Comment on "Alfvén Instability in a Compressible Flow"

The Alfvén wave has been a popular subject in various studies in the past and, in particular, in studies dealing with the heating of the solar corona and the consequent generation or acceleration of solar wind. However, finding a widespread source for the wave appears to be a major problem. In a recent paper [1], it is claimed that a new MHD instability of the Alfvén wave has been found: incompressible Alfvén modes propagating in a compressible spatially varying flow were apparently exponentially amplified. Bearing in mind the importance of the Alfvén wave and the possible consequences in case of such a widespread source for its generation, we here reexamine the model used in Ref. [1].

We have found that the results obtained in the work [1] are wrong. This is because the "equilibrium" used in Ref. [1] is only assumed, instead of being self-consistently determined from the equations, and it does not exist. Below, we explain this in more detail.

The general starting equations should read

$$\rho\left(\frac{\partial}{\partial t} + \vec{V}\cdot\nabla\right)\vec{V} = -\nabla\left(p + \frac{B^2}{2\mu_0}\right) + \frac{(\vec{B}\cdot\nabla)}{\mu_0}\vec{B}, \quad (1)$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla\times(\vec{V}\times\vec{B}), \quad (2)$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{V}) = 0. \quad (3)$$

In Ref. [1] it is assumed that $\vec{B}_0 = B_0\vec{e}_z$, while $B_0 = \mathrm{const}$, and the equilibrium velocity is $\vec{u}_0 = u_0(z)\vec{e}_z$, while the pressure term is omitted. The model thus implies a spatially accelerated/decelerated equilibrium (or background) plasma, although the source for this effect is missing.

In Ref. [1] only the continuity equation (3) is used to describe the equilibrium, yielding

$$\rho_0(z)u_0(z) = c_1, \quad (4)$$

where c_1 is a constant. Taroyan makes a mistake here by *assuming* an equilibrium velocity, i.e., by setting it "to have a step function profile", instead of obtaining it from Eq. (4) and from the momentum equation, Eq. (1), when it is used to describe the equilibrium, or some other form of the momentum. We stress that the spatial variation of the plasma velocity is well known in the models dealing with the solar wind (as clear from the references cited in [1]). However, in those models it follows *self-consistently* from the momentum and continuity equations.

As a matter of fact, it is seen that without the pressure term, for a time-independent equilibrium or background plasma, the right-hand side in Eq. (1) vanishes so that

$$\frac{\rho_0}{2}\frac{\partial u_0^2}{\partial z} \equiv c_1\frac{\partial u_0}{\partial z} = 0. \quad (5)$$

However, this contradicts the condition (4) where u_0 is assumed to be depending on z. Clearly, one way out of this is to keep the pressure term in Eq. (1) when it is used to describe the equilibrium. This in fact is equivalent to taking into account the compressibility in the equilibrium, as done inconsistently in Ref. [1]. This yields the second condition for the equilibrium:

$$u_0(z)^2 + 2c_s^2\ln\rho_0(z) = c_2 = \mathrm{const}. \quad (6)$$

Here, $c_s^2 = \kappa T/m$. Equation (6) is to be used together with Eq. (4) in order to self-consistently determine the possible profiles for $\rho_0(z)$ and $u_0(z)$.

In other words, the equilibrium plasma flow considered by Taroyan contains a step function velocity, i.e., a (steady) shock. This is all right as long as the Rankine-Hugoniot conditions (all of them) are satisfied across the discontinuity (see, e.g., [2], Chap. 4, p. 170). The Rankine-Hugoniot condition following from the continuity equation is the one considered by Taroyan, viz. $[\![\rho_0 u_0]\!] = 0$; i.e., $\rho_0 u_0$ has to be constant across the discontinuity. However, the momentum equation also yields a Rankine-Hugoniot jump condition. In the simplified setup considered, this condition reduces to $[\![\rho_0 u_0^2]\!] = 0$, which is equivalent to the condition (5) mentioned above. Hence, the combination of the two conditions results in $[\![u_0]\!] = 0$, i.e., if the pressure is ignored, the equilibrium velocity cannot jump. But if the pressure is kept then both Eqs. (4) and (6) must be used.

Hence, the equilibrium velocity cannot be just "assumed" to have an arbitrary step profile. The step has to satisfy all of the Rankine-Hugoniot conditions. Moreover, to obtain a "steady" shock, the shock speed should be zero, which is clearly not the case in the plasma flow considered by Taroyan. The results obtained in Ref. [1] are thus wrong and the conclusions should be disregarded.

These results are obtained in the framework of the projects G.0304.07 (FWO-Vlaanderen), C 90347 (ESA Prodex 9), and GOA/2009-009 (K. U. Leuven). Financial support by the European Commission through the SOLAIRE Network (MTRN-CT-2006-035484) is gratefully acknowledged.

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Received 5 January 2009; published 30 June 2009
DOI: 10.1103/PhysRevLett.103.019501
PACS numbers: 52.35.Py

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