An Attack on Two Hash Functions by Zheng-Matsumoto-Imai

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Abstract. In [ZMI89,ZMI90] two constructions for a collision resistant hash function were proposed. The first scheme is based on a block cipher, and the second scheme uses modular arithmetic. It is shown in this paper that both proposals have serious weaknesses.

1 Introduction

For an informal definition of a collision resistant hash function the reader is referred to [PGV92]. The following model will be used to describe iterated hash functions:

$$H_i = f(X_i, H_{i-1}) \quad i = 1, 2, \dots, t.$$

Here f is the round function, X_i are the t message blocks, H_i are the chaining variables, H_0 is equal to the initial value, that should be specified together with the scheme, and H_t is the hashcode. It was shown by I. Damgård [Dam89] that if the round function f is a collision resistant function, h is a collision resistant hash function. The authors of [ZMI89,ZMI90] claim that their constructions yield a collision resistant round function. It will be demonstrated that in both cases the round function is not collision resistant, and that in some cases collisions for h can be constructed.

2 The Hash Function Based on a Block Cipher

The round function f compresses a 224-bit input to a 128-bit output and is based on xDES¹. This block cipher is one of the extensions of DES [Fi46] that has been proposed in [ZMI89b]. xDES¹ is a three round Feistel cipher with block length 128 bits, key size 168 bits and with the F function equal to DES. One round is defined as follows:

$$C1_{i+1} = C2_i \quad \text{and} \quad C2_{i+1} = C1_i \oplus \text{DES}(K_i, C2_i) \quad i = 0, 1, 2.$$

The variables $C1_i$ and $C2_i$ are 64-bit blocks, and K_i are 56-bit keys. The block cipher is then written as

$$C2_3 \parallel C1_3 = \text{xDES}^1(K_1 \parallel K_2 \parallel K_3, C1_0 \parallel C2_0).$$

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Here $C1_0$ and $C2_0$ are the first and second part of the plaintext, and $C2_3$ and $C1_3$ are the first and second part of the ciphertext. The collision resistant function consists of 2 xDES¹ operations:

$$f(Y1\|Y2) = \text{xDES}^1(\text{chop}_{72}(\text{xDES}^1(\beta\|Y1, \alpha))\|Y2, \alpha).$$

Here $Y1$ and $Y2$ are 112-bit blocks, α is a 128-bit constant, β is a 56-bit initialization variable and chop_r drops the r least significant (or rightmost) bits of its argument. The complete hash function has the following form: $H_i = f(H_{i-1}\|X_i)$, where H_{i-1} is a 128-bit block, and X_i is a 96-bit block. The rate of this scheme is equal to 4, which means that 4 DES encryptions are required to hash 64 bits.

The scheme has two weaknesses, that allow to produce collisions for the round function f . First only 56 bits are kept from the first xDES¹ encryption, and hence a birthday attack will require only 2^{29} operations to produce a collision for the intermediate value and hence for the function f . The second problem is that if $\beta = K_1$ and $Y_1 = K_2\|K_3$, one can use the key collision search algorithm described in [QD89] to produce key collisions for the DES plaintext equal to the second part of α . This yields a collision for f in about 2^{33} operations.

The scheme can be strengthened however by distributing β equally over K_1 , K_2 , and K_3 , and by increasing the size of β [Zhe92]. It will be shown that independently of the size of β , the security level can not be larger than 44 bits. If the size of β is equal to v bits (in the original proposal $v = 56$), the number of fixed bits of β that enter the key port of a single DES block is equal to $v/3$ (it will be assumed that v is divisible by 3). It can be shown that the rate of this scheme is then equal to $R = \frac{6 \cdot 64}{208 - 2v}$. The number of bits of Y_1 that enter the key port will be denoted with y , hence $y + v/3 = 56$. Two attacks are now considered.

For the fixed value of the right part of α and of the first $v/3$ bits of β , one can calculate and store a set of 2^z different ciphertexts. The probability that a collision will be found in this set is approximately equal to 2^{2z-65} . If $y > 32$, implying $v < 72$, a value of $z = 33$ is clearly sufficient to obtain a collision. If on the other hand $y \leq 32$, one will take $z = y$, and the probability of success is smaller than one. One can however repeat this procedure, (e.g., if one attacks a DES block different from the first one, a different value can be chosen for the value of the bits of Y_1 that enter the first DES), and the expected number of operations for a single collision is equal to 2^{65-y} , while the required storage is equal to 2^y . An extension of the Quisquater algorithm [QD89] could be used to eliminate the storage. If the security level S is expressed in bits, it follows that $S = \max\{65 - y, 33\}$. With the relation between y and v , one obtains $S = \max\{9 + v/3, 33\}$.

A second attack follows from the observation that only v bits are kept from the output of the first xDES¹ operation (hence the chop operation is chopping $128 - v$ bits). It is clear that finding a collision for the remaining v bits requires only $2^{v/2+1}$ operations, or $S \leq v/2 + 1$ bits. This attack is more efficient than the first attack if $v < 64$ bits.

The relation between S and v can be summarized as follows: if $v < 64$ then $S = v/2 + 1$, if $64 \leq v < 72$ then $S=33$, and if $72 \leq v < 104$ then $S = v/3 + 9$.

One can conclude that producing a collision for the proposed round function requires less than 2^{44} operations. Depending on the allocation of the bits of X_i and H_{i-1} to Y_1 and Y_2 , it might also be feasible to produce a collision for the hash function with a fixed initial value: it is certainly possible to produce a collision for the hash function if there is a single DES block where all key bits are selected from X_i .

3 The Hash Function Based on Modular Arithmetic

In this case the round function f consists of 2 modular squarings with an n -bit modulus (with $n = 500$):

$$f(Y1\|Y2) = (\text{chop}'_{450}((\beta\|Y1)^2 \bmod N) \| Y2)^2 \bmod N,$$

where $\text{chop}'_r(x)$ drops the r most significant bits of x , $Y1$ and $Y2$ are 450-bit blocks, and β is a 50-bit initialization variable. The complete hash function has the following form: $H_i = f(H_{i-1}\|X_i)$, where H_{i-1} is a 500-bit block, and X_i is a 400-bit block. The security of this scheme is based on the fact that $O(\log N)$ bits of squaring modulo N is hard if N is a Blum integer, i.e., $N = pq$ with $p \equiv q \equiv 3 \bmod 4$. From this it is wrongly concluded that finding two integers such that their squares agree at the 50 least significant positions is hard (a trivial collision for x is $x' = -x$). As only 50 bits of the first squaring are used as input to the second squaring, it follows that collisions can be found with a birthday attack in 2^{26} operations. It can be shown that one can find a second preimage and hence a collision for f even if $k = n/4$ bits are selected, or $3n/4$ bits are chopped. The algorithm is the same as the one presented in [Gir87] to break a related scheme with redundancy in the least significant positions.

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