Diederik Verscheure

Division PMA,
Department of Mechanical Engineering,
Katholieke Universiteit Leuven,
Belgium
diederik.verscheure@mech.kuleuven.be

Inna Sharf

Department of Mechanical Engineering,
McGill University,
Montreal, Quebec,
Canada,
inna.sharf@mcgill.ca

Herman Bruyninckx

Jan Swevers

Joris De Schutter

Division PMA,
Department of Mechanical Engineering,
Katholieke Universiteit Leuven,
Belgium
{herman.bruyninckx, jan.swevers,
joris.deschutter}@mech.kuleuven.be

Identification of Contact Parameters from Stiff Multi-point Contact Robotic Operations

Abstract

Computer simulations play an important role in the design and validation of constrained robotic operations. The fidelity of these simulations, however, depends on the specification of contact dynamics parameters, which often need to be determined experimentally. In this paper we investigate the identification of contact parameters from complex stiff multi-point contact scenarios encountered in typical robotic operations using a recently developed least-squares-based method. This method is extended to deal with geometric uncertainties by employing a non-linear separable least-squares formulation. The latter is solved using a variable projection method, and allows simultaneous identification of contact parameters and geometric uncertainties. The conditions for observability of geometric uncertainties are derived and a regularized formulation is proposed in case the identification of geometric uncertainties is ill-conditioned. The

applicability of the original method and the benefits of the extended method with identification of geometric uncertainties for the identification of the contact parameters are illustrated by means of experimental data measured with the Special Purpose Dexterous Manipulator (SPDM) Task Verification Facility (STVF) manipulator at the Canadian Space Agency (CSA).

KEY WORDS—contact parameters, identification, estimation, contact dynamics, least-squares, multi-point contact

1. Introduction

Robots are increasingly used to perform complex tasks, which often involve contact with the environment. Constrained robotic tasks remain challenging, however, because the dynamic behavior of the payload and the environment affect the accuracy and stability of the low-level controllers. Therefore, computer simulations (Vukobratovic and Velkjo 1999; Velkjo and Vukobratovic 1999; Van Vliet et al. 2000) play an essential role in the design and validation of constrained robotic operations and are especially important for tasks where prior

testing is difficult, if not impossible, such as space operations (Ma and Carr 1998a,b). While finite-element-based simulations are generally impractical, owing to their high computational cost and significant modeling overhead, simulations based on simplified models and model order reduction techniques are widely used and allow reliable results to be obtained (Ma et al. 1997; Ma 2004).

Specifically, the dynamics of the contact between objects, which in general consist of the *structural dynamics* of the objects and the *contact dynamics* between the objects, are often simplified using the *surface compliance* approach (Ma et al. 1997). In this approach, objects are considered to be *rigid* and contact forces are modeled as a function of *local interference distances* between the objects. While this method has been proven to be effective (Ma et al. 1997; Ma 2004) for computer simulations, particularly when interference distances are significantly smaller than object dimensions, the fidelity of these simulations also depends on the accurate specification of model parameters. Owing to the combination of structural and contact dynamics in a single surface compliance model, the parameter specification is of particular difficulty. As a result, these model parameters, which we also loosely refer to as contact parameters, often need to be determined experimentally. To avoid time-consuming experiments with dedicated hardware, such as material testing machines, the development of methods for the identification of contact parameters from *typical robotic operations* using *robotic hardware* is a topic of on-going research. In the context of space robotic applications, these methods are useful in identifying contact parameters from on-Earth robotic tests¹, as well as to identify contact parameters from the actual space robotic operations in a later stage.

Initial efforts in the identification of environment and payload contact dynamics were motivated by the need to improve contact stability and force tracking in the context of force and impedance control (Hogan 1985; Yabuta et al. 1988; Lin and Yae 1992; Singh and Popa 1995; Love and Book 1995; Seraji and Colbaugh 1997; Blouin et al. 1999; Park et al. 2003; De Gersem 2005). In this context, the combined dynamics of the environment and the payload are approximated using the surface compliance approach, as a linear spring, spring-damper or mass-spring-damper model and the model parameters are estimated on-line using adaptive control methods (Yabuta et al. 1988; Singh and Popa 1995; Seraji and Colbaugh 1997), (extended) Kalman filtering (Lin and Yae 1992; Blouin et al. 1999; De Gersem 2005), recursive least squares (Love and Book 1995) or active observers (Park et al. 2003). Although often negligible, the identified dynamics also contain the structural dynamics of the robotic system itself.

Unlike these control-inspired approaches, where only a reasonable estimate of the environment and payload contact dynamics is needed, recent work has increasingly focused on *accurate* identification of contact parameters, for purposes such as computer simulations or detection and perception of environment or object properties (Kikuuwe and Yoshikawa 2002, 2003). As a result, a number of performance investigations (Agar 2004; Agar et al. 2005) have been carried out and various on- and off-line algorithms have been developed and refined (Erickson et al. 2003; Patel 2003; Agar 2004; Agar et al. 2005; Diolaiti et al. 2005; Mu and Sharf 2007; Verscheure et al. 2009). Erickson et al. (2003) compared the performance of some of the early on-line methods (Love and Book 1995; Singh and Popa 1995; Seraji and Colbaugh 1997). In addition to the three on-line methods of Love and Book (1995), Singh and Popa (1995) and Seraji and Colbaugh (1997), a frequency-domain method is presented by Erickson et al. (2003), while a number of other frequency-domain methods are also discussed by Patel (2003). Agar (2004) and Agar et al. (2005) compared several on- and off-line time- and frequency-domain methods for the estimation of contact dynamics parameters for stiff robotic payloads. Diolaiti et al. (2005), Mu and Sharf (2007) and Verscheure et al. (2009) presented both on- and off-line methods for non-linear contact models.

However, all of these methods consider only *geometrically simple, single-point contact* scenarios. An off-line least-squares-based method for *complex multi-point contact* scenarios is developed and investigated by Weber et al. (2002, 2006) by means of simulations. In this paper, this off-line method is investigated using experimental data gathered from *typical robotic operations* involving rather stiff contact. The experimental investigation is particularly challenging because of the stiff contact, the limited freedom for experiment design, and the presence of modeling errors and mismatches between the geometric models and the real objects. To deal with small geometric uncertainties, the method of Weber et al. (2002, 2006) is extended, resulting in simultaneous identification of the contact parameters and the geometric uncertainties. The conditions for observability of the geometric uncertainties are derived and a regularized formulation is proposed in case the identification of geometric uncertainties is ill-conditioned. The experimental results illustrate the viability of the extended approach.

The outline of this paper is as follows. In Section 2, we discuss the modeling of contact dynamics, the contact geometry, sensor and geometric uncertainties. Section 3 is dedicated to the identification algorithm for identification of contact parameters and the extension for identification of both contact parameters and geometric uncertainties. Also, the observability conditions are discussed and we propose regularization to deal with ill-conditioning. Section 4 discusses the experimental test case, the robotic facility and the experimental procedure, while Section 5 describes the modeling of the test case. In Section 6, we describe the identification implementation and subsequently, Section 7 presents the experimental results. Fi-

1. Note that these tests may be carried out using other robotic hardware than the space robotic hardware, for which prior testing may be difficult or impossible.

nally, Section 8 formulates conclusions and points to future work.

2. Modeling

2.1. Preliminaries

Although there is no restriction on the number of objects, this paper focusses on contact between two objects. The two objects are accompanied by corresponding frames $o1$ and $o2$, while the world is fixed by a well-chosen reference frame w .

The position and orientation of objects 1 and 2 with respect to the world frame w , called the “pose” in the remainder of this paper, are expressed by homogeneous transformation matrices ${}^{o1}_w\mathbf{T}$ and ${}^{o2}_w\mathbf{T}$. For example, ${}^{o1}_w\mathbf{T}$ is defined as

$${}^{o1}_w\mathbf{T} = \begin{pmatrix} {}^{o1}_w\mathbf{R} & {}^{o1}_w\mathbf{p} \\ \mathbf{0}_{1 \times 3} & 1 \end{pmatrix}, \quad (1)$$

where ${}^{o1}_w\mathbf{p}$ represents the position vector from the origin of w to the origin of $o1$, expressed in frame w , and the matrix ${}^{o1}_w\mathbf{R}$ represents the orientation of $o1$ relative to w .

The velocities of objects 1 and 2 are described by six-dimensional column vectors ${}^{o1}_w\mathbf{t}$ and ${}^{o2}_w\mathbf{t}$ which express the translational and angular velocity of the origin of frames $o1$ and $o2$, expressed in, and with respect to, world frame w .

The forces and moments exerted onto objects 1 and 2 are expressed as six-dimensional force/moment vectors.

For example, ${}^{o2}_w\mathbf{w}$ is the force/moment vector exerted by object 2 onto object 1 at the origin of frame $o1$, expressed in frame $o1$.

2.2. Contact Dynamics Modeling

The relation between kinematic and dynamic quantities is given by the contact dynamics equations. Similarly to Weber et al. (2006), a surface compliance approach is adopted, whereby objects are considered to be *rigid* and contact forces are modeled as a function of *local interference distances* between the objects². While the treatment of Weber et al. (2006) is more elaborate, this paper limits itself to a linear undamped force-deformation law in the normal direction and a Coulomb model of sliding friction. All quantities are expressed in and with respect to either object frame $o1$ or $o2$, hence all subscripts and superscripts are omitted in this section for clarity.

2. The surface compliance approach has proven to be effective (Ma et al. 1997; Ma 2004), although it is mainly limited to contact between non-conforming objects, where the contact area is small relative to the dimensions of the contacting objects. These conditions are met reasonably well for the test case considered in this paper.

Fig. 1. Undeformed object geometry (solid line), deformed object geometry (schematically, dash-dotted line), interference distance δ_i , contact point position $\mathbf{p}_i$, normal contact vector $\mathbf{e}_{n,i}$ and tangential contact vector $\mathbf{e}_{t,i}$.

The magnitude of the normal force at contact point i , can be expressed as a function of the interference distance δ_i as

$$F_{n,i} = \begin{cases} K_i \delta_i, & \text{if } \delta_i \geq 0, \\ 0, & \text{if } \delta_i < 0. \end{cases} \quad (2)$$

Similarly, the magnitude of the friction force $F_{t,i}$ at a sliding contact point i , can be expressed as

$$F_{t,i} = \mu_i F_{n,i}, \quad (3)$$

where μ_i is the sliding coefficient of friction at contact point i .

The force/moment vector $\mathbf{w}$ exerted *onto* the considered object at the origin of that object frame, expressed in the corresponding object frame, can be written as

$$\mathbf{w} = \begin{pmatrix} -\sum_{i=1}^N (F_{n,i} \mathbf{e}_{n,i} + F_{t,i} \mathbf{e}_{t,i}) \\ -\sum_{i=1}^N \mathbf{p}_i \times (F_{n,i} \mathbf{e}_{n,i} + F_{t,i} \mathbf{e}_{t,i}) \end{pmatrix}, \quad (4)$$

where N is the number of contact points, $\mathbf{e}_{n,i}$ is the outward normal to the considered object at contact point i and $\mathbf{e}_{t,i} = -\mathbf{v}_{t,i} / \|\mathbf{v}_{t,i}\|_2$ is defined opposite to the relative tangential velocity $\mathbf{v}_{t,i}$ of the other object with respect to the considered object at contact point i . Here $\mathbf{e}_{n,i}$, $\mathbf{e}_{t,i}$, $\mathbf{p}_i$ are all expressed in the object frame of the considered object, as illustrated in Figure 1 for body 1.

Substituting for $F_{n,i}$ and $F_{t,i}$ using (2) and (3) and assuming $K_i = K$ and $\mu_i = \mu$ for $i = 1 \dots N$, a linear equation for the parameters is obtained

$$\begin{aligned} \mathbf{w} &= \begin{pmatrix} -\sum_{i=1}^N (K \delta_i \mathbf{e}_{n,i} + \mu K \delta_i \mathbf{e}_{t,i}) \\ -\sum_{i=1}^N \mathbf{p}_i \times (K \delta_i \mathbf{e}_{n,i} + \mu K \delta_i \mathbf{e}_{t,i}) \end{pmatrix} \\ &= \phi^T \boldsymbol{\theta}^0, \end{aligned} \quad (5)$$

where

$$\phi = \begin{pmatrix} -\sum_{i=1}^N \delta_i \mathbf{e}_{n,i} & -\sum_{i=1}^N \delta_i \mathbf{e}_{t,i} \\ -\sum_{i=1}^N \delta_i \mathbf{p}_i \times \mathbf{e}_{n,i} & -\sum_{i=1}^N \delta_i \mathbf{p}_i \times \mathbf{e}_{t,i} \end{pmatrix}^T, \quad (6)$$

$$\theta^0 = \begin{pmatrix} K \\ \mu K \end{pmatrix}. \quad (7)$$

2.3. Contact Geometry Modeling

Although it is not required, in this paper, we consider objects that can be described as a union $\mathcal{P} = \cup_{p=1 \dots Q} \mathcal{P}_p$ of convex polyhedra, where $\mathcal{P}_p = \{\mathbf{x} \in \mathcal{R}^3 \mid \mathbf{F}_p \mathbf{x} + \mathbf{g}_p \leq 0\}$ is defined as a set of linear inequalities. In this case, the interference distances δ_i , the locations of the contact points $\mathbf{p}_i$ and the normals $\mathbf{e}_{n,i}$ can be easily determined from ${}^w_1\mathbf{T}_w$, ${}^w_2\mathbf{T}_w$ and the object descriptions $\mathcal{P}_{o1}$ and $\mathcal{P}_{o2}$, using the minimal distance approach described by Nahon (1994). Also, the tangential directions $\mathbf{e}_{t,i}$ are easily obtained from ${}^w_1\mathbf{T}_w$, ${}^w_2\mathbf{T}_w$, ${}^w_1\mathbf{t}_w$, ${}^w_2\mathbf{t}_w$, $\mathbf{p}_i$ and $\mathbf{e}_{n,i}$. This procedure is called “mindist” in the remainder of this paper.

2.4. Uncertainty Modeling

In practice, a number of uncertainties affect $\mathbf{w}$ and the calculation of ϕ in (5). The force/moment vector $\mathbf{w}$ on the one hand, which is typically measured directly with a force/moment sensor, is corrupted with noise $\Delta \mathbf{w}$, such that only an estimate

$$\hat{\mathbf{w}} = \mathbf{w} + \Delta \mathbf{w} \quad (8)$$

is available. The noise $\Delta \mathbf{w}$ is assumed to be zero-mean and uncorrelated in time, and to have a diagonal covariance matrix $\mathbf{R}$. The regressor matrix ϕ , on the other hand, consists of the combination of time-varying variables δ_i , $\mathbf{p}_i$, $\mathbf{e}_{n,i}$ and $\mathbf{e}_{t,i}$, which in turn are calculated from the measured quantities ${}^w_1\mathbf{T}_w$, ${}^w_2\mathbf{T}_w$, ${}^w_1\mathbf{t}_w$, ${}^w_2\mathbf{t}_w$ and from the nominally constant geometric object specifications $\mathcal{P}_{o1}$ and $\mathcal{P}_{o2}$.

The quantities ${}^w_1\mathbf{T}_w$, ${}^w_2\mathbf{T}_w$, ${}^w_1\mathbf{t}_w$ and ${}^w_2\mathbf{t}_w$ are affected by measurement errors due to quantization, sensor noise and *systematic error sources*. Examples of systematic error sources in constrained robotic tasks, are the misspecification of the pose of an object with respect to the world frame w or with respect to the robot end-effector frame ee due to gripping or fixture inaccuracies.

The geometric object specifications $\mathcal{P}_{o1}$ and $\mathcal{P}_{o2}$ are affected by geometric errors caused by non-zero manufacturing tolerances and geometric simplifications.

Usually, the effect of systematic error sources and deviations from the object specifications on ϕ is *larger* than the effect of measurement errors due to quantization or sensor noise and very often, systematic errors and geometric deviations are

also *constant*. Therefore, the effect of quantization and sensor noise is neglected, while the systematic errors and geometric deviations are modeled by a set of uncertainty variables $\mathbf{x}_u = (x_{u,1}, \dots, x_{u,j}, \dots, x_{u,M})^T$. The uncertainty variables $\mathbf{x}_u$ can consist of a wide range of parameterizable transformations such as scaling, rotation, translation of individual object features or of the objects as a whole, and even non-linear transformations. Furthermore, they are assumed to have a *nominal* or *design value* of $\mathbf{x}_u^d$, while their true value is unknown and equal to $\mathbf{x}_u^0$. The uncertainty variables affect the regressor matrix, such that only an estimate $\hat{\phi} = \phi(\mathbf{x}_u^d)$ of the true regressor matrix $\phi^0 = \phi(\mathbf{x}_u^0)$ is known. Very often, the relation between ϕ and $\mathbf{x}_u$ is not known explicitly or analytically, although ϕ can be evaluated numerically for a given value of $\mathbf{x}_u$.

3. Identification

3.1. Identification of Contact Parameters

By collecting a set of L measurements $\hat{\mathbf{w}}_k$ and $\phi_k(\mathbf{x}_u)$ in a $6L$ -dimensional output vector $\hat{\mathbf{Y}}$ and a $2 \times 6L$ -dimensional regressor matrix $\Phi(\mathbf{x}_u)$, while assuming the parameter vector θ to be time-independent, an overdetermined linear system

$$\hat{\mathbf{Y}} = \mathbf{Y} + \Delta \mathbf{Y} = \Phi(\mathbf{x}_u)^T \theta, \quad (9)$$

in θ is obtained. From (9), an estimate $\hat{\theta}$ of θ^0 , which is the solution of

$$\min_{\theta} \frac{1}{6L} \left\| \hat{\mathbf{Y}} - \Phi(\mathbf{x}_u)^T \theta \right\|_2^2, \quad (10)$$

can be calculated using least squares techniques. For simplicity, we limit the discussion to ordinary least squares (OLS) which considers the standard 2-norm $\|\cdot\|_2 = \sqrt{(\cdot)^T(\cdot)}$. Extension to weighted least squares (WLS) which considers the weighted 2-norm $\|\cdot\|_w = \sqrt{(\cdot)^T \mathbf{W}^{-1}(\cdot)}$ with positive semi-definite weighting matrix $\mathbf{W}$ is straightforward, yet notationally tedious.

The least-squares estimate equals (Kailath et al. 2000)

$$\hat{\theta} = (\Phi(\mathbf{x}_u)^T \Phi(\mathbf{x}_u))^{-1} \Phi(\mathbf{x}_u)^T \hat{\mathbf{Y}}. \quad (11)$$

The estimate $\hat{\theta}$ is called a *global estimate* when $\hat{\mathbf{Y}}$ and $\Phi(\mathbf{x}_u)$ contain all available measurements, and an *instantaneous estimate* when $\hat{\mathbf{Y}}$ and $\Phi(\mathbf{x}_u)$ contain only $\hat{\mathbf{w}}_k$ and $\phi_k(\mathbf{x}_u)$ from time-step k . Since the true value $\mathbf{x}_u^0$ of $\mathbf{x}_u$ is generally unknown, $\mathbf{x}_u$ is typically replaced by the design value $\mathbf{x}_u^d$ in (11). However, in general $\mathbf{x}_u^d \neq \mathbf{x}_u^0$, and the resulting solution for $\hat{\theta}$ may be biased.

3.2. Identification of Geometric Uncertainties

In contrast, by minimizing

$$\min_{\theta, \mathbf{x}_u} V_L(\theta, \mathbf{x}_u) = \frac{1}{6L} \left\| \hat{\mathbf{Y}} - \Phi(\mathbf{x}_u)^T \theta \right\|_2^2, \quad (12)$$

unbiased estimates³ $\hat{\boldsymbol{\theta}}$ and $\hat{\mathbf{x}}_u$ of both $\boldsymbol{\theta}^0$ and $\mathbf{x}_u^0$ can be obtained. Namely, as $L \rightarrow \infty$, $V_L(\boldsymbol{\theta}, \mathbf{x}_u) \rightarrow V_\infty(\boldsymbol{\theta}, \mathbf{x}_u)$, which has a global minimum at $\boldsymbol{\theta} = \boldsymbol{\theta}^0$ and $\mathbf{x}_u = \mathbf{x}_u^0$ and which is defined as

$$\begin{aligned} V_\infty(\boldsymbol{\theta}, \mathbf{x}_u) &= \lim_{L \rightarrow \infty} \frac{1}{6L} \left\| \hat{\mathbf{Y}} - \boldsymbol{\Phi}(\mathbf{x}_u)^T \boldsymbol{\theta} \right\|_2^2 \\ &= \lim_{L \rightarrow \infty} \frac{1}{6L} \left\| \mathbf{Y} + \Delta \mathbf{Y} - \boldsymbol{\Phi}(\mathbf{x}_u)^T \boldsymbol{\theta} \right\|_2^2 \\ &= \text{trace}(\mathbf{R}) + \lim_{L \rightarrow \infty} \frac{1}{6L} \left\| \mathbf{Y} - \boldsymbol{\Phi}(\mathbf{x}_u)^T \boldsymbol{\theta} \right\|_2^2, \quad (13) \end{aligned}$$

where $\mathbf{R}$ is the covariance matrix of the components of $\Delta \mathbf{Y}$, as defined below (8). As a result, $\hat{\boldsymbol{\theta}}$ and $\hat{\mathbf{x}}_u$ which minimize $V_L(\boldsymbol{\theta}, \mathbf{x}_u)$ converge to $\boldsymbol{\theta}^0$ and $\mathbf{x}_u^0$ as $L \rightarrow \infty$.

The parameter estimation problem (12) is a separable⁴ non-linear least-squares problem (Golub and Pereyra 1973, 2003), which is linear in $\boldsymbol{\theta}$ and non-linear in $\mathbf{x}_u$. Hence, it can be solved efficiently by minimizing the variable projection functional (Golub and Pereyra 1973, 2003)

$$\min_{\mathbf{x}_u} \frac{1}{6L} \left\| \hat{\mathbf{Y}} - \boldsymbol{\Phi}(\mathbf{x}_u)^T (\boldsymbol{\Phi}(\mathbf{x}_u)^T \boldsymbol{\Phi}(\mathbf{x}_u))^{-1} \boldsymbol{\Phi}(\mathbf{x}_u) \hat{\mathbf{Y}} \right\|_2^2, \quad (14)$$

obtained by replacing $\boldsymbol{\theta}$ in $V_L(\boldsymbol{\theta}, \mathbf{x}_u)$ by the least-squares solution of (11).

3.3. Conditions for the Observability of Geometric Uncertainties

The requirements for estimating only $\boldsymbol{\theta}$, namely that $\boldsymbol{\Phi}(\mathbf{x}_u)$ is of full rank, are discussed in detail by Weber et al. (2006). Conversely, the requirements for estimating $\boldsymbol{\theta}$ and $\mathbf{x}_u$ simultaneously are more elaborate as shown here. Assuming for simplicity small variations of the geometric uncertainties $\Delta \mathbf{x}_u = \mathbf{x}_u - \mathbf{x}_u^0$ such that $\boldsymbol{\Phi}(\mathbf{x}_u)$ can be approximated as

$$\boldsymbol{\Phi}(\mathbf{x}_u) = \boldsymbol{\Phi}(\mathbf{x}_u^0) + \sum_{j=1}^M \frac{\partial \boldsymbol{\Phi}(\mathbf{x}_u)}{\partial x_{u,j}} \bigg|_{\mathbf{x}_u=\mathbf{x}_u^0} \Delta x_{u,j}, \quad (15)$$

while $\hat{\mathbf{Y}} = \mathbf{Y}$, the first-order optimality conditions for $V_L(\boldsymbol{\theta}, \mathbf{x}_u)$, are

$$-\frac{2}{6L} \boldsymbol{\Phi}(\mathbf{x}_u) (\mathbf{Y} - \boldsymbol{\Phi}(\mathbf{x}_u)^T \boldsymbol{\theta}) = \mathbf{0}, \quad (16)$$

$$-\frac{2}{6L} \left(\boldsymbol{\theta}^T \frac{\partial \boldsymbol{\Phi}(\mathbf{x}_u)}{\partial x_{u,j}} \bigg|_{\mathbf{x}_u=\mathbf{x}_u^0} \right) (\mathbf{Y} - \boldsymbol{\Phi}(\mathbf{x}_u)^T \boldsymbol{\theta}) = \mathbf{0}, \quad (17)$$

for $j = 1 \dots M$.

Equation (16) holds if $\boldsymbol{\theta}$ is equal to the least-squares solution $\hat{\boldsymbol{\theta}}$ given by (11), while in general, (17) holds if $(\mathbf{Y} - \boldsymbol{\Phi}(\mathbf{x}_u)^T \boldsymbol{\theta}) = \mathbf{0}$. If both of these conditions are satisfied and assuming that $\boldsymbol{\Phi}(\mathbf{x}_u)$ is of full rank (Weber et al. 2006), $\boldsymbol{\Phi}(\mathbf{x}_u) = \boldsymbol{\Phi}(\mathbf{x}_u^0)$ and $\boldsymbol{\theta} = \boldsymbol{\theta}^0$. Furthermore, when $\frac{\partial \boldsymbol{\Phi}(\mathbf{x}_u)}{\partial x_{u,j}} \bigg|_{\mathbf{x}_u=\mathbf{x}_u^0}$ for $j = 1, \dots, M$ are also mutually linearly independent and linearly independent from $\boldsymbol{\Phi}(\mathbf{x}_u)$, there is a unique solution to $\boldsymbol{\Phi}(\mathbf{x}_u) = \boldsymbol{\Phi}(\mathbf{x}_u^0)$, namely $\Delta \mathbf{x}_u = \mathbf{0}$ or in other words $\mathbf{x}_u = \mathbf{x}_u^0$.

Conversely, when either of the latter conditions is not satisfied, there is a non-finite subspace of solutions for $\Delta \mathbf{x}_u$ and optimization problem (14) is ill-posed in the sense of Hadamard⁵. For example, if $(\partial \boldsymbol{\Phi}(\mathbf{x}_u) / \partial x_{u,l}) \big|_{\mathbf{x}_u=\mathbf{x}_u^0} = \mathbf{0}$, $\boldsymbol{\Phi}(\mathbf{x}_u)$ can be made equal to $\boldsymbol{\Phi}(\mathbf{x}_u^0)$ irrespective of the value of $\Delta x_{u,l}$. In this case, the geometric uncertainty variable $x_{u,l}$ is called *unobservable*. More generally, when $(\partial \boldsymbol{\Phi}(\mathbf{x}_u) / \partial x_{u,j}) \big|_{\mathbf{x}_u=\mathbf{x}_u^0}$ for $j = 1, \dots, M$ are not linearly independent, certain elements or combinations of $\mathbf{x}_u$ are unobservable. In addition, when $(\partial \boldsymbol{\Phi}(\mathbf{x}_u) / \partial x_{u,l}) \big|_{\mathbf{x}_u=\mathbf{x}_u^0}$ is linearly related to $\boldsymbol{\Phi}(\mathbf{x}_u)$, (17) for $j = l$ is always zero if $\boldsymbol{\theta}$ equals the least-squares estimate $\hat{\boldsymbol{\theta}}$, in accordance with (16). Consequently, certain elements or combinations of $\mathbf{x}_u$ are unobservable.

Although the above-mentioned conditions leading to unobservability are only rarely met *exactly*, in practice, they are relevant even if they are met only approximately. This is often the case for *typical robotic operations*, such as those considered in this paper. In that case, $V_L(\boldsymbol{\theta}, \mathbf{x}_u)$ may not have a *well-defined* global minimum in the neighborhood of $\mathbf{x}_u = \mathbf{x}_u^0$ such that the uncertainties $\mathbf{x}_u$ are only weakly observable or even unobservable and that problem (14) is ill-conditioned or even ill-posed (Hansen 1998).

3.4. Ill-conditioned Identification of Geometric Uncertainties

A well-known approach to deal with ill-posed or ill-conditioned problems is regularization (Tikhonov and Arsenin 1977; Hansen 1998). In this paper, we consider Tikhonov regularization (Tikhonov and Arsenin 1977; Hansen 1998), resulting in the following objective function

$$\begin{aligned} \min_{\mathbf{x}_u} \frac{1}{6L} \left\| \hat{\mathbf{Y}} - \boldsymbol{\Phi}(\mathbf{x}_u)^T (\boldsymbol{\Phi}(\mathbf{x}_u)^T \boldsymbol{\Phi}(\mathbf{x}_u))^{-1} \boldsymbol{\Phi}(\mathbf{x}_u) \hat{\mathbf{Y}} \right\|_2^2 \\ + \frac{\gamma}{M} \left\| \mathbf{x}_u - \mathbf{x}_u^d \right\|_2^2, \quad (18) \end{aligned}$$

where γ is a suitably chosen weighting factor and where $\mathbf{x}_u^d$ is the best prior guess for $\mathbf{x}_u$, or in other words, the design or nominal value of $\mathbf{x}_u$. The regularization biases the estimates of

3. Under the assumption that the output vector $\hat{\mathbf{Y}}$ is generated by model (9).

4. The separability aspect comes from the idea that the linear variables can be separated or eliminated from the problem, while the resulting problem is non-linear in the remaining variables.

5. Problems which model physical phenomena are well-posed if they have a solution which is *unique* and which depends continuously on the data, in some reasonable topology. Otherwise, they are ill-posed.

the geometric uncertainties towards their design values, while still allowing reasonable estimates to be obtained when there is significant evidence that the actual geometric uncertainties differ from their design values. In a Bayesian context, γ is chosen as the ratio of the variance of the noise in the output $\hat{\mathbf{Y}}$ and the variance in the design value $\mathbf{x}_u^d$. However, as the choice of the variance in $\mathbf{x}_u^d$ may involve some arbitrariness, γ can also be chosen using L-curve analysis (Hansen 1998). In the L-curve analysis method, the magnitudes of both terms of objective function (18) are evaluated with respect to each other, for different values of γ , and a γ is chosen which makes both terms relatively small.

4. Experimental Test Case, Setup and Procedure

As part of the multi-national collaboration to build the International Space Station (ISS), Canada has contributed two manipulator systems: the Space Station Remote Manipulator System (SSRMS) and the Special Purpose Dexterous Manipulator (SPDM) (Ma et al. 1997). The SPDM is responsible for maintenance replacement and handling of specific station elements, called Orbital Replacement Units (ORUs). Motivated by the stringent requirement of complete planning and verification of tasks, the SPDM Task Verification Facility (STVF) (Piedbœuf et al. 1999; Zhu et al. 2002), which emulates the dynamics of the SPDM end-effector using a hardware-in-the-loop simulation scheme (De Carufel et al. 2000), was developed as the on-Earth verification facility by CSA and is used to develop and verify contact tasks such as insertion and extraction of ORUs, and to conduct experimental contact parameter estimation (Lange and Gonthier 2004). The Arm Computer Unit (ACU), a main control element of the SSRMS, is a typical example of an ORU.

The experimental case employed in this paper to demonstrate the identification procedure, considers the on-Earth validation of a space insertion task, using mock-ups of the ACU and the ACU docking receptacle shown in Figure 2. The experimental setup consists of the STVF manipulator and its worksite shown in Figure 3, which is equipped with a force plate. The ACU docking receptacle is mounted onto an aluminum plate which is in turn mounted onto the force plate, while the ACU itself is manipulated by the STVF manipulator by means of a micro-fixture mounted onto the ACU.

Four different tests of the insertion task are carried out, named tests 1a, 1b, 2a and 2b hereafter. Each test starts with an insertion of the ACU into the receptacle with a contact force around 30 N normal to the force plate to establish a reference position. This first insertion is hereafter referred to as the “reference insertion”. Subsequently, for each test, the ACU is retracted normal to the force plate over a distance of 2.7×10^{-2} m, followed by a motion parallel to the force plate. For tests 1a and 1b, the ACU motion consists of a rotation

Fig. 2. Arm Computer Unit (ACU) (left) and ACU docking receptacle (right) mock-ups.

Fig. 3. STVF manipulator (left) and STVF worksite (SMT) (right).

by 2° around the normal to the force plate, while for tests 2a and 2b, the ACU motion consists of a forward translation by 2.7×10^{-2} m parallel to the force plate. Then, the test continues with the actual insertion, whereby the ACU is lowered until contact is made with the receptacle and gradually inserted into the receptacle. The latter insertion is named the “test insertion” for each test. The test insertions are carried out at low speeds of about 2×10^{-3} m s⁻¹ and during the insertions, the contact force normal to the force plate is kept approximately constant at around 50 N.

The STVF end-effector position ${}^{ee}\mathbf{T}_w$ (Figure 4) is calculated from the forward rigid-object kinematics of the STVF manipulator, with an effective resolution of 2×10^{-6} m, while the end-effector velocity ${}^{ee}\mathbf{t}_w$ (Figure 4) is calculated by finite differences with an effective resolution of 2×10^{-4} m s⁻¹. The force ${}^{o2}_{fp}\mathbf{w}$ (Figure 4) is measured using the force plate with a force resolution of 1.5 N. These position, velocity and force measurements are collected at 1,000 Hz and downsampled to 50 Hz for the identification.

Fig. 4. ACU insertion with world (w), end-effector (ee), force plate (fp), object 1 ($o1$) and object 2 ($o2$) frames.

Fig. 5. ACU and ACU docking receptacle geometric models.

5. ACU and Receptacle Modeling

5.1. ACU and Receptacle Geometry and Frames

In the four tests, most of the contact occurs between the ACU V-guide faces and the receptacle V-groove faces, and to a limited extent, between the rounded ACU V-guide edges and the receptacle V-groove faces. As a result, these features are modeled fairly accurately. For the other features, the only requirement is that they are modeled such that they are *unable* to cause contact in the geometric models.

The geometries of the ACU and the receptacle are modeled as unions of convex polyhedra shown in Figure 5. The ACU V-guides of the mock-up are rounded with a radius of curvature of 3.8×10^{-3} m. In the geometric model, these roundings are approximated using triply chamfered V-guides. While the accuracy can be further improved, the main purpose of the incorporation of the rounding is to prevent contact between the otherwise sharp edges of the ACU V-guides and the faces of the receptacle V-grooves in the geometric model.

The ACU and its receptacle are named objects 1 and 2, with reference frames $o1$ and $o2$, respectively (Figure 4). The frames $o1$ and $o2$ are chosen such that if ${}^{o1}_w\mathbf{T} = {}^{o2}_w\mathbf{T} = \mathbf{I}_{4 \times 4}$, the ACU and the receptacle are fully mated, in such a way that the ACU V-guide faces and receptacle V-groove faces are touching with zero intersection. As a result, when the ACU and its receptacle are fully mated with zero contact force, ${}^{o1}_{o2}\mathbf{T} = \mathbf{I}_{4 \times 4}$.

5.2. ACU and Receptacle Geometric Uncertainties

5.2.1. ACU and Receptacle Feature Uncertainties

To model geometric uncertainties, variables $\mathbf{x}_u$ accounting for the uncertainty of the object features are considered. The choice of the uncertainty variables is not straightforward and requires some physical insight. Here, we consider only variables which represent simple transformations such as translations and rotations.

First, it is only meaningful to consider uncertainties for the features that undergo contact, otherwise the uncertainty variables are by definition *unobservable*. Second, even for features that undergo contact, some uncertainty variables may be unobservable, while some variables are only observable in certain combinations. For example, for tests 2a and 2b⁶, mainly face-face contact occurs between the faces of the ACU V-guides and receptacle V-grooves. Consequently, as illustrated in Figure 6, V-guide shifts $x_{u,j}$ along the V-guides are clearly unobservable, since $(\partial\phi(\mathbf{x}_u)/\partial x_{u,j})|_{\mathbf{x}_u=\mathbf{0}} = \mathbf{0}$, while transverse shifts and rotations in the YZ -plane are observable under some mild conditions. As another example, scaling of the ACU V-guides or translations of the ACU V-guides normal to the YZ -plane are only observable in combination, because they both result in translation of the ACU V-guide faces along their normals, such that the derivatives $(\partial\phi(\mathbf{x}_u)/\partial x_{u,j})|_{\mathbf{x}_u=\mathbf{0}}$ for these variables are linearly dependent. Similarly, small transverse shifts of the positions of both the ACU V-guides and the receptacle V-grooves are only observable relative to each other. Owing to this, and following the principle of Ockham's razor⁷ (Ariew 1976), it is recommended to keep geometric feature uncertainties to a minimum and possibly decide in an iterative manner whether or not to incorporate them, based on their explanatory value.

As the ACU V-guides and receptacle V-grooves are fixed in an adjustable manner onto the ACU and the receptacle respectively, we consider four-dimensional vectors $\mathbf{x}_{u,t}$ and $\mathbf{x}_{u,r}$ accounting for transverse shifts and rotations of each of the four ACU V-guides in the YZ -plane with respect to their centers

6. For tests 1a and 1b, some edge-face contact occurs between the edges of the ACU V-guides and the faces of the receptacle V-grooves, such that V-guide shifts $x_{u,j}$ along the V-guides may be weakly observable.

7. The principle of Ockham's razor states that the explanation of any phenomenon should make as few assumptions as possible, eliminating those that make no difference.

Fig. 6. Observable and unobservable uncertainty variables for ACU V-guides.

(Figure 6). Conversely, uncertainties such as rotations or translations *outside* the YZ -plane are believed to be much smaller, since adjustment of these degrees of freedom is not possible for the ACU and the receptacle. Furthermore, because the position of the ACU V-guides and receptacle V-grooves are only observable in combination, only the ACU V-guides are assumed to be uncertain, while the positions of the receptacle V-grooves are assumed to be equal to their design positions. Here $\mathbf{x}_{u,t}$ and $\mathbf{x}_{u,r}$ are chosen such that the ACU V-guides are at their design positions for $\mathbf{x}_{u,t} = \mathbf{0}$ and $\mathbf{x}_{u,r} = \mathbf{0}$.

5.2.2. ACU and Receptacle Pose Uncertainties

In addition to geometric feature uncertainties of the two objects, it is usually difficult to accurately determine the pose of the objects with respect to the end-effector frame ee or the world frame w . In our case, uncertainties in ${}^{o1}_{ee}\mathbf{T}$ and ${}^{o2}_w\mathbf{T}$ are due to fixture or gripping inaccuracies and due to the inaccurately known location of the worksite with respect to the world frame w , respectively.

To reduce the effect of these uncertainties, it is typical to assume one of these quantities to be known and to calculate the other quantity from a situation where the relative pose ${}^{o1}_{o2}\mathbf{T}$ between the two objects is known fairly accurately. In this paper, ${}^{o2}_w\mathbf{T}$ is assumed to be perfectly known and constant while ${}^{o1}_{ee}\mathbf{T}$ is calculated from a well-chosen end-effector pose ${}^{ee}_{w,0}\mathbf{T}$ and from an estimate of ${}^{o1}_{o2}\mathbf{T}$ in this pose. We hereafter call ${}^{ee}_{w,0}\mathbf{T}$ the “reference pose”, and it is chosen in such a way that the error on the estimated ${}^{o1}_{o2}\mathbf{T}$ is as small as possible.

The different tests reveal that the end-effector pose for which the ACU is fully inserted and at which the forces are approximately the same, varies within 2×10^{-4} m and 2×10^{-4} rad for position and angular directions, between the reference insertion and the test insertion. This variation is observed both *within* a single test and *between* different tests. As a result, it is difficult to make a good choice for a single reference pose ${}^{ee}_{w,0}\mathbf{T}$.

Therefore, the following approach is used. For each test, the end-effector pose at the end of the test insertion, when the force

normal to the force plate is approximately 50 N, is taken to be the reference pose ${}^{ee}_{w,0}\mathbf{T}$. In the reference pose, it is assumed that objects 1 and 2 are fully mated in all directions, except that there is a resultant displacement δ_r of object 1 with respect to object 2 in the X -direction of object frame $o2$ (Figure 5), such that objects 1 and 2 are intersecting. In other words, it is assumed that

$${}^{o1}_{o2}\mathbf{T} = \begin{pmatrix} 1 & 0 & 0 & \delta_r \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (19)$$

where δ_r is estimated by extrapolating the relation between the position and the normal force during the retraction phase of the reference insertion. In the retraction phase of the reference insertion, the contact force normal to the force plate varies from 30 N to 0 N, while the end-effector position normal to the force plate changes by a measured amount $\hat{\delta}_{r,30}$. Therefore, δ_r is estimated as $\hat{\delta}_r = (50 \text{ N}) / (30 \text{ N}) \cdot \hat{\delta}_{r,30}$ which yields a value in the range $[7.0, 10.0] \times 10^{-5}$ m for all tests. Accordingly, the mid-point value $\hat{\delta}_r = 8.5 \times 10^{-5}$ m is taken for all tests as a *nominal value* and the actual displacement is assumed to be equal to $\delta_r = \hat{\delta}_r + x_{u,\delta}$, where $x_{u,\delta}$ is an uncertainty variable to be estimated by the identification algorithm.

Finally, for each test, ${}^{o1}_{ee}\mathbf{T}$, which is needed to calculate ${}^{o1}_w\mathbf{T}$, is determined from ${}^{ee}_{w,0}\mathbf{T}$ and ${}^{o2}_w\mathbf{T}$ as

$${}^{o1}_{ee}\mathbf{T}(x_{u,\delta}) = ({}^{ee}_{w,0}\mathbf{T})^{-1} \cdot {}^{o2}_w\mathbf{T} \cdot {}^{o1}_{o2}\mathbf{T}(x_{u,\delta}). \quad (20)$$

6. Identification Implementation

To identify both contact parameters and geometric uncertainties, a WLS objective function, augmented with a weighted regularization term is chosen, namely

$$\min_{\boldsymbol{\theta}, \mathbf{x}_u} \tilde{V}_L(\boldsymbol{\theta}, \mathbf{x}_u) = \frac{1}{6L} \left\| \hat{\mathbf{Y}} - \boldsymbol{\Phi}(\mathbf{x}_u)^T \boldsymbol{\theta} \right\|_{\mathbf{W}_Y}^2 + \frac{\gamma}{M} \left\| \mathbf{x}_u - \mathbf{x}_u^d \right\|_{\mathbf{W}_{x_u}}^2, \quad (21)$$

where $\hat{\mathbf{Y}}$ and $\boldsymbol{\Phi}(\mathbf{x}_u)$ are expressed in and with respect to frame $o1$, $\mathbf{x}_u$ is a nine-dimensional vector of the geometric uncertainty variables defined in Section 5, namely $\mathbf{x}_u = (x_{u,\delta}, \mathbf{x}_{u,t}, \mathbf{x}_{u,r})^T$, $\mathbf{W}_Y$ is a $6L \times 6L$ -dimensional diagonal weighting matrix with $(F_{\max}^2, F_{\max}^2, F_{\max}^2, M_{\max}^2, M_{\max}^2, M_{\max}^2)^T$ repeated L times on the diagonal⁸. Here $F_{\max} = 50$ N

8. In theory, $\mathbf{W}_Y$ should be block diagonal with the measurement noise covariance matrix $\mathbf{R}$ repeated on the diagonal. However, for the considered case, modeling errors are more important than sensor noise. The chosen weighting assumes the *relative* effect of modeling errors on the forces and moments to be the same and realizes an equal treatment of forces and moments relative to their maximum values.

Fig. 7. Identification algorithm.

and $M_{\max} = 6 \text{ Nm}$ are approximately equal to the maximum force and moment values of the measured output vector $\hat{\mathbf{Y}}$. The 9×9 -dimensional matrix $\mathbf{W}_{\mathbf{x}_u}$ is chosen as a diagonal matrix with the square of $3 \times 10^{-5} \text{ m}$, $(10, 10, 10, 10)^T \times 10^{-5} \text{ m}$ and $((10, 10, 10, 10)^T \times 10^{-5} \text{ m}) / (1.6 \times 10^{-2} \text{ m rad}^{-1})$ on the diagonal, corresponding to *educated guesses* for the standard deviations of $x_{u,\delta}$, $\mathbf{x}_{u,t}$ and $\mathbf{x}_{u,r}$, respectively. The factor $1.6 \times 10^{-2} \text{ m}$ represents half the length of the ACU V-guides and effectively converts the standard deviations on $\mathbf{x}_{u,t}$ to standard deviations on $\mathbf{x}_{u,r}$ in radians. The purpose of the weighting matrices is to normalize both terms of the objective function and to obtain an interpretable magnitude of both error terms⁹. The first term in objective function (21) can be interpreted as the mean square force/moment error *relative* to the maximum force/moment, while the second term can be interpreted as the mean square mismatch between $\mathbf{x}_u$ and $\mathbf{x}_u^d$ relative to the guessed standard deviations of the mismatch.

The identification algorithm is implemented as shown schematically in Figure 7 with the main steps described below. The steps related to the identification of the geometric uncertainties $\mathbf{x}_u$ are shown in dotted lines.

(1) Non-linear optimization step

As discussed in Section 3, optimization problem (21) can be rewritten as an optimization problem with $\mathbf{x}_u$ as optimization variables by replacing $\boldsymbol{\theta}$ by the least-squares solution (11).

The resulting optimization problem is solved using the Matlab “fminsearch” function, which is based on the Nelder–Mead simplex method (Nelder and Mead 1965). Unlike finite differences based methods (Elster and Neumaier 1997), the Nelder–Mead method (Nelder and Mead 1965) is recommended by Nocedal (1992), Fletcher (1987) and Powell (1987) for optimizing noisy functions such as the first term of objective function (21). In addition, this method requires only function evaluations and no knowledge of the derivatives of objective function (21) with respect to the geometric uncertainties $\mathbf{x}_u$.

(2) Calculation of the geometric models

Once the geometric uncertainties have been specified, either as a user-defined guess, usually equal to the design value $\mathbf{x}_u^d$, or *iteratively* by the non-linear optimization step, the geometric models $\mathcal{P}_{o1}$ and $\mathcal{P}_{o2}$ are updated. In this paper, only $\mathcal{P}_{o1}$ depends on the uncertainty variables $\mathbf{x}_{u,t}$ and $\mathbf{x}_{u,r}$, while $\mathcal{P}_{o2}$ does not depend on any of the uncertainties $\mathbf{x}_u$.

(3) Calculation of the object kinematics and forces

In addition, after the geometric uncertainties have been specified, the kinematic quantities $^{o1}_w \mathbf{T}$, $^{o2}_w \mathbf{T}$, $^{o1}_w \mathbf{t}$, $^{o2}_w \mathbf{t}$ required by the “mindist” procedure and the force/moment vectors $^{o2}_{o1} \mathbf{w}$ and $^{o1}_{o2} \mathbf{w}$ for the linear least-squares identification are calculated as follows.

For all tests, the same estimate of $^{o2}_w \mathbf{T}$ is used for the fixed pose of the receptacle in the world frame, while $^{o1}_w \mathbf{T}$ is calculated from the measured end-effector pose $^{ee}_w \mathbf{T}$ and $^{ee} \mathbf{T}(x_{u,\delta})$ given by (20) which is evaluated using the current value of $x_{u,\delta}$, that is

$$^{o1}_w \mathbf{T}(x_{u,\delta}) = ^{ee}_w \mathbf{T} \cdot ^{o1}_{ee} \mathbf{T}(x_{u,\delta}). \quad (22)$$

Furthermore, since $^{o2}_w \mathbf{T}$ is assumed to be constant, $^{o2}_w \mathbf{t} = \mathbf{0}_{6 \times 1}$, while $^{o1}_w \mathbf{t}$ is calculated from the measured end-effector velocity $^{ee}_w \mathbf{t}$ as

$$^{o1}_w \mathbf{t} = \begin{pmatrix} \mathbf{I}_{3 \times 3} & [\mathbf{p}^\times] \\ \mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3} \end{pmatrix} \cdot ^{ee}_w \mathbf{t}, \quad (23)$$

where $[\mathbf{p}]^\times$ is the skew-symmetric matrix of a vector $\mathbf{p}$. Furthermore, $^{o2}_{o1} \mathbf{w}$ and $^{o1}_{o2} \mathbf{w}$ are calculated from the measured force/moment vector $^{o2}_{fp} \mathbf{w}$ as

$$^{o2}_{o1} \mathbf{w} = \begin{pmatrix} ^{fp}_{o1} \mathbf{R} & \mathbf{0}_{3 \times 3} \\ [\mathbf{p}_{o1}]^\times & ^{fp}_{o1} \mathbf{R} \end{pmatrix} \cdot ^{o2}_{fp} \mathbf{w},$$

$$^{o1}_{o2} \mathbf{w} = - \begin{pmatrix} ^{fp}_{o2} \mathbf{R} & \mathbf{0}_{3 \times 3} \\ [\mathbf{p}_{o2}]^\times & ^{fp}_{o2} \mathbf{R} \end{pmatrix} \cdot ^{o2}_{fp} \mathbf{w}, \quad (24)$$

9. Although the choice of the weighting may seem arbitrary, it is not required and not very critical. However, it is usually worthwhile to incorporate as much prior knowledge as possible, no matter how crude it may be.

where ${}^{fp}_{o1}\mathbf{p}$, ${}^{fp}_{o1}\mathbf{R}$, ${}^{fp}_{o2}\mathbf{p}$ and ${}^{fp}_{o2}\mathbf{R}$ are calculated from

$$\begin{aligned} {}^{fp}_{o1}\mathbf{T} &= ({}^{o1}_w\mathbf{T})^{-1} \cdot {}^{fp}_w\mathbf{T}, \\ {}^{fp}_{o2}\mathbf{T} &= ({}^{o2}_w\mathbf{T})^{-1} \cdot {}^{fp}_w\mathbf{T}. \end{aligned} \quad (25)$$

Note that the effect of $x_{u,\delta}$ on ${}^{o1}_w\mathbf{t}$ and ${}^{o2}_w\mathbf{w}$ is very small. Therefore, it is ignored in (23)–(25).

(4) Linear Least-squares Step

After applying the “mindist” routine, explained in Section 3, the stacked regressor matrix $\Phi(\mathbf{x}_u)$ and output vector $\hat{\mathbf{Y}}$ are constructed with $\hat{\mathbf{w}}_k$ and $\phi_k(\mathbf{x}_u)$ from all time-steps, except those where the “mindist” routine yields zero contact points. To reduce the effect of the velocity measurement noise on the tangential unit vectors $\mathbf{e}_{t,i} = -\mathbf{v}_{t,i} / \|\mathbf{v}_{t,i}\|_2$, which increases *tremendously* with decreasing tangential velocity $\mathbf{v}_{t,i}$, $\mathbf{e}_{t,i}$ is replaced by $\mathbf{0}_{3 \times 1}$ in the regressor matrices $\phi_k(\mathbf{x}_u)$, whenever $\|\mathbf{v}_{t,i}\|_2 < v_{t,m}$. This is equivalent to assuming sticking behavior below $v_{t,m}$ and not estimating the coefficient of friction at the sticking contact points. In practice, the threshold velocity $v_{t,m}$ is chosen of the order of magnitude of the velocity resolution. The least-squares estimate $\hat{\boldsymbol{\theta}}$ is then determined from $\Phi(\mathbf{x}_u)$ and $\hat{\mathbf{Y}}$ using (11), from which the estimated stiffness $\hat{K}$ and coefficient of friction $\hat{\mu}$ can be calculated as

$$\hat{K} = \hat{\theta}_1, \quad (26)$$

$$\hat{\mu} = \frac{\hat{\theta}_2}{\hat{\theta}_1}. \quad (27)$$

Subsequently, the value of objective function (21) is calculated at $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}$ and passed back to the non-linear optimization step.

7. Identification Results

For all tests, the identification procedure is applied to the data from the test insertions and carried out by first assuming $\mathbf{x}_u = \mathbf{x}_u^d = \mathbf{0}$ and estimating only the stiffness and friction parameters. Subsequently, the procedure is also carried out *with* the non-linear optimization step to estimate both contact parameters and geometric uncertainties. To facilitate the interpretation of the results, visualizations of the test insertions and the locations of the contact points during the insertions are shown in Extensions 1 and 2.

7.1. Identification of Contact Parameters

The global and instantaneous estimates of the contact parameters for tests 1a and 2a are shown in Figures 8 and 9, respectively, while the global estimates are also summarized in

Fig. 8. Estimated parameters for test 1a without identification of geometric uncertainties.

Fig. 9. Estimated parameters for test 2a without identification of geometric uncertainties.

Table 1 for all tests. As discussed by Weber et al. (2006), except in some special situations, the global identification problem is usually well posed, since $\mathbf{W}_Y^{-1/2}\Phi$ is of full rank. This also applies for the instantaneous identification, unless all contact points are sticking. In that case, only $\hat{\theta}_1$ can be calculated.

The measured and reconstructed forces for tests 1a and 2a are shown in Figures 10 and 11, respectively. The reconstructed forces are calculated by means of equation (9), using the global estimate $\hat{\boldsymbol{\theta}}$. The number of contact points for tests 1a

Table 1. Estimated contact parameters for all tests without identification of geometric uncertainties.

Test	$\hat{K}$ (10^5 N/m)	$\hat{\mu}$ (-)
1a	1.82	0.08
1b	1.59	0.07
2a	1.93	0.22
2b	1.71	0.28

Fig. 10. Measured and reconstructed forces for test 1a without identification of geometric uncertainties.

and 2a are shown in Figures 12 and 13. The figures and results for tests 1b and 2b are very similar to those of tests 1a and 2a and are therefore omitted.

As can be seen in Figure 8, the instantaneous stiffness estimates for test 1a appear to be reasonably constant, as would be expected. The instantaneous estimates of the coefficient of friction are rather noisy, with tangential contact point velocities $\mathbf{v}_{t,i}$ which are on average not higher than 5×10^{-4} m s $^{-1}$, and often close to or below the velocity resolution. Nevertheless, the global estimates $\hat{K}$ and $\hat{\mu}$ are representative and result in a reasonable agreement between the measured and reconstructed forces, shown in Figure 10. After 36 s, the ACU is almost fully inserted and the contact point velocities decrease even further, remaining below the threshold velocity for most

Fig. 11. Measured and reconstructed forces for test 2a without identification of geometric uncertainties. **Fig. 12. The number of contact points for test 1a without identification of geometric uncertainties.**

of the time, apart from some intermittent misdetections due to noise. As a result, in this phase, the instantaneous estimates of the coefficient of friction are no longer meaningful.

As can be seen in the top part of Figure 12, during a large part of the insertion for test 1a, the number of contact points is around four, consisting of contact between the outer upright edges of the ACU V-guides and the faces of the receptacle V-grooves. When the ACU is almost fully inserted, the number of contact points increases to eight, consisting of contact between the faces of the ACU V-guides and the faces of the receptacle V-grooves.

Table 2. The ratio of the standard deviations of the columns of $\mathbf{W}_Y^{-1/2} (\partial \Phi(\mathbf{x}_u) / \partial x_{u,j})|_{\mathbf{x}_u=\mathbf{0}}$ with respect to the standard deviations of the columns of $\mathbf{W}_Y^{-1/2} \Phi(\mathbf{0})$ for $j = 1, \dots, M$ for tests 1a and 2a.

Test	column	$x_{u,\delta}$ (10^4 m $^{-1}$)	$\mathbf{x}_{u,t}$ (10^3 m $^{-1}$)	$\mathbf{x}_{u,r}$ (10^1 rad $^{-1}$)
1a	1	1.08	(4.35, 3.82, 3.59, 3.92) ^T	(10.53, 8.52, 7.40, 9.06) ^T
1a	2	1.09	(3.58, 3.49, 3.40, 3.71) ^T	(11.14, 8.86, 8.16, 9.32) ^T
2a	1	1.10	(3.66, 3.70, 5.08, 5.42) ^T	(6.23, 9.63, 9.78, 10.82) ^T
2a	2	1.15	(2.27, 2.95, 4.13, 4.55) ^T	(5.81, 11.06, 8.55, 9.98) ^T

$$\mathbf{P}_2 = \begin{pmatrix} \text{grid pattern} \end{pmatrix}, \quad (28)$$

where $\blacksquare$, $\bullet$ and $\cdot$ denote a correlation coefficient with an absolute value in $[1, 0.85]$, $[0.85, 0.5]$ and $[0.5, 0]$ respectively. For test 2a, $\mathbf{P}_1$ and $\mathbf{P}_2$ are given schematically by

[illegible]

The results for tests 1b and 2b are similar to those for tests 1a and 2a and are therefore omitted. It can be seen that for test 1a, there is a strong correlation between different $\mathbf{x}_u$ for both columns. The correlation coefficients corresponding to $\mathbf{W}_Y^{-1/2}\Phi(\mathbf{0})$ and the derivatives related to $x_{u,\delta}$ are especially high. The main reason for this strong correlation is the fact that the contact force normal to the force plate is nearly constant during the test insertion, while $x_{u,\delta}$ is accounting for uncertainty normal to the force plate. Only by varying the contact force normal to the force plate does it become possible to identify both $\boldsymbol{\theta}$ and $x_{u,\delta}$ more accurately. Furthermore, the correlation coefficients corresponding to the derivatives related to the *same components* of $\mathbf{x}_{u,t}$ and $\mathbf{x}_{u,r}$ are especially high. This strong correlation is caused by the fact that variation of the variables corresponding to the same components of $\mathbf{x}_{u,t}$ and $\mathbf{x}_{u,r}$ has a similar effect on the regressor matrix $\mathbf{W}_Y^{-1/2}\Phi(\mathbf{0})$. In addition, there is also significant mutual correlation between other derivatives related to $x_{u,\delta}$, $\mathbf{x}_{u,t}$ and $\mathbf{x}_{u,r}$, and with $\mathbf{W}_Y^{-1/2}\Phi(\mathbf{0})$.

For test 2a, the same very high correlations are also observed, while the mutual correlation between other derivatives related to $x_{u,\delta}$, $\mathbf{x}_{u,t}$ and $\mathbf{x}_{u,r}$, and with $\mathbf{W}_Y^{-1/2}\Phi(\mathbf{0})$ in test 2a is

less pronounced than for test 1a. A major reason for this difference, is that the contact points in test 1a are located *almost symmetrically*, such that the effect of variations of $\mathbf{x}_{u,t}$ and $\mathbf{x}_{u,r}$ tends to cancel out in $\mathbf{W}_{\mathbf{Y}}^{-1/2} \Phi(\mathbf{x}_u)$. This is not the case for test 2a, where the insertion phase features only two contact points.

7.2.2. L-curve analysis

From the covariance matrices $\mathbf{P}_1$ and $\mathbf{P}_2$, it can be seen that the identification of geometric uncertainties is ill-conditioned, especially for tests 1a and 1b. Therefore, a regularized approach with a non-zero γ in objective function (21) is adopted. The optimal $\gamma = \gamma_{\text{opt}}$ is determined using L-curve analysis (Hansen 1998). Namely, objective function (21) is minimized for different values of γ such that for each value of γ , a different solution $\hat{\mathbf{x}}_u$ is obtained. The magnitude of $(1/6L) \left\| \hat{\mathbf{Y}} - \Phi(\hat{\mathbf{x}}_u)^T (\Phi(\hat{\mathbf{x}}_u)^T \Phi(\hat{\mathbf{x}}_u))^{-1} \Phi(\hat{\mathbf{x}}_u) \hat{\mathbf{Y}} \right\|_{\mathbf{W}_Y}^2$, hereafter called the “force cost”, and $(1/M) \left\| \hat{\mathbf{x}}_u - \mathbf{x}_u^d \right\|_{\mathbf{W}_{x_u}}^2$, hereafter called the “geometric cost”, is then evaluated for each solution $\hat{\mathbf{x}}_u$ and the evolution of both terms with respect to each other is compared graphically in a so-called “L-curve” (Hansen 1998). In our case, γ is varied from 10^{-4} , corresponding to almost no regularization, to 10^3 , corresponding almost to identification of contact dynamics without the consideration of geometric uncertainties.

The L-curves for tests 1a and 2a are shown in Figure 14. When γ is large, the mean relative force error, which is the square root of the force cost is around 18% and 24% for tests 1a and 2a, respectively. For low γ , the mean relative force error is reduced to around 10% for both tests. However, as γ tends to zero, corresponding to no regularization, the geometric cost becomes exceedingly large, while the associated force cost does not decrease significantly beyond a certain γ . This behavior is known as “overfitting” and it is caused by the combination of the ill-conditioning of the geometric identification problem and the presence of modeling errors. By choosing a γ beyond which the force cost no longer decreases significantly, but which still keeps the geometric cost relatively small, the degree of overfitting can be kept in check.

The optimal value γ is chosen to be $\gamma_{\text{opt}} = 0.0203$ and is taken the same for all tests. The corresponding optimal points

Table 3. Estimated contact parameters and geometric uncertainties for all tests.

Test	$\hat{K}$ (10^5 N m $^{-1}$)	$\hat{\mu}$ (—)	$\hat{x}_{u,\delta}$ (10^{-4} m), $\hat{\mathbf{x}}_{u,t}$ (10^{-4} m) and $\hat{\mathbf{x}}_{u,r}$ (10^{-4} rad)
1a	1.55	0.08	$\hat{x}_{u,\delta} = 0.087$ $\hat{\mathbf{x}}_{u,t} = (-0.82, 0.28, 0.36, 0.43)^T$ $\hat{\mathbf{x}}_{u,r} = (3.69, -3.41, 1.43, -0.15)^T$
1b	1.48	0.07	$\hat{x}_{u,\delta} = 0.0894$ $\hat{\mathbf{x}}_{u,t} = (-0.76, -0.13, 0.22, 0.56)^T$ $\hat{\mathbf{x}}_{u,r} = (2.83, -0.09, 1.72, -1.86)^T$
2a	1.81	0.19	$\hat{x}_{u,\delta} = -0.0160$ $\hat{\mathbf{x}}_{u,t} = (1.04, -0.40, 0.81, -0.88)^T$ $\hat{\mathbf{x}}_{u,r} = (-2.29, -1.95, 0.34, 4.21)^T$
2b	1.65	0.22	$\hat{x}_{u,\delta} = 0.0112$ $\hat{\mathbf{x}}_{u,t} = (0.37, 0.07, 1.01, -0.90)^T$ $\hat{\mathbf{x}}_{u,r} = (-3.76, -2.03, 0.72, 22.38)^T$

Fig. 14. The L-curves for tests 1a and 2a.

are indicated on the L-curves in Figure 14 for tests 1a and 2a. For all tests, γ_{opt} realizes a mean relative force error of about $\sqrt{0.015} \approx 12\%$.

7.2.3. Regularized Results

The global and instantaneous estimates of the contact parameters for tests 1a and 2a with identification of geometric uncertainties and with regularization with γ_{opt} are shown in Figures 15 and 16, while the global estimates are also summarized in Table 3 for all tests. For test 1a, the progression of the instantaneous estimates of the stiffness and the coefficient of friction with time, shown in Figure 15 looks similar to that in Figure 8. Conversely for test 2a, the instantaneous estimates of the stiffness and the coefficient of friction shown in Figure 16 are much more constant than in Figure 9.

Fig. 15. Estimated parameters for test 1a with identification of geometric uncertainties.

For all tests, the stiffness estimates are somewhat lower than without the identification of geometric uncertainties. This can be explained by the fact that $x_{u,\delta}$ is essentially unobservable, as can be seen in the covariance matrices (28) and (29), while the other components of $\mathbf{x}_u$ are weakly observable. In other words, there is no value of $x_{u,\delta}$ which causes a significant decrease in objective function (21), while there are values of $\mathbf{x}_{u,t}$ and $\mathbf{x}_{u,r}$ which do decrease the objective function (21). Adjustment of $\mathbf{x}_{u,t}$ or $\mathbf{x}_{u,r}$ tends to increase the interference distances, while the regularization keeps $x_{u,\delta}$ close to zero. Therefore, the estimated stiffness will tend to decrease. The regularization keeps

Fig. 16. Estimated parameters for test 2a with identification of geometric uncertainties.

this overfitting behavior within acceptable bounds, while still allowing some freedom to reduce objective function (21).

The measured and reconstructed forces for tests 1a and 2a are shown in Figures 17 and 18. The reconstructed forces for test 1a look relatively similar to those without the identification of geometric uncertainties shown in Figure 10. Indeed, owing to the ill-conditioning of the identification of geometric uncertainties for tests 1a and 1b, the effect of these uncertainties is literally *unobservable*, in other words, it cannot be seen in the measurements. Conversely, for test 2a, the reconstructed forces in Figure 18 match a lot better with the measured forces, compared with those without the identification of geometric uncertainties shown in Figure 11.

The estimated geometric uncertainties $\hat{\mathbf{x}}_u$ are summarized in the right part of Table 3 for all tests, while their effect, magnified by a factor of 100, is shown in Figures 19 and 20 for tests 2a and 2b, respectively. It can be seen that the identified uncertainties are not the same. Owing to the ill-conditioning of the identification of geometric uncertainties, especially for tests 1a and 1b, this is not unexpected. Nevertheless, there is some qualitative agreement between the identified uncertainties for tests 2a and 2b (Figures 19 and 20). Namely, the transverse shifts of the V-guides are in the same directions for both tests. The bottom left V-guide in Figure 19 is mostly translated for test 2a, while the same V-guide appears to be both translated and rotated for test 2b. This reflects the fact that the two degrees of freedom for this V-guide are observable to some extent, although their effect cannot be separated very accurately.

Fig. 17. Measured and reconstructed forces for test 1a with identification of geometric uncertainties.

7.3. Discussion

The results with the identification of the geometric uncertainties, especially for tests 2a and 2b, demonstrate improved agreement between measured and reconstructed forces, as well as more consistent instantaneous and global estimates of the stiffness and the coefficient of friction.

Namely, *without* the identification of the geometric uncertainties, it is *merely coincidental* that the global stiffness estimates for tests 2a and 2b are close to those of tests 1a and 1b. As can be seen in Figure 9, the longer the two contact point phase of the insertion would take, the closer the global estimate $\hat{K}$ would be to $4.0 \times 10^5 \text{ N m}^{-1}$.

Conversely, *with* the identification of geometric uncertainties there is a reasonable agreement between both global and instantaneous stiffness estimates for all tests and also with both global and instantaneous stiffness estimates of tests 1a and 1b *without* the identification of geometric uncertainties. This provides a good indication that the true contact stiffness is close to $1.8 \times 10^5 \text{ N m}^{-1}$.

The differences in instantaneous and global estimates of the coefficient of friction between tests 1a and 1b, and tests 2a and 2b, seem to be mainly related to the different insertion procedure used, with lower contact point velocities for tests 1a

Fig. 18. Measured and reconstructed forces for test 2a with identification of geometric uncertainties.

Fig. 19. Nominal and estimated ACU V-guide positions for test 2a, exaggerated by a factor of 100.

and 1b, and not so much to whether or not identification of geometric uncertainties is considered. However, for tests 2a and 2b with the identification of geometric uncertainties, instantaneous estimates and global estimates of the coefficient of friction are also more consistent. For tests 2a and 2b, the instantaneous estimates are less noisy, both without and with the identification of geometric uncertainties, thus providing a good indication that the true coefficient of friction is approxi-

Fig. 20. Nominal and estimated ACU V-guide positions for test 2b, exaggerated by a factor of 100.

mately 0.2, at least for velocities well above the velocity resolution.

Despite the ill-conditioning of the geometric identification, the high stiffness of the contacting objects, and the linear contact dynamics and Coulomb friction model approximation, the approach with the identification of geometric uncertainties performs surprisingly well in terms of force reconstruction and consistency of the contact dynamics estimates.

By careful experimental design (Lefebvre et al. 2005), the observability of geometric uncertainties can be improved. However it should be noted that in many cases, and especially for insertion tasks, the freedom for experiment design may be quite limited and, more importantly, specially designed experiments would not represent “typical robotic operations”, which is the main focus of this paper. In fact, we are only interested in accurate identification of geometric uncertainties insofar as it *improves* the identification of contact parameters. The experimental results seem to indicate that it is beneficial to *recognize* that there is uncertainty, but do not demonstrate that *accurate* identification of geometric uncertainties is really necessary.

Nevertheless, we favor a cautious interpretation and refrain from general claims concerning the accuracy of the contact parameter estimates with identification of geometric uncertainties. Rather, our extension with identification of geometric uncertainties serves as a useful tool to investigate the effect of geometric uncertainties on contact parameter estimation. The actual quality of the contact parameter estimates should be judged by a careful comparison of the results *without and with* identification of geometric uncertainties.

8. Conclusions and Future Work

In this paper we have investigated the identification of contact parameters from multi-point contact robotic operations. The method developed by Weber et al. (2002, 2006) has been extended to deal with small geometric uncertainties. The extension is based on a separable least-squares formulation, which

is solved using a variable projection method, and allows us to simultaneously identify contact dynamics parameters and geometric uncertainties. The conditions for observability of geometric uncertainties have been derived, which allow us to assess the conditioning of the identification of the geometric uncertainties, and a regularized formulation has been proposed to deal with ill-conditioning.

The method with and without identification of geometric uncertainties has been applied to experimental data gathered from the on-Earth verification of a space insertion task using the STVF manipulator at the Canadian Space Agency. The use of practical experimental data allows us to identify a number of important modeling and implementation aspects which are only encountered in experimental investigations, and a simple and decoupled implementation of the identification of geometric uncertainties has been presented.

The results without identification of geometric uncertainties illustrate the applicability of the original method (Weber et al. 2006) and also predict the ballpark values for the contact stiffness and the coefficient of sliding friction. However, there are a number of discrepancies in the global and instantaneous contact parameters estimates between different tests, which are likely to be caused by the presence of geometric uncertainties. The results with identification of geometric uncertainties, whereby regularization is used to deal with ill-conditioning, support this theory and exhibit an improved agreement between measured and reconstructed forces, and an improved consistency between global and instantaneous estimates for different tests. Considered together, the two sets of results provide a fairly narrow range for the possible values of the contact parameters. Further research consists of investigating the benefit of the method with identification of geometric uncertainties for identification of contact parameters for other experimental test cases.

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Appendix: Index to Multimedia Extensions

The multimedia extension page is found at <http://www.ijrr.org>

Table of Multimedia Extensions

Extension	Type	Description
1	Video	Visualization of test insertion 1a.
2	Video	Visualization of test insertion 2a.

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