Acceleration of soliton by nonlinear Landau damping of dust-helical waves

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The problem of nonlinear Landau damping of helicon waves in dusty plasma in particular emphasis to the acceleration of soliton is presented here. This in the framework of a collisionless, anisotropic homogeneous dusty plasma in one dimension, can be well described by two coupled dynamical equations of the generalized Zakharov type, with one extra nonlocal term coming from Landau damping. Nonlinear-nonlocal term gives rise to essential contributions relative to the local term. Then under different conditions, kinetic nonlinear Schrödinger equation is constructed and nonlinear decrement is obtained for two cases. It is noticed that the time dependant term in the ponderomotive force plays a significant role for this kind of damping. Additionally, it is shown that nonlinear Landau damping leads to the amplitude modulation of dust helicon waves, further modulational instability, and maximal growth rate is obtained when the group velocity of the helicon wave reaches the dust-acoustic speed. It is demonstrated that how the nonlinear Landau damping leads to the acceleration of soliton, which is eventually slowed down after transferring some of its energy to the wave. Emission of dust-acoustic wave by accelerated soliton is discussed briefly. © 2009 American Institute of Physics. [DOI: 10.1063/1.3127711]

I. INTRODUCTION

Several studies related to the propagation of a circularly polarized electromagnetic wave or so called helicon wave along an external magnetic field B_0 in ordinary plasmas have been done.^{1–6} Problem of their possible modulational instability, which arises when helical wave propagates along the external magnetic field, was considered and it was shown that growth rate for the modulational instability was much higher when slow waves were propagating perpendicularly than in case of parallel propagation.⁵ Later on, the effect of helicon waves on the weakly ionized plasma and its filamentation and self-focusing was also considered.⁶

As is well known, the physical mechanism of Landau damping is related to the exchange of energy between the waves and particles, when phase velocity of waves resonate the velocity of particles, and it proved to be a milestone having applications both in laboratory and space plasmas.^{7–10} However, later on the study of nonlinear effects in wave particle interaction became a topic of growing interest and both simulations and analytical treatment have been used to study this problem in various ways.^{11–13} In 1965, O'Neil observed that after a specific characteristic oscillation time the resonant particles give energy back to the wave, which they had gained, and eventually the damping reduces exponentially.¹² In 1972, Davidson studied the nonlinear Landau damping of electron plasma waves in unmagnetized plasmas.¹³ The same year Porkolab and Chang considered instabilities and induced scattering due to nonlinear Landau damping of electrostatic plasma waves in magnetized

plasma.¹⁴ Sugaya considered electron beam acceleration and cross field acceleration in relativistic magnetized plasmas.¹⁵ In 1982, Shatashvili and Tsintsadze¹⁶ described the nonlinear propagation of Langmuir waves in a collisionless plasma using the kinetic approach. A set of generalized Zakharov's equations was obtained taking into account the relativistic effects on nonlinear Landau damping. It was shown that the Landau damping term with nonlocal nonlinearity can also lead to the modulational instability. Fla *et al.*¹⁷ 1989 considered the modulational instability of the MHD due to nonlinear Landau damping. Nonlinear Landau damping and particle trapping in Alfvén wave of finite amplitude were also considered.¹⁸ This effect was considered in auroral (E-region) Farley–Buneman turbulence and the bounced motion of protons in solar wind was also investigated to describe the formation of dissipative structures.^{8,10} In 1996, Isichenko reported that the Landau damping decay is algebraic ($E \propto t^{-1}$) rather than exponential.¹⁹ Since then several numerical kinetic simulation studies have been performed, which demonstrate that algebraic dissipation is never observed.^{20–23} However in the present work, the physical mechanism underlying this so-called collisionless Landau damping is rather different, we will show analytically how the nonlinear term in the average kinetic Vlasov equation leads to the nonlinear Landau damping effect, which is exponential in nature.

To the best of our knowledge, investigation of nonlinear propagation of helicon waves in dusty plasmas has not been undertaken earlier. We wish to consider the propagation of transverse helicon waves in a collisionless dusty plasma. However it is clear that in our consideration linear damping of such waves is very small since the gyroradius of ions is

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much less than the length of helicon waves, i.e., $\rho_i \ll \lambda$ where $\rho_i = v_{ti}/\omega_{ci}$. On the other hand nonlinear interaction of helicon waves with plasma (wave-wave-particle interactions) leads to the appearance of plasma-wave beats with frequencies $\omega - \omega'$ and wave vectors $\mathbf{k} - \mathbf{k}'$. The resonant interaction between these beats and the particles $\omega - \omega' / |\mathbf{k} - \mathbf{k}'| = v_\alpha$ leads to nonlinear Landau damping, (here v_α is the thermal velocity of the species α). Since mechanisms involving nonlinear interaction of radiations with some plasma modes are of great interests and there is a wide range of applications, e.g., the auroral kilometric radiation, solar radio bursts,^{24,25} etc. In the present work, we shall discuss an important property of soliton that when it moves with acceleration, it behaves like particle and emits dust-acoustic type waves.

To consider one dimensional propagation of the helicon waves, we obtained a generalized set of Zakharov equations with both local and nonlocal nonlinearities, where the nonlocal term enters only through the nonlinear Landau damping. In different cases, kinetic nonlinear Schrödinger (KNLS) equation with local and nonlocal nonlinearity is obtained. When the group velocity of helicon waves is close to the dust acoustic velocity, then in KNLS equation, only nonlocal nonlinear term remains. However, it is found that space dependent term in the ponderomotive force expression plays a main role for obtaining Landau damping rates. Then it is shown that the nonlinear Landau damping term leads to modulational instability and growth rate was obtained for the general case.

This article is organized in the following manner. Section II gives the basic formulation for the helicon waves propagating in dusty plasma. In Sec. III, we present the derivation of density oscillation of electrons, ions, and dust particles and show that damping is important only for ions, while a KNLS equation with local and nonlocal nonlinearities has also been obtained here. In Sec. IV, nonlinear decrement is obtained for two cases. In Sec. V, the amplitude modulation of the helicon wave is given and the growth rate for modulational instability through the nonlinear Landau damping is examined. Section VI discusses how soliton is accelerated due to nonlinear Landau damping and emission of dust-sound waves by the accelerated soliton envelope takes place. Finally, conclusions are given in Sec. VII.

II. DISPERSION RELATION FOR HELICON WAVES

We consider a situation in which a circularly polarized EM wave is applied to anisotropic, homogeneous three component plasma containing electrons, ions, and massive dust grains having constant static negative charge. We know that for circularly polarized transverse EM waves propagating along external magnetic field $\mathbf{B}_o(0, 0, B_o)$, we have the following relation:

$$\frac{k^2 c^2}{\omega^2} \Big|_{\pm} = 1 - \frac{\omega_{pe}^2}{\omega(\omega \pm \omega_{ce})} - \frac{\omega_{pi}^2}{\omega(\omega \mp \omega_{ci})}, \quad (1)$$

where $\omega_{p\alpha}^2 = \sqrt{4\pi e^2 n_{0\alpha}/m_\alpha}$ and $\omega_{c\alpha} = eB_o/m_\alpha c$ are the Langmuir and cyclotron frequencies of the α species, respectively.

Using neutrality condition ($n_{io} = Z_d n_{do} + n_{eo}$), we obtain a dispersion relation under the conditions $k^2 c^2 / \omega_{pd}^2 \gg 1$; $\omega_{cd}, \omega_{pd} \ll \omega$,

$$\omega = \frac{k^2 c^2}{\omega_{pd}^2} \omega_{cd}, \quad (2)$$

substituting it into the dispersion relation and treating ω and k as operators given by^{26,27}

$$\omega = \omega_o + i \frac{\partial}{\partial t}, \quad k = k_o - i \frac{\partial}{\partial z}, \quad (3)$$

we obtain NLSE as

$$i \left(\frac{\partial}{\partial t} + \mathbf{V}_g \cdot \frac{\partial}{\partial \mathbf{z}} \right) A + \frac{v_g^*}{2} \frac{\partial^2 A}{\partial z^2} + \omega_o \left(\frac{Z_d \delta n_d}{n_{od}} \right) A = 0. \quad (4)$$

Here $v_g = 2k_o c^2 \omega_{cd} / \omega_{pd}^2$ is the group velocity, which is twice more than phase velocity of the circularly polarized electromagnetic (helicon) wave ω_o / k_o , and $v_g^* = c^2 \omega_{cd} / \omega_{pd}^2$.

III. BASIC EQUATIONS AND FORMULATION

Our aim is to study the nonlinear Landau damping of dust helicon waves, for this write down the average Vlasov's equation

$$\frac{\partial}{\partial t} \langle f_\alpha \rangle + (\mathbf{v} \cdot \nabla) \langle f_\alpha \rangle + \mathbf{F} \frac{\partial \langle f_{0\alpha} \rangle}{\partial \mathbf{p}} = 0, \quad (5)$$

$$\mathbf{F} = e_\alpha \left[\langle E \rangle + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right] + \mathbf{F}_{NL},$$

where $\langle E \rangle = -\nabla \phi$ is the electrostatic field due to the ponderomotive force $\mathbf{F}_{NL}$. Equation (5) is known as kinetic equation for the slowly varying distribution function of particles, which embodies a new time dependent force due to the intense EM field, in addition to the usual Lorentz force²⁸

$$\mathbf{F}_{NL} = -m_\alpha c^2 \left[\frac{\omega_o}{\omega_o \pm \omega_{c\alpha}} \nabla \left(\frac{e^2 |A|^2}{(m_\alpha c^2)^2} \right) \pm \frac{\omega_{c\alpha} \mathbf{k}_o}{(\omega_o \pm \omega_{c\alpha})^2} \frac{\partial}{\partial t} \times \left(\frac{e^2 |A|^2}{(m_\alpha c^2)^2} \right) \right],$$

where $\pm$ represents different polarizations. For the condition $\omega \ll \omega_{ci}$, ω_{ce} and $\omega \gg \omega_{cd}$, as we shall see later, ponderomotive force effect is equal on both ions and electrons,

$$F_{\text{Pond}}^i \approx F_{\text{Pond}}^e \gg F_{\text{Pond}}^d. \quad (6)$$

However, we will see only ions play vital role for the excitation of very low frequency wave. To define the density variations, we shall linearize Eq. (5) and use $\langle f_\alpha \rangle = f_{0\alpha} + \delta f_\alpha$ (Zakharov's approximation). Here we shall assume equilibrium distribution function $f_{0\alpha}$ to be Maxwellian, since propagation is considered in the z -direction only, so

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_z \frac{\partial}{\partial z}\right) \delta f_\alpha - \left[\frac{\partial}{\partial z} \left(e_\alpha \Phi - \frac{\omega_o}{\omega_{c\alpha}} \psi_\alpha^2 \right) - \frac{k_o}{\omega_{c\alpha}} \frac{\partial}{\partial t} \psi_\alpha^2 \right] \frac{\partial f_{0\alpha}}{\partial p_z} = 0. \quad (7)$$

Here ponderomotive potential is written in the energy units ($\psi^2 = e^2 A^2 / m_i c^2$).

We shall represent δf_α , Φ , δn_α , and ψ_α^2 in Fourier representation as $\int (dk/2\pi) \int (d\omega/2\pi) e^{i(kz - \omega t)}$

$\times [\delta f_\alpha(k, \omega), \Phi(k, \omega), \psi_\alpha^2(k, \omega)]$. Density variations are defined as

$$\delta n_\alpha(z, t) = \int d\mathbf{v} \int \frac{dk}{2\pi} \int \frac{d\omega}{2\pi} e^{i(kz - \omega t)} \delta f_\alpha(k, \omega, \mathbf{p}).$$

The Fourier component will be

$$\delta n_\alpha(k, \omega) = \int d\mathbf{v} \cdot \delta f_\alpha(k, \omega, \mathbf{p}). \quad (8)$$

Solving Eq. (7) in Fourier representation, we obtain

$$\delta f_\alpha(k, \omega, \mathbf{p}) = - \frac{\{k[e_\alpha \Phi(k, \omega) - (\omega_o/\omega_{c\alpha}) \psi_\alpha^2(k, \omega)] + (k_o/\omega_{c\alpha}) \omega \psi_\alpha^2(k, \omega)\}}{\omega - kv_z} \frac{1}{m_\alpha} \frac{\partial f_{0\alpha}}{\partial v_z}. \quad (9)$$

After substitution of Eq. (9) into Eq. (8) using Maxwellian's distribution function, we get

$$\delta n_\alpha(k, \omega) = - \frac{n_{0\alpha}}{T_\alpha} \left\{ \left[e_\alpha \Phi(k, \omega) - \frac{\omega_o}{\omega_{c\alpha}} \psi_\alpha^2(k, \omega) \right] + \left(\frac{k_o}{\omega_{c\alpha}} \frac{\omega}{k} \psi_\alpha^2 \right) \right\} \left[1 - I_{+\alpha} \left(\frac{\omega}{kv_{ti}} \right) \right], \quad (10)$$

where the function $I_{+\alpha}(x) = x e^{-x^2/2} \int_{i\infty}^x d\tau e^{\tau^2/2}$ has asymptotic expressions²⁹

$$I_{+\alpha}(x) = 1 + \frac{1}{x^2} + \frac{3}{x^4} + \dots - i \sqrt{\frac{\pi}{2}} x e^{-x^2/2}$$

$$\text{for } |\operatorname{Re} x| \gg |\operatorname{Im} x|,$$

$$I_{+\alpha}(x) = -i \sqrt{\frac{\pi}{2}} x \quad \text{for } |x| \ll 1.$$

To take into account the low frequency dust acoustic waves, we use the inequality

$$\mathbf{v}_{dt} \ll \frac{\omega}{k} \ll v_{te}, v_{ti}. \quad (11)$$

Now we can separately describe the expressions for the number density of electrons, ions and dust grains from Eq. (10) as

$$\delta n_e(k, \omega) = \frac{n_{0e}}{T_e} \left[e\Phi(k, \omega) - \frac{\omega_o}{\omega_{ce}} \left(1 - \frac{k_o \omega}{k \omega_o} \right) \psi^2 \right], \quad (12)$$

$$\delta n_i(k, \omega) = - \frac{n_{oi}}{T_i} \left(Z_i e\Phi(k, \omega) - \frac{(\omega_o k + k_o \omega)}{k \omega_{ci}} \psi^2(k, \omega) \right) \times \left(1 + i \sqrt{\frac{\pi}{2}} \frac{\omega}{k v_{ti}} \right), \quad (13)$$

similarly for dust,

$$\delta n_d(k, \omega) = - \frac{n_{0d} Z_d e\Phi(k, \omega)}{m_d} \frac{k^2}{\omega^2}. \quad (14)$$

Using the quasineutrality condition in terms of Fourier representation and assuming the condition $n_{oe}/n_{oi} \ll T_e/T_i$,

$$\delta n_i(k, \omega) = Z_d \delta n_d(k, \omega). \quad (15)$$

Substituting Eq. (13) into Eq. (15), we obtain

$$e\Phi(k, \omega) = - \frac{T_i}{(1 + i\Pi)} \left(\frac{Z_d \delta n_d(k, \omega)}{n_{oi}} \right) + \frac{1}{\omega_{ci}} \left(\omega_o - k_o \frac{\omega}{k} \right) \psi^2(k, \omega), \quad (16)$$

where $\Pi = \sqrt{\pi/2} \omega / k v_{ti}$. Substituting Eq. (16) into Eq. (14) and taking inverse Fourier transformation, we obtain a Zakharov type of equation

$$\left(\frac{\partial^2}{\partial t^2} - U_o^2 \frac{\partial^2}{\partial z^2} \right) \frac{\delta n_d}{n_{0d}} + \frac{1}{\sqrt{2\pi}} \left(\frac{U_o^2}{v_{ti}} \right) \frac{\partial^2}{\partial z^2} P \int_{-\infty}^{\infty} \frac{dz'}{z - z'} \int_{-\infty}^{z'} dz'' \frac{\partial}{\partial t} \frac{\delta n_d(z'', t)}{n_{0d}} = - \left(\frac{Z_d \omega_o}{m_d \omega_{ci}} \right) \frac{\partial^2}{\partial z^2} \psi^2(z, t) - \left(\frac{Z_d k_o}{m_d \omega_{ci}} \right) \frac{\partial^2}{\partial t \partial z} \psi^2(z, t), \quad (17)$$

where $U_o = \sqrt{(Z_d^2 n_{od}/n_{oi})(T_i/m_d)}$ is the dust-acoustic speed. The second term in the left hand side (l.h.s.) of Eq. (17) is known as a nonlocal term. While arriving at Eq. (17) we used the following formula $|k|/k = (1/i\pi) P \int_{-\infty}^{\infty} (dz'/z) e^{ikz}$, here P denotes the Cauchy principal part.¹⁶

IV. DAMPING RATES

In the nonlinear Schrödinger Eq. (4), it is necessary to express the dust density variations $[\delta n_d(z, t)]$ through the ponderomotive potential. For this, we first consider the quasistationary limit (4) and (17) by assuming

$$\psi = \psi(\xi, t), \delta n_d = \delta n_d(\xi, t), \quad (18)$$

$$\xi = z - u_p \xi, \frac{\partial}{\partial t} \ll v_g \frac{\partial}{\partial \xi}.$$

Equation (17) takes the form

$$\frac{\delta n_d(z, t)}{n_{od}} - \frac{1}{\sqrt{2\pi}} \left(\frac{U_o^2}{v_{ti}} \right) \frac{v_g}{(v_g^2 - U_o^2)} P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} \frac{\delta n_d(\xi', t)}{n_{od}} \\ = - \frac{Z_d(\omega_o - k_o v_g)}{m_d \omega_{ci} (v_g^2 - U_o^2)} \psi^2(\xi, t).$$

It is obvious from the above equation that time dependent term in the ponderomotive force is of more significance than the usual space dependent term; ignoring this may not lead to Landau damping. Since the group velocity of a circularly polarized electromagnetic wave is twice more than the phase velocity (ω_o/k_o), so the combination $(\omega_o - k_o v_g) = -\omega_o$. We thus receive

$$\frac{\delta n_d(z, t)}{n_{od}} = \beta P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} \frac{\delta n_d(\xi', t)}{n_{od}} \\ + \frac{Z_d \omega_o}{m_d \omega_{ci} (v_g^2 - U_o^2)} \psi^2(\xi, t), \quad (19)$$

where $\beta = (1/\sqrt{2\pi})(v_g/v_{ti})(U_o^2/(v_g^2 - U_o^2))$. Multiplying the above equation with the operator $M = 1 + \beta P \int_{-\infty}^{\infty} (d\xi'/\xi' - \xi)$ and using the well known Poincaré–Bertand formula,¹⁶

$$P \int_{-\infty}^{\infty} \frac{dz'}{z' - z} P \int_{-\infty}^z \frac{dz''}{z'' - z'} \frac{F(z'')}{z'' - z'} = (i\pi)^2 F(z).$$

We obtain the expression for dust density,

$$\frac{\delta n_d}{n_{od}} = \frac{Z_d \omega_o \beta v_g}{\sqrt{2\pi} m_d \omega_{ci} v_{ti} U_o^2} \\ \times \left[|\psi(\xi, t)|^2 + \beta P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} |\psi(\xi', t)|^2 \right]. \quad (20)$$

Substituting Eq. (20) into the Schrödinger equation, where $\psi = eA/\sqrt{m_i c}$, we obtain the new kinetic nonlinear Schrödinger equation with nonlocal nonlinearity.

$$i \frac{\partial \psi}{\partial t} + v_g' \frac{\partial^2 \psi}{\partial \xi^2} + F |\psi|^2 + F \beta P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} |\psi(\xi', t)|^2 \psi = 0, \quad (21)$$

where $F = Z_d^2 \omega_o^2 v_g \beta / \sqrt{2\pi} m_d \omega_{ci} v_{ti} U_o^2$. The last term of the Eq. (21) represents the nonlinear Landau damping of circularly polarized EM waves on ions.

Now we find an expression for nonlinear decrement by assuming $\psi = \psi_0 e^{i(k\xi - \omega t)} + \psi_0^* e^{-i(k\xi - \omega t)}$ with constant ψ_0 ,

$$\text{Im } \omega = - \sqrt{\frac{\pi}{2}} \frac{U_o^2}{v_g^3 v_{ti}} \left(\frac{Z_d^2 \omega_o^2}{m_d \omega_{ci}} \right) \left(1 - \frac{U_o^2}{v_g^2} \right)^{-2} |\psi_0|^2. \quad (22)$$

The above expression clearly shows that the nonlinear Landau damping decay is exponential and the presence of $[1 - (U_o^2/v_g^2)]^{-2}$ makes it independent of both the subsonic and the supersonic regimes.

Now, we shall consider a case when the group velocity becomes almost equal to the dust-sound speed, i.e., ($v_g \approx U_o$), we then see that the Zakharov Eq. (17) gives

$$\frac{Z_d \omega_o}{m_d \omega_{ci}} \psi^2 - \frac{U_o^2}{\sqrt{2\pi}} \left(\frac{v_g}{v_{ti}} \right) P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} \frac{\delta n_d(\xi', t)}{n_{od}} = 0. \quad (23)$$

Multiplying both sides of Eq. (23) by the operator $M = P \int (d\xi'/\xi' - \xi)$, we obtain the expression for δn_d as

$$\frac{\delta n_d}{n_{od}} = \frac{\sqrt{2}}{\pi^{3/2}} \left(\frac{v_{ti}}{v_g} \right) \frac{Z_d \omega_o}{m_d U_o^2 \omega_{ci}} P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} |\psi(\xi', t)|^2, \quad (24)$$

substituting into Schrödinger equation,

$$i \frac{\partial \psi}{\partial t} + v_g' \frac{\partial^2 \psi}{\partial \xi^2} + \mu P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} |\psi|^2 \psi = 0, \quad (25)$$

where $\mu = (\pi^{3/2} \sqrt{2}/m_d \omega_{ci})(v_{ti}/v_g)(Z_d \omega_o/U_o)^2$, which is always >0 .

In this case, we obtain the nonlinear Landau damping rate

$$\text{Im } \omega = - \sqrt{\frac{2}{\pi T_i m_i}} \left(\frac{n_{oi} Z_d^2 \omega_o^2}{n_{od} v_g \omega_{ci}} \right) |\psi_0|^2. \quad (26)$$

From Eqs. (22) and (26), we conclude that for the Landau damping of helicon waves, the time dependent term in ponderomotive expression is more dominant than the space dependent term.

V. MODULATIONAL INSTABILITY OF THE SCHRÖDINGER EQUATION

Nonlinear Landau damping term in Eq. (21) drastically changes the physical picture of the modulational instability of the helicon waves. To study the stability, we shall use in Eq. (25) the Madelung's representation of the complex amplitude ψ of the circularly polarized electromagnetic wave,^{30,31}

$$\psi = a(\xi, t) e^{i\varphi(\xi, t)},$$

where the amplitude $a(\xi, t)$ and the phase φ are real quantities. Substituting Eq. (26) into Eq. (25), we obtain from the real and imaginary parts, respectively,

$$v_g' \frac{\partial^2 a}{\partial \xi^2} - a \frac{\partial \varphi}{\partial t} - a v_g' \left(\frac{\partial \varphi}{\partial \xi} \right)^2 \\ + \mu P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} |a(\xi', t)|^2 a(\xi, t) = 0, \quad (27)$$

$$\frac{\partial a^2}{\partial t} + v_g' \frac{\partial}{\partial \xi} \left(a^2 \frac{\partial \varphi}{\partial \xi} \right) = 0. \quad (28)$$

We then linearize the above equations with respect to

the perturbations in the amplitude a and the phase φ as $a = a_0 + \delta a$, $\varphi = \varphi_0 + \delta\varphi$ and where a_0, φ_0 denote the equilibrium values and $\delta a, \delta\varphi$ are small perturbations. After linearization of Eqs. (27) and (28), we seek plane wave solutions proportional to $\exp i(q\xi - \Omega t)$. For the perturbation δa and $\delta\varphi$, we obtain the following dispersion equation:

$$\Omega^2 = v_g' q^2 \left[\frac{v_g' q^2}{2} + i\pi\mu a_o^2 \right] \quad (29)$$

or

$$\Omega = \pm v_g' q^2 / \sqrt{2} [1 + i\Gamma]^{1/2}, \quad (30)$$

where $\Gamma = 3\pi\mu a_o^2 \omega_{pd}^2 / c^2 \omega_{cd} q^2$. Solutions of above are

$$\text{Re } \Omega = \frac{v_g' q^2}{2} (\sqrt{1 + \Gamma^2} + 1)^{1/2} \quad (31)$$

and

$$\text{Im } \Omega = \frac{v_g' q^2}{2} (\sqrt{1 + \Gamma^2} - 1)^{1/2}, \quad (32)$$

since $\text{Re } \Omega \gg \text{Im } \Omega$, so we have oscillatory instability.

VI. ACCELERATION OF SOLITONS

Now we shall demonstrate how the nonlinear Landau term leads to the acceleration of solitons, which can radiate dust-acoustic wave. For this, writing down Schrödinger equation in the dimension of energy,

$$\begin{aligned} & i \frac{\partial \psi(\xi, t)}{\partial t} + \frac{\partial^2 \psi(\xi, t)}{\partial \xi^2} \\ & + \chi \left[|\psi(\xi, t)|^2 \psi(\xi, t) + \beta P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} |\psi(\xi', t)|^2 \right] \psi(\xi, t) \\ & = 0, \end{aligned} \quad (33)$$

where $\chi = Z_d^2 \omega_o^2 v_g \beta / \sqrt{2} \pi m_d \omega_{ci} v_{ti} U_o^2 \omega_{ci}$ and the space and time coordinates ξ and t are normalized, respectively, by ω_{pd}/c and ω_{cd} . We write down the solution of above equation in the form

$$|\psi| = F[\xi - \bar{\xi}(t)] \exp[i\varphi(\xi, t)], \quad (34)$$

where F and φ are real quantities. $\bar{\xi}(t)$ is the coordinate of center of localization of the helicon wave and is generally a nonlinear function of time. Using Eqs. (33) and (34), we obtain a couple of equations,

$$\begin{aligned} & \frac{\partial^2 F}{\partial \xi^2} - \left[\frac{\partial \varphi}{\partial t} + \left(\frac{\partial \varphi}{\partial \xi} \right)^2 \right. \\ & \left. - \chi \left(F^2 + \beta P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} |F(\xi', t)|^2 \right) \right] F = 0, \end{aligned} \quad (35)$$

$$\frac{\partial F^2}{\partial t} + 2 \frac{\partial}{\partial \xi} \left(\frac{\partial \varphi}{\partial \xi} \right) F^2 = 0. \quad (36)$$

As the amplitude of F is described in the form of a function, which depends upon the self-similar argument $z = \xi - \bar{\xi}(t)$, we

then find its values that vanish at infinity ($z \rightarrow \pm \infty, F \rightarrow 0$), from Eq. (36),

$$\varphi = \dot{\bar{\xi}}(t) \xi + f(t), \quad (37)$$

where $f(t)$ is an arbitrary function of time. Substitution of Eq. (37) into Eq. (35) gives

$$\begin{aligned} & \frac{\partial^2 F}{\partial \xi^2} - \left[f(t) + \ddot{\bar{\xi}}(t) \xi + (\dot{\bar{\xi}}(t))^2 \right. \\ & \left. - \chi \left(F^2 + \beta P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} |F(\xi', t)|^2 \right) \right] F = 0. \end{aligned} \quad (38)$$

We obtain for the first integral of Eq. (38),

$$\begin{aligned} & \left(\frac{\partial F}{\partial \xi} \right)^2 - \left[f(t) + \ddot{\bar{\xi}}(t) \xi + (\dot{\bar{\xi}}(t))^2 - \frac{1}{2} \chi F^2 \right. \\ & \left. - \chi \beta P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} |F(\xi', t)|^2 \right] F^2 + \frac{1}{2} F^2 \\ & + \int_{-\infty}^{\xi} d\xi'' \left(\ddot{\bar{\xi}}(t) + \chi \beta P \frac{d}{d\xi''} \right. \\ & \left. \times \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi''} |F(\xi', t)|^2 \right) F^2(\xi'', t) = 0. \end{aligned} \quad (39)$$

It follows from above that

$$\int_{-\infty}^{\xi} d\xi'' \left(\ddot{\bar{\xi}}(t) + \chi \beta P \frac{d}{d\xi''} \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi''} |F(\xi', t)|^2 \right) F^2 = 0. \quad (40)$$

If we assume that the maximum value of the amplitude $F = F_m$ corresponds to the point $\xi = \bar{\xi}(t)$, then we can find the expression for the function $f(t)$ from Eq. (39),

$$\begin{aligned} & f(t) + (\dot{\bar{\xi}}(t))^2 = \frac{1}{2} \chi \beta F_m^2 - \ddot{\bar{\xi}}(t) \bar{\xi}(t) - \chi \beta P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \bar{\xi}} F_m^2 \\ & - \frac{1}{F_m^2} \int_{-\infty}^{\bar{\xi}} d\xi'' \left(\ddot{\bar{\xi}}(t) - \chi \gamma P \frac{d}{d\xi''} \right. \\ & \left. \times \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi''} |F(\xi', t)|^2 \right) F^2(\xi'', t). \end{aligned} \quad (41)$$

Equation (40) takes the form after the use of Eq. (43),

$$\frac{\partial^2 F}{\partial \xi^2} - \left[\begin{aligned} & \ddot{\bar{\xi}}(t)[\xi - \bar{\xi}(t)] + \frac{1}{2}\chi F_m^2 + \chi\beta P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} F_m^2 + \chi\beta P \int_{-\infty}^{\infty} \frac{d\xi' F^2(\xi', t)}{\xi' - \xi} \\ & + \frac{1}{F_m^2} \int_{-\infty}^{\bar{\xi}} d\xi'' \left[\ddot{\bar{\xi}}(t) + \chi\beta P \frac{d}{d\xi''} \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi''} F(\xi', t)^2 \right] F^2(\xi'', t) \end{aligned} \right] F + \chi F^3 = 0. \quad (42)$$

Introducing

$$\chi\beta P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi''} F^2(\xi', t) \equiv \mathcal{O}(\xi'', t), \quad (43)$$

we can limit ourselves in the expansion to first three terms only,

$$\begin{aligned} \mathcal{O}(\xi) &= \mathcal{O}(\bar{\xi}) + (\xi - \bar{\xi}) \left. \frac{\partial \mathcal{O}(\xi)}{\partial \xi} \right|_{\bar{\xi}} \\ &+ \frac{1}{2}(\xi - \bar{\xi})^2 \left. \frac{\partial^2 \mathcal{O}(\xi)}{\partial \xi^2} \right|_{\bar{\xi}} + \dots \end{aligned} \quad (44)$$

If the conditions

$$F_m^2 \gg \Delta l^2 \frac{\partial^2}{\partial \xi^2} \mathcal{O}(\xi) \Big|_{\bar{\xi}} \frac{1}{F_m^2} \int_{-\infty}^{\bar{\xi}} d\xi'' (\xi'' - \bar{\xi}) F^2 \quad (45)$$

are satisfied, we obtain the well-known equation for the amplitude

$$\frac{\partial^2 \Psi}{\partial \xi^2} - \frac{1}{2} \Psi_m^2 \Psi + \Psi^3 = 0, \quad (46)$$

where $\Psi = \chi^{-1/2} F$. The above equation has solution of the form

$$\Psi = \Psi_m \operatorname{sech} \frac{1}{\Delta l} [\xi - \bar{\xi}(t)], \quad (47)$$

while the width of the soliton is equal to $\Delta l = \sqrt{2}/\Psi_m$ and the coordinate of the center of soliton, according to Eqs. (41), (43), and (44) can be determined as

$$\ddot{\bar{\xi}}(t) = \beta \chi^2 \left[\frac{d}{d\xi} P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} \Psi^2(\xi', t) \right]_{\xi=\bar{\xi}}. \quad (48)$$

Substituting Eq. (47) into Eq. (48),

$$\ddot{\bar{\xi}}(t) = -\frac{21\beta}{\sqrt{\pi}} \chi^{3/2} \Psi_m^3 \zeta(3), \quad (49)$$

where $\zeta(3)$ is Reimann zeta function. We see that condition (45) for Eq. (47) is satisfied since

$$\begin{aligned} \left. \frac{\partial^{2m-1} \mathcal{O}}{\partial \xi^{2m-1}} \right|_{\xi=\bar{\xi}} &= (-1)^m \pi \beta \left(\frac{2}{\pi} \right)^{2m+1/2} \\ &\times \chi^2 \frac{\Psi_m^2}{(\Delta l)^{2m-1}} \int_0^{\infty} dx \frac{x^{2m}}{\sinh x} \end{aligned} \quad (50)$$

or

$$\begin{aligned} \left. \frac{\partial^{2m-1} \mathcal{O}}{\partial \xi^{2m-1}} \right|_{\xi=\bar{\xi}} &= (-1)^m \pi \beta \left(\frac{2}{\pi} \right)^{2m+1/2} \\ &\times \chi^2 \frac{\Psi_m^2}{(\Delta l)^{2m-1}} \frac{2^{2m+1} - 1}{2^{2m}} \Gamma(2m+1) \\ &\times \zeta(2m+1), \end{aligned} \quad (51)$$

and

$$\left. \frac{\partial^{2m} \mathcal{O}}{\partial \xi^{2m}} \right|_{\xi=\bar{\xi}} = 0. \quad (52)$$

Equation (49) shows that during the nonlinear interaction soliton transfers some of its energy to ions and eventually its motion are slowed down while its amplitude (Ψ_m) remains constant. Now, we would like to briefly discuss the emission of dust-acoustic wave by the soliton moving with acceleration, for that we first rewrite Eq. (17) in the dimensionless form

$$\begin{aligned} & \left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial \xi^2} \right) \frac{\delta n_d}{n_{od}} \\ & + \frac{1}{\sqrt{2\pi}} \frac{U_o}{v_{ti}} \frac{\partial}{\partial \xi} P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi' - \xi} \int_{-\infty}^{\xi} d\xi'' \frac{\partial}{\partial t} \frac{\delta n_d(\xi'', t)}{n_{od}} \\ & = -\frac{\partial^2}{\partial \xi^2} \psi^2(\xi, t) - \left(\frac{k_0 U_o}{\omega_o} \right) \frac{\partial^2}{\partial t \partial \xi} \psi^2(\xi, t). \end{aligned} \quad (53)$$

We used $t' = t\omega_{ci}$, $\xi' = (\omega_{ci}/U_o)\xi$, and $\psi'^2 = (Z_d^2 \omega_o^2 / m_d \omega_{ci} v_g^2 \omega_{cd}) \psi^2$ in above expression, for our convenience primes are dropped. Now, following Kaw *et al.*, we can separate the part of the density perturbation, which is concentrated in the region of localization of the helicon wave oscillations, where the quantity N will characterize the perturbation of the density by the dust-acoustic wave outside the soliton and assuming the velocity of the source of radiation of the soliton to be much larger than the sound velocity, we can obtain an important expression for a radiating soliton moving with acceleration. From Eqs. (36) and (49), we may determine the spatial distributions of the perturbation of the velocity by the radiated dust-acoustic wave. Kaw *et al.* analyzed the radiation field for specific forms of the density profile of an inhomogeneous plasma for two cases,³² in the same way, for this problem for a linear profile $n(z) = 2\alpha z$. The motion of the center of the soliton is described according to Eq. (17) by the formula

$$\bar{z}(t) = \bar{z}(0) + ut - \frac{1}{2}\alpha t^2, \quad (54)$$

where $\bar{x}(0)$ is the initial value of the coordinate, u is the initial velocity, and the accelerator is constant and equals to $-\alpha$. Similarly in the second case concerning a quadratic inhomogeneity, $n(z) = \alpha^2 z^2$, we can show that the soliton executes harmonic oscillations around the point $z=0$ and does not change its shape. One may investigate Eq. (54) as was done by Kaw *et al.* for the helicon soliton.³² However, the present work was aimed to show how the accelerated soliton can radiate dust-acoustic waves and this problem will be considered in the future.

VII. CONCLUSIONS

In the present work, we considered the nonlinear propagation of high-frequency circularly polarized electromagnetic waves (helicons) in a collisionless dusty plasma, taking into account the nonlinear Landau damping effect. For the one dimensional propagation of the nonlinear helicon waves, we obtained a generalized set of Zakharov equations containing both local and nonlocal nonlinear terms, later coming through the nonlinear Landau damping. Then, we obtained the kinetic nonlinear Schrödinger (KNLS) equation for the two cases. It has been shown that when the group velocity of helicon waves is close to the dust-acoustic speed, then in the KNLS equation remains only a nonlocal nonlinear term arising due to nonlinear Landau damping. The nonlinear decrement is obtained for each case. It is shown that the nonlinear Landau damping term leads to the modulational instability and maximum growth rate is obtained. Finally, it is discussed the wave-wave particle interaction slows down the accelerated soliton. Emission of dust acoustic wave by accelerated soliton is discussed shortly.

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