INFLUENCE OF PLUG FLOW WHEN TESTING SHEAR THICKENING POWDER TYPE SELF-COMPACTING CONCRETE IN A WIDE-GAP CONCENTRIC CYLINDER RHEOMETER

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Abstract

When testing powder type self-compacting concrete (SCC) in a wide-gap concentric cylinder rheometer, sometimes a plug state arises, introducing an error in the obtained rheological flow parameters. In this paper, the classification of plug state inside a wide-gap concentric cylinder rheometer is illustrated for a nonlinear (Herschel-Bulkley) flow behaviour, which is not seldom observed in the case of powder type SCC. For a linear (Bingham) flow behaviour, the classification of plug inside a concentric cylinder rheometer is already well described in literature. The applied methodology is adapted to the nonlinear case.

With a plug state, a solid state arises inside the sheared test material, so that it is rotating as a rigid body. When applying a stepwise decreasing rotational velocity sequence, plug will begin at the outer, rotating cylinder and propagates towards the inner, stationary cylinder as the velocity of the outer cylinder will further decrease. This means that, with a plug state and assuming no slippage in the transition zone from the viscoplastic to the solid state, the outer boundary condition of the integration equations of the Couette inverse problem solution must be corrected to the rigid body velocity at the boundary between the viscoplastic and the solid state (i.e. the plug radius R_p).

For each rotational velocity of the outer cylinder N_p, the corresponding plug radius R_p can be calculated. However, these calculations are based on the assumption that the calculated rheological parameters are correct to begin with. Nevertheless, it was found that even if plug was occurring in some of the measurements when testing powder type SCC, it did not introduce a large error to the rheological parameters. In fact, the error generated by plug flow on the rheological flow parameters always remained within their 95% confidence intervals in case of the shear thickening powder type SCC mixes tested.

1. INTRODUCTION

Powder type self-compacting concrete (SCC) is often found to exhibit a shear thickening flow behaviour [1-6]. During the experimental part of a Belgian research project concerning the influence of mineral additions and chemical admixtures in SCC on microcracking and durability [7], the rheological properties of about 30 powder type SCC mixes were determined by means of the 'CONTEC VISCOMETER 5'. A detailed description of all the test results can be found in [5]. Besides, the influence of mineral additions on the rheological parameters of SCC is studied more into detail in the framework of a current K.U.Leuven MSc dissertation [8]. For some of those mixes, a plug was formed during the rheological measurements, especially at the lower rotational velocities. In this paper, the classification of plug and the error generated by plug flow will be looked at in detail for the Herschel-Bulkley flow model.

2. CONCRETE RHEOLOGY

The 'CONTEC VISCOMETER 5' used for the experiments in this work can be seen as an example of a wide-gap concentric cylinder rheometer. It is one of the most recent updates of the 'CONTEC BML VISCOMETER 3' [9], well designed for testing both (self-compacting) concrete and mortar (Figure 1). The measuring system consists of an outer cylinder (R_o = 145 mm) rotating at an angular velocity $\Omega_o = 2\pi N$ and an inner cylinder (R_i = 100 mm) being stationary and registering the applied torque T from the test material. To avoid slippage between the test material, both inner and outer cylinder are provided with protruding vanes. More details about the CONTEC VISCOMETER 5 can be found in [3,9,10,11].

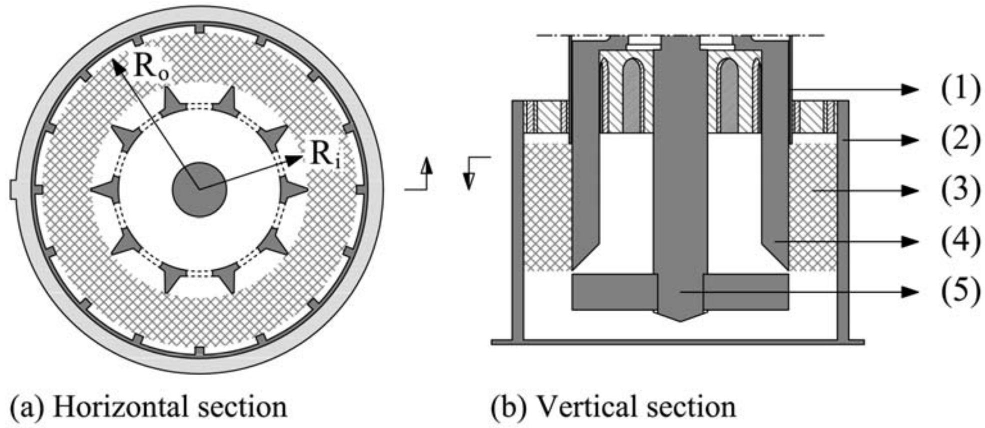

Figure 1: The CONTEC VISCOMETER 5 [3]: (1) top ring in order to ensure a constant height of the sheared test material; (2) outer cylinder, mounted on a rotating disk; (3) sheared test material taken into consideration during measurement/calculation; (4) inner cylinder - upper unit, free to rotate against a load cell, registering the applied torque T from the test material; (5) inner cylinder - bottom unit, fixed at the mounting point of the inner cylinder, virtually eliminating the effect of 3D shearing at the bottom.

'Absolute' rheometry involves the rheological measurements in 'absolute units of physics': it involves the determination of the flow behaviour in terms of shear stress τ (Pa) and shear rate $\dot{\gamma}$ (s^{-1}) instead of torque T (N·m) and rotational velocity N (rps). In case of a concentric cylinder rheometer, the derivation of the flow curve $\tau(\dot{\gamma})$ from the torque measurements $T(N)$ is often called the 'Couette inverse problem'. A detailed description can be found in [3,5].

The viscoplastic flow behaviour, i.e. after exceeding the yield stress, of a Herschel-Bulkley fluid is generally described by:

$$\tau = \tau_{0,\mathrm{HB}} + K\dot{\gamma}^n \tag{1}$$

with: $\tau_{0,\mathrm{HB}}$ the Herschel-Bulkley yield stress (Pa), K the consistency coefficient (Pa·s^n) and n the flow index (-). For a flow index n = 1, K is called the 'plastic viscosity' (denoted as μ, Pa·s) and the Bingham viscoplastic flow behaviour is found.

3. GENERAL CONSIDERATIONS ABOUT PLUG FLOW

With a plug, a solid state arises, so the shear rate becomes zero: $\dot{\gamma} \equiv 0$ (i.e. a so-called rigid body rotation). When applying a stepwise decreasing rotational velocity sequence (as it is the case with the ConTec Viscometer 5), the condition $\dot{\gamma} = 0$, and so plug, will begin at the outer cylinder and propagates towards the inner cylinder as the angular velocity Ω_o (rad/s) of the outer cylinder is further decreased [10]. The location of the boundary between the viscoplastic and the solid state is defined by the plug radius R_p, which can be calculated from:

$$R_\mathrm{p}^2 = \frac{T}{2\pi h \tau_0} \tag{2}$$

with τ_0 the yield stress, according to the viscoplastic model concerned (since $\dot{\gamma} = 0$ implies that the shear stress is only defined by the 'yield' stress, and so independent of the visco-plastic model used for $\dot{\gamma} > 0$).

Note that Eq. (2) is directly related to the equivalence between the applied torque and shear stress on a cylindrical shell, expressed by:

$$\tau(r) = \frac{T}{2\pi r^2 h} \tag{3}$$

with h the height of the cylindrical shell (m).

For $R_\mathrm{p} \geq R_\mathrm{o}$, no plug state inside the test material is present. By putting $R_\mathrm{p} = R_\mathrm{o}$ in Eq. (2) and by using Eq. (4) as the expression of the torque measurements $T(N)$ for a Herschel-Bulkley fluid [3], Eq. (2) can be solved for the minimum rotational velocity in avoiding plug. The latter rotational velocity will be designated as N_p^* further on in this paper, i.e. the plug rotational velocity beneath which a plug is formed inside the test material.

$$T = \frac{4\pi h \tau_{0,\mathrm{HB}}}{\left(\frac{1}{R_\mathrm{i}^2} - \frac{1}{R_\mathrm{o}^2}\right)} \ln\left(\frac{R_\mathrm{o}}{R_\mathrm{i}}\right) + \frac{2^{2n+1}\pi^{n+1} h K}{n^n \left(\frac{1}{R_\mathrm{i}^{2/n}} - \frac{1}{R_\mathrm{o}^{2/n}}\right)^n} N^n \tag{4}$$

Care should be taken when eliminating a plug state by simply increasing the rotational velocity (so that $N \geq N_\text{p}^*$), since such action also increases the difference in shear rate $\nabla\dot{\gamma}$ within the test material, resulting in a larger likelihood of particle migration [10].

For $R_\text{p} < R_\text{o}$, the outer boundary condition of the integration equations of the Couette inverse problem solutions must be changed from "$r = R_\text{o} \Rightarrow \omega = \Omega_\text{o} \equiv 2\pi N$" to "$r = R_\text{p} \Rightarrow \omega = 2\pi N_\text{p}$", with N_p the rotational velocity resulting in a plug radius at $r = R_\text{p} < R_\text{o}$. However, it should be kept in mind that this altered boundary condition may be incorrect due to possible slippage in the transition zone from viscoplastic to solid state: when slippage occurs, the rheometer measures a smaller torque than expected.

4. CLASSIFICATION OF PLUG FOR A BINGHAM FLUID

The classification of plug for a Bingham fluid is well described in [10]. The same methodology will hereafter be used in order to classify plug for a Herschel-Bulkley fluid. In this way, the obtained derivations in this paper can also be directly related to a Bingham fluid when the flow index equals unity: $n = 1$.

5. CLASSIFICATION OF PLUG FOR A HERSCHEL-BULKLEY FLUID

The classification of plug for a Herschel-Bulkley fluid can be treated similar to the classification of plug for a Bingham fluid.

Assuming no slippage in the transition zone from viscoplastic to solid state and keeping in mind the above mentioned outer boundary condition alteration, Eq. (4) can be recalculated as:

$$T = \frac{4\pi h \tau_{0,\text{HB}}}{\left(\frac{1}{R_\text{i}^2} - \frac{1}{R_\text{p}^2}\right)} \ln\left(\frac{R_\text{p}}{R_\text{i}}\right) + \frac{2^{2n+1}\pi^{n+1} hK}{n^n \left(\frac{1}{R_\text{i}^{2/n}} - \frac{1}{R_\text{p}^{2/n}}\right)^n} N_\text{p}^n \tag{5}$$

In this way, the plug rotational velocity $N_\text{p} \leq N_\text{p}^*$ for a Herschel-Bulkley fluid can be calculated by combining Eqs. (2) and (5) into:

$$R_\text{p}^2 = \frac{1}{2\pi h \tau_{0,\text{HB}}} \left(\frac{4\pi h \tau_{0,\text{HB}}}{\left(\frac{1}{R_\text{i}^2} - \frac{1}{R_\text{p}^2}\right)} \ln\left(\frac{R_\text{p}}{R_\text{i}}\right) + \frac{2^{2n+1}\pi^{n+1} hK}{n^n \left(\frac{1}{R_\text{i}^{2/n}} - \frac{1}{R_\text{p}^{2/n}}\right)^n} N_\text{p}^n \right) \tag{6}$$

And thus:

$$N_\text{p} = \left(\frac{n}{4\pi}\right)\left(\frac{1}{R_\text{i}^{2/n}} - \frac{1}{R_\text{p}^{2/n}}\right)\left(\frac{\tau_{0,\text{HB}}}{K}\left(R_\text{p}^2 - 2\left(\frac{1}{R_\text{i}^2} - \frac{1}{R_\text{p}^2}\right)^{-1} \ln\left(\frac{R_\text{p}}{R_\text{i}}\right)\right)\right)^{\frac{1}{n}} \tag{7.a}$$

and, by putting $R_\text{p} \equiv R_\text{o}$:

$$N_\mathrm{p}^* = \left(\frac{n}{4\pi}\right)\left(\frac{1}{R_\mathrm{i}^{2/n}} - \frac{1}{R_\mathrm{o}^{2/n}}\right)\left(\frac{\tau_\mathrm{0,HB}}{K}\left(R_\mathrm{o}^2 - 2\left(\frac{1}{R_\mathrm{i}^2} - \frac{1}{R_\mathrm{o}^2}\right)^{-1}\ln\left(\frac{R_\mathrm{o}}{R_\mathrm{i}}\right)\right)\right)^{\frac{1}{n}} \quad (7.b)$$

So, by creating a vector representing the potential plug radii $\mathbf{R}_\mathrm{p} = [R_\mathrm{i},\ldots,R_\mathrm{o}]$ and by putting it into Eq. (7.a), a corresponding vector of rotational velocities where a plug state is active, is found: $\mathbf{N}_\mathrm{p}(\mathbf{R}_\mathrm{p}) = [N_\mathrm{p}(R_\mathrm{i}),\ldots,N_\mathrm{p}(R_\mathrm{o})]$. Note that the last element in the vector $\mathbf{N}_\mathrm{p}(\mathbf{R}_\mathrm{p})$, i.e. $N_\mathrm{p}(R_\mathrm{o})$, represents the rotational velocity when a plug starts to form at the outer cylinder $r = R_\mathrm{o}$: $N_\mathrm{p}(R_\mathrm{o}) \equiv N_\mathrm{p}^*$.

Putting the two vectors $\mathbf{R}_\mathrm{p}$ and $\mathbf{N}_\mathrm{p}(\mathbf{R}_\mathrm{p})$ in Eq. (5), produces the corresponding torque vector $\mathbf{T}(\mathbf{R}_\mathrm{p},\mathbf{N}_\mathrm{p})$. Plotting $\mathbf{T}(\mathbf{R}_\mathrm{p},\mathbf{N}_\mathrm{p})$ as a function of $\mathbf{N}_\mathrm{p}(\mathbf{R}_\mathrm{p})$ produces the torque profile when the plug state is extending from the outer cylinder R_o towards the inner cylinder R_i. An example of such torque profile is shown in Figure 2 for one of the powder type SCC mixes actually tested (for all test results: see [5]). In order to illustrated the magnitude of error generated by plug flow (see further), mix SCC274 [5], having the highest N_p^* value, is chosen as the most representative mix for this paper.

From the vector $\mathbf{N}_\mathrm{p}(\mathbf{R}_\mathrm{p})$, it is clear that each element in the vectors $\mathbf{R}_\mathrm{p}$ and $\mathbf{N}_\mathrm{p}$ corresponds to each other. The corresponding plug radius R_p for a given rotational velocity N_p is calculated by applying the Newton-Raphson iteration algorithm on Eq. (7.a), or more precisely on Eq. (8), shown below:

$$f(R_\mathrm{p}) = \left(\frac{n}{4\pi}\right)\left(\frac{1}{R_\mathrm{i}^{2/n}} - \frac{1}{R_\mathrm{p}^{2/n}}\right)\left(\frac{\tau_\mathrm{0,HB}}{K}\left(R_\mathrm{p}^2 - 2\left(\frac{1}{R_\mathrm{i}^2} - \frac{1}{R_\mathrm{p}^2}\right)^{-1}\ln\left(\frac{R_\mathrm{p}}{R_\mathrm{i}}\right)\right)\right)^{\frac{1}{n}} - N_\mathrm{p} = 0 \quad (8)$$

Basically, this algorithm consist of iterating Eq. (9) until $\left|R_\mathrm{p}^{k+1} - R_\mathrm{p}^k\right|$ is less than some specific value (here, a value of $5\cdot10^{-5}$ mm is used), with k the iteration index. The first guess usually consists of either the inner $R_\mathrm{p}^0 = R_\mathrm{i}$ or the outer $R_\mathrm{p}^0 = R_\mathrm{o}$ radius.

$$R_\mathrm{p}^{k+1} = R_\mathrm{p}^k - \frac{f(R_\mathrm{p}^k)}{\partial f(R_\mathrm{p}^k)/\partial R_\mathrm{p}} \quad (9)$$

It should be mentioned explicitly that the above given calculations of R_p and N_p are based on the assumption that the calculated Herschel-Bulkley parameters $\tau_\mathrm{0,HB}$, K and n were correct to begin with. In general, the error generated by plug flow on these parameters is depending on:

- the number of rotational velocity steps actually tested beneath the plug rotational velocity N_p^*, and
- the degree of deviation between the torque profiles with or without plug (see Figure 2, respectively the black and the grey line).

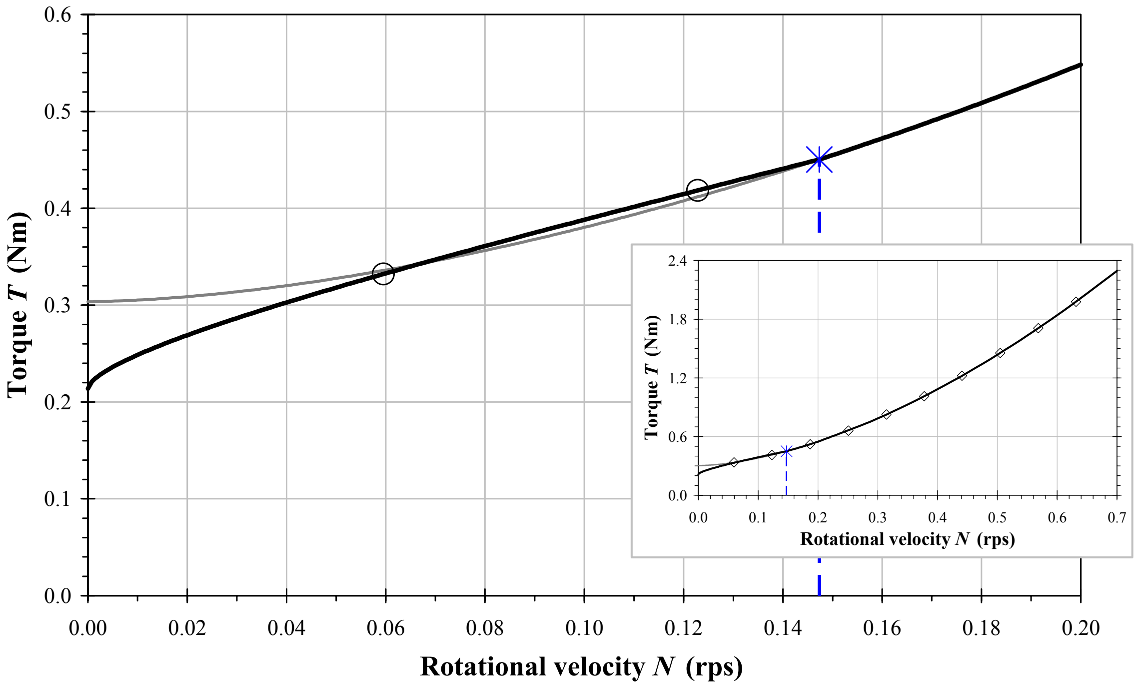

Figure 2: Plot of **T**(**R**$_p$,**N**$_p$) as a function of **N**$_p$(**R**$_p$) for SCC274 (black line). The grey line is an extrapolation from the torque, according to Eq. (4), when no plug is occurring, which is readily seen with the incorporated figure. The geometry consist of (R_i, R_o, h) = (0.100, 0.145, 0.125) m. The rheological parameters are $\tau_{0,HB}$ = 27.27 Pa, K = 2.84 Pa·s$^{1.67}$ and n = 1.67 and thus, according to Eq (7.b), N_p^* = 0.147 rps. The corresponding torque $T(N_p^*)$ is represented by the cross marker (t).
The circle markers on the solid line (○) represent the average torque points, as should be measured when the plugged zone is extending towards the inner cylinder and if the rheological parameters $\tau_{0,HB}$, K and n were correct to begin with. The diamond markers (◊) in the incorporated figure represent the rotational velocities actually tested. From this, it is clear that a plug state is active for the two lowest rotational velocity steps: at N = 0.06 rps and N = 0.12 rps.

In case of SCC274, the error generated by plug is (very) small: for the two rotational velocity steps actually tested beneath the plug rotational velocity N_p^*, the differences between the black and the grey line (Figure 2) are very small, resulting in a small error (Table 1). The term "Error" shown in this table represents the percentage difference between (a) the values for the Herschel-Bulkley parameters obtained from the $T(N)$ measurement (grey line in Figure 2) and (b) the values for the Herschel-Bulkley parameters after "recalculation" (i.e. after a new nonlinear regression analysis, based on the average torque values on the black line in Figure 2), relative to the values obtained from the $T(N)$ measurement. This order of magnitude of error on the converted model parameters (< 1%) was found for all powder type SCC mixes tested in this work [5,8], when described by a nonlinear flow behaviour.

Table 1: Calculation of the error generated by plug flow on the Herschel-Bulkley flow parameters for SCC274 (see [3,5] for the determination of G_{HB}, H_{HB}, J and $\tau_{0,\mathrm{HB}}$, K and n).

		Measurement	**Recalculation**	**Error (%)**
Plug rotational velocity N_{p}^{*}	(rps)	0.147	0.147	-
Average torque at N_{p} = 0.06 rps	(N·m)	0.336	0.332	-
Average torque at N_{p} = 0.12 rps	(N·m)	0.412	0.418	-
Flow resistance G_{HB}	(N·m)	0.304	0.304	-
Viscosity factor H_{HB}	(N·m·s)	3.615	3.611	-
Flow index factor J	(-)	1.673	1.672	-
Herschel-Bulkley yield stress $\tau_{0,\mathrm{HB}}$	(Pa)	27.27 ± 1.43 (*)	27.31	0.13
Consistency coefficient K	(Pa·s)	2.84 ± 0.33 (*)	2.83	-0.09
Flow index n	(-)	1.67 ± 0.05 (*)	1.67	-0.06

(*) For the converted model parameters based on the T(N) measurements, the 95% confidence interval half-widths are also mentioned.

6. CONCLUSIONS

It can be concluded that, even if plug was occurring in some of the measurements done in this work, plug flow did not introduce a very large error to the Herschel-Bulkley flow parameters $\tau_{0,\mathrm{HB}}$, K and n of powder type SCC mixes. This finding is in agreement with [10], where it was stated that "*it is not to be expected that the plug flow will generate any error of dramatic magnitude*" if the ratio yield stress/plastic viscosity is sufficient low (≤ 100 s^{-1}), as it is the case for most (powder type) SCC mixes.

In fact, it can be stated that the error generated by plug flow on the converted Herschel-Bulkley model parameters remained within their corresponding 95% confidence interval half-widths for all the measurements done in this work (see [5,8] for a list of all test results).

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