given a communication channel, what is the maximal number of tokens in the channel during the execution?

```
for (i = 0; i < N; ++i)
  for (j = 0; j < N - i; ++j)
    a[i+j] = f(a[i+j]);
```

Number of tokens in FIFO at (i,j):

$$n(i,j) = \begin{cases} j-1 & \text{if } i=0 \wedge 1 \leq j < N \\ N-i & \text{if } j=0 \wedge 1 \leq i < N \\ N-i-1 & \text{if } 1 \leq i < N \wedge 1 \leq j < N-i \end{cases}$$

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Maximal number of tokens:

$$\max\left(\max_{1 \leq j < N} j-1, \max_{1 \leq i < N} N-i, \max_{1 \leq i < N \wedge 1 \leq j < N-i} N-i-1\right)$$

Computing Parametric Upper Bounds

Maximal number of tokens:

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⇒ can be approximated using Bernstein expansion

- Ideally:
 - ▶ maximum over
 - ▶ integer points
- Bernstein expansion gives
 - ▶ upper bound over
 - ▶ rational points

In practice very accurate

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Upper bound of parametric polynomial over parametric polytope
(bernstein)

Outline

1 Introduction and Motivation

2 Polyhedral Process Networks

- Polytope Model
- Dataflow Analysis
- Parametric Counting
- Computing Parametric Upper Bounds

3 Conclusion

Conclusion

Derivation of Polyhedral Process Networks requires

- Parametric Integer Programming (lexmax)
⇒ available in `isl`, `piplib` and PPL (soon)
- Enumeration of parametric polytopes
⇒ available in `barvinok`
- Bounds on polynomial over parametric polytopes
⇒ available in `bernstein`

Prototype implementation available from

<http://www.kotnet.org/~skimo/loop/isa-0.09.tar.bz2>

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`bernstein` library included in `barvinok`, available from

<http://freshmeat.net/projects/barvinok/>

`isl` available from <http://freshmeat.net/projects/isl/>