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A THEORY OF STRUCTURAL MODEL VALIDITY IN SIMULATION

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Abstract

During the last decennia, the practice of simulation has become increasingly popular among many system analysts, model builders and general scientists for the purpose of studying complex systems that surpass the operability of analytical solution techniques. As a consequence of the pragmatic orientation of simulation, a vital stage for a successful application is the issue of validating a constructed simulation model. Employing the model as an effective instrument for assessing the benefit of structural changes or for predicting future observations makes validation an essential part of any productive simulation study. The diversity of the employment field of simulation however brings about that there exists an irrefutable level of ambiguity concerning the principal subject of this validation process. Further, the literature has come up with a plethora of ad hoc validation techniques that have mostly been inherited from standard statistical analysis. It lies within the aim of this paper to reflect on the issue of validation in simulation and to present the reader with a topological parallelism of the classical philosophical polarity of objectivism versus relativism. First, we will position validation in relation to verification and accreditation and elaborate on the prime actors in validation, i.e. a conceptual model, a formal model and behaviour. Next, we will formally derive a topological interpretation of structural validation for both objectivists and relativists. As will be seen, recent advances in the domain of fuzzy topology allow for a valuable metaphor of a relativistic attitude towards modelling and structural validation. Finally, we will discuss several general types of modelling errors that may occur and examine their repercussion on the natural topological spaces of objectivists and relativists. We end this paper with a formal, topological oriented definition of structural model validity for both objectivists and relativists. The paper is concluded with summarising the most important findings and giving a direction for future research.

1. Introduction

Evaluating the truthfulness and the amount of realism of a simulation model soon turns out to be one of the most cumbersome tasks in the life cycle of a simulation study. Building reliable and credible simulation models is however a vital prerequisite for a successful application of simulation to any kind of managerial problem situation. Due to the complexity of reality and the accompanying difficulties that arise when modelling real systems, building accurate simulation models is rather a matter of art [Tocher 1963] instead of applying an outlined, straightforward set of methodological rules. In addition, the pragmatic orientation of simulation brings about that a majority of work on validation that has been published is based on case studies and personal experiences in simulating grand, complicated real world systems [for a list of recent case studies, see Kleijnen 1995]. Also, since simulation is definitely one of the most interdisciplinary methodologies, the collection of publications on validity of real life simulation models is spread out over a spectrum of fields, ranging from military simulations, applications in production facilities, power plant simulation, simulation in chemistry, biology and physics, aircraft simulation, analysis of high speed communication networks, studies of economic systems, artificial intelligence simulation, general system dynamics and so on. The diversity of simulation applications has made that a solid, regulated and objective theory on model validation is still lacking. Also, since simulation brings together a group of people with distinct backgrounds and differing goals of applying simulation, inevitably, different interpretations of the concept of validation and the process validation should actually be comprised of exist. As a consequence, the level of divergence in validity interpretations makes that a debate on validation is echoed along the foundations of many epistemological schools [Kleindorfer 1998, Déry 1993, Barlas 1990]. Although comprehensive, theoretical reflections on the issue of model validity in simulation are certainly not absent in the literature [Balci 1998a, Kelton 1991, Banks 1984, Zeigler 1976], the model builder will feel he is rather overwhelmed by a multitude of ad hoc validation methods, often greatly inspired on classic statistical theory [Kleijnen 1998, Martens 1998, Barlas 1989, Balci 1984, Fishman 1967]. In addition, little of those practical validation techniques empower him with a formal instrument to express a *degree* of validity or an *amount* of model accuracy instead of the usual *binary* like validity assessment. As a matter of fact, many of the published statistical validation techniques approach the phenomenon of validation by means of a test in a rather pass/fail fashion. Models that fail to pass the test are labelled as invalid, models that pass are accepted[1] but there is no indication at all of the proportion of their (in)validity.[2] In search of a way out of this dilemma, we feel it would be appropriate, not to approach the issue of model validity from a binary, statistical point of view, but to come up with a formalism that allows to express the accuracy of a model on a scale of model validity. Since in practice, model validity is in fact highly context dependent, the objective uniqueness of such a scale is not guaranteed. In that respect, it might be worthwhile to investigate how the relative aspect of model validity could be blended with a technique that allows for a vague, imprecise, fuzzy and relative statement on model validity. It lies within the aim of this paper to reflect on this confrontation between an objective, binary and a relative, fuzzy approach to the issue of validation. We will demonstrate that the binary-fuzzy polarity resembles the classical philosophical duality of foundationalism versus anti-foundationalism. Further, irrespective of ones philosophical beliefs, we will formulate the act of validation as a measurement activity following a topological and platonic view of reality. As will be seen, insights of both classical and fuzzy topology allow for a useful mathematical counterpart of epistemological groundwork. In that respect, model validity becomes a distance that is estimated using a natural metric, representing ones proper philosophical beliefs.

The paper is organised as follows. In an initial section, we will discuss different notions of model validity and relate them to a general philosophy of science. Further, we will elaborate on the classical philosophical duality of objectivism versus relativism in the context of modelling and validation. Also, we will introduce the principal players in the act of validation, i.e. the conceptual or structural model, the formal model and the model's behaviour. The following paragraph then elaborates on a topological approach to structural model validation. Through a sequence of postulates, we will introduce a general topological interpretation of conceptual modelling and approach the issue of validation from a metric, distance-oriented avenue. After a derivation of the conceptual topologies that represent both a foundational and an anti-foundational attitude towards structural validation, we will address ourselves to a study of general modelling errors and an analysis of their repercussion on both conceptual model and universe. Finally, we end our monograph by a formal definition of structural model validity within the context of our metric, topological approach to validation. The paper is concluded by summarising

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[1] Notice that models that pass may be accepted but their validity can never be proven.

[2] As a result, we feel there is a highly uncomfortable level of ambiguity surrounding the application of statistical tests for validating a model. On the one hand, one accepts in general modelling theory that any model will contain at least some errors while on the other hand it is just the absence of errors that is formulated as the hypothesis to be verified. Consequently, a verdict on model validity using a statistical instrument becomes a matter of sample size. If only enough samples are found, the difference between reality and model, no matter its magnitude, will be brought to light. We feel that overall this is a dangerous attitude towards validation since, for the particular test applied, the sensitivity of invalidity detection towards sample size is usually unknown in advance.

the most important research findings and giving a direction for a collection of future research activities.

2. Validation, its kinds and their relation to the philosophy of science

Reflecting on the issue of validation, an early dilemma to overcome yet is no doubt an elucidation of what the term *validation* actually stands for. Put differently, what is the meaning of a phrase that goes like *"This particular model is a valid model of that phenomenon in reality"*? Searching for a passage out of this predicament, it seems inescapable to have a look at the major terms that comprise this phrase, i.e. a model, a phenomenon in reality and the issue of being valid. Although only a complete understanding of reality can give rise to an impeccable theory of validity, it would be far to ambitious in view of the continual epistemological debate that is held among modern scientists to work out an entire philosophy of existence. However, in order to justify any theory of model validity that is proposed, it is vital to clear up any confusion surrounding the partaking subjects in the act of validation. In the next paragraph, we will discuss therefore the classical philosophical duality of objectivism versus relativism that has become almost a standard topic in any debate on validation. Next, we will arrive at different kinds of validation and discuss their relation to nearby concepts as there are verification and accreditation. Inspired by the philosophical reflections on validation, we will end this paragraph with an elaboration on different kinds of models and discuss their role in both verification and validation.

2.1. A polarity of validation roots

Throughout the literature on simulation, it is found that many researchers exhibit a level of dissimilarity concerning the term *validation* and hence of the focus and the essential objectives of the associated validation process. Not only there exists a level of ambiguity on what the process of validation should actually focus on, some researchers utter also some amount of confusion with closely related concepts as there are *verification* and *accreditation*. It will come as no surprise that a reflection on validation of simulation models is closely related to the theory of verifying the veracity of an hypothesis, a statement or a general conjecture. As a matter of fact, depending on ones philosophical beliefs, some will approach the matter of validation on a pure rational (rationalism) or empirical (empiricism) basis, while others may interpret validation in general terms of model usefulness (instrumentalism), the ability to survive (falsificationism) or its social acceptance (Kuhnianism). In accordance with recent publications on validation [Kleindorfer 1998, Barlas 1996], we will adhere to the notion of a philosophical polarity ranging from an attitude of pure objectivism or foundationalism to a position of strong anti-foundationalism or relativism. In an *objectivistic* approach, models are either true or false. For a rationalist, the truthfulness or falsehood of such a model is revealed through a process of logical deductions, unsuitable for empirical verification. For an empiricist, no assumptions or postulates are accepted that cannot be fully verified using a solid, empirical scientific procedure [see also Naylor 1967]. Or, as is stated by Landry et al. [Landry 1983], where a rationalist concentrates on syntactical aspects and logic of a model, an empiricist will focus more on semantical aspects and facts. At the heart of this foundationalist approach, that embraces both rationalists and empiricists, lies the recognition of a basis or foundation in which the validity of any model is brought to light. In a way, rationalists share the conviction that the intellect or "ratio" is the ultimate source for knowledge in their quest to this foundation. Empiricists on the other hand claim it is by observations of facts and the studying of real phenomena that the basis of truth can be revealed. In any case, in this objective context, validation of a simulation model is defined as the process during which the truthfulness and the accuracy of a model are confirmed in regard to the foundation. Notice that the final outcome of such an objective validity study is always binary: there is no "truth" at the partial confirmation of a theory. Models are either fully correct or fully wrong; no state of partial validity exists. Where rationalists and empiricists will differ in their conviction of the archetypal search method in their global quest of the foundation, they will agree on the objective outcome of a verdict on the truth-value of a theory in regard to the foundation. It requires little argument to recognise that this binary and objective approach to model validation can count on only little adherence in practical situations. As a result of an impracticable sphere surrounding a binary verdict on model validity, many researchers will follow a *relativistic* pathway in assessing a model's validity. At the heart of this anti-foundationalism lies the agreement that models may be more or less valid representations of some phenomenon in reality, partially depending on the particular world view of the model stakeholders and the goal of the modelling study itself. Since everyone has its own worldview and image of reality, there is no consensus on the identity (and even the existence) of an objective foundation. Further, the search for this foundation is in fact a needless contemplation of epistemè since validity of a theory is decided upon using a relative basis that represents the idiosyncrasy of the particular validation circumstances. The concept of absolute validity that symbolises a foundational attitude is henceforth translated to a matter of practical value, model acceptance and model robustness. For an instrumentalist, and a positive economist in particular [Friedman 1953], validity equals then the ability to predict future or historical observations using the model. The model is employed as a forecasting instrument of some behaviour in reality that is of particular interest. The model's capability of reproducing this behaviour or extrapolating it towards the future makes up its usefulness and defines its validity. On the other hand, addressing the issue of validation through the notion of model acceptance brings us in contact with a model's credibility and relates validation to accreditation (establishing credibility amongst a model's stakeholders). In light of Kuhn's theory of evolving paradigms [Kuhn 1970], validation is a process with both social and historical dimensions where a model's validity is judged upon based on an explicit set of methodological rules that are considered being the projection of a paradigm in evolution on a geometry of acceptance. Interpreting validation as accreditation brings validation from a pure scientific occupancy to a matter of managerial interest. A model is valid if its stakeholders have accepted it. Finally, approaching the matter of validity in terms of model robustness associates validation with the ability to survive a collection of tests, intended to falsify a conjecture or a theory [Popper 1959]. In summary, where rationalism and empiricism form the corner stones of the foundationalist's approach to validation, instrumentalism, falsificationism and Kuhnianism constitute the pillars of an anti-foundationalist's view. It comes as no surprise that this polarity of objectivism versus relativism has an enormous impact on the interpretation of validation and an assessment of model validity in general.

2.2. Structural validity, behavioural validity and their interrelation

Passing the paragraph on a polarity of validation roots, it has become clear that ones philosophical convictions play a central role in ones attitude towards validation. In search of a definition of validation in the context of simulation, let us have a look at a recent, widely accepted book on simulation theory [Kelton 1991]. According to Kelton et al., validation is defined as being primarily related to the conceptual model underlying a simulation study. More specific, validation is the process of verifying the truthfulness of the conceptual model. This conceptual model, according to the book, is further described as the collection of assumptions that are made during the simulation study. Notice that, unfortunately, the concept of a *conceptual model* is not unanimously defined in the literature. In light of this paper, we will follow the interpretation of a conceptual model as being a mental image of reality [Landry 1983] and state that a conceptual model embraces both objects and their description as well as causal relationships among these objects. It is believed that in major parts of the simulation literature [Banks 1998, Oral 1993], this description of a conceptual model is the most widely accepted and commonly used. In that respect, a conceptual model remains an abstract, intangible reflection of reality on a mental geometry that is articulated through usage of an object-relation paradigm.

Definition 1: *(A conceptual model)*
A conceptual model of a phenomenon in reality is an abstract, mental reflection of the participating objects, their description and the driving causal interrelationships.

Notice that the definition of validation, being concerned about the conceptual model, does not exclude validation from being active at another level of interest. Depending on the particular modelling scenario, it may very well not be the conceptual model one is interested in, but the behaviour that can be noticed when observing the model in action. As a result, validation shifts from the domain of conceptual models to the universe of conceptual behaviour, observations or stochastic processes. In accordance with the area of interest, the literature has clearly distinguished between conceptual validity on one hand and operational validity on the other hand [see for example Balci 1998b]. In parallel to the terminology of conceptual and operational validity, one often applies the terms structural and behavioural validity. Structural validity is generally defined as the amount of accuracy of the structure of a model, i.e. the correctness of the network of entities and their causal relations. Behavioural validity is concerned about the agreement of real and simulated (stochastic) processes of interest. Although a level of discussion may arise concerning the parallelism of these terms, we will however further consider conceptual and operational validity as synonyms for respectively structural and behavioural validity.

As a natural consequence of the distinction between structure and behaviour, one of the early questions that poses it is whether structural validity automatically induces behavioural validity. In other words, does the validity of a model from a conceptual point of view automatically imply its validity in every operational context? Looking ahead on a formal reflection on conceptual models that is about to come, the answer to this question turns out be *yes*. Structural conceptual validity undoubtingly enforces a validity of behaviour. The reason for this being the conceptual model the ultimate source for any behaviour that might be observed in reality. Since no behaviour may be observed that is not accountable for in the conceptual model, structural accuracy implies behavioural accuracy. But what about the inverse proposition? Does behavioural validity imply structural validity? Let us first define the working object of behavioural validation, i.e. behaviour as follows:

<u>*Definition 2:*</u> *(Behaviour)*
Behaviour of a conceptual model is the collection of all possible observations that can be made, either directly or indirectly from observing the model.

Notice that behaviour consists of *all* immediate and combined observations[3]. To what extent is a model capable of reproducing behaviour as it may be observed in reality? At this moment, it is imperative to make a distinction between a deterministic and a stochastic model. For a deterministic approach to reality, there is only one possible behaviour. Reproducing behaviour equals in that case the creation of a perfect replica of the behaviour as it occurred in reality. As expected, a stochastic approach induces a collection of behaviours equipped with some probability function. For a deterministic model, operational validity implies an exact imitation of behaviour in reality.[4] Where deterministic behaviour validity could be rephrased as a point-by-point validity, this is certainly not true for stochastic validity. For a stochastic model, it matters that the model's behaviour obeys the same probability law as its true equivalent. Returning to the starting point of our discussion, whether behaviour validity implies structural validity, the answer to this question is *no* for both deterministic and stochastic models. To see why, let us have a look at the following example.

<u>*Example 1*</u>*: Structural versus behavioural validity*

Consider the following fictive portrayal of a phenomenon in reality. The phenomenon consists of a machine (M) and a single queue (Q). The machine operates at a constant speed of one entity per minute. Entities arrive at the machine at a constant interarrival time of half a minute. At the very moment that the queue contains more than five elements, the service rate of the machine is quadrupled until the queue is completely empty[5]. A causal relationship exists thus between queue length and machine speed. A collection of interesting behaviour from this system might be the speed of the machine (M_ω), the length of the queue (Q_λ), the sequence of waiting times and service times as entities leave ($E_\omega^{(1)}, E_\omega^{(2)}, \ldots$ and $E_\sigma^{(1)}, E_\sigma^{(2)}, \ldots$), the sequence of interarrival times ($Q_\iota^{(1)}, Q_\iota^{(2)}, \ldots$), etc.... Now, suppose that a system analyst is asked to study the phenomenon and to come up with a conceptual model. Let us suppose that after some time he comes up with a model that is in no way different from the true model of the phenomenon, except in one: based on the deterministic behaviour of the phenomenon, he has concluded that the machine is programmed to increase its service rate during some repetitive amount of time. In essence, in accordance with his

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[3] In a queuing environment, an immediate observation might be the sequence of waiting times of customers as they leave the system. Combined observations are created applying some mathematical formula to immediate observations. Averages, variances, distributions, ratios, etc... are all examples of combined observations.

[4] Notice that the technique of trace-driven simulation is a typical example of removing the stochastic element in a model in order to approach the validation problem from a deterministic point of view.

[5] At that moment, it is assumed the service rate resumes its normal value.

perceptions, the causal influence from queue length to service rate has shifted towards time. So conceptually (or structurally) his model is wrong. But what about the model's behaviour? Careful examination of the deterministic identity of the phenomenon reveals both conceptual models induce exactly the same behaviour. From a behavioural point of view, they are indistinguishable. Hence, for deterministic models behaviour validity does not necessarily imply structural validity.

Let us now introduce a level of non-determinism in the model. Suppose the arrivals follow a prespecified probability distribution (say exponential with mean half a minute). After making the necessary changes to the system, the analyst is once again invited to observe the phenomenon and to construct a conceptual model. After a thorough investigation, he claims we altered the deterministic interarrival process and changed it to exponential. In asking whether the relation of time on service rate is still valid, he comes up with the statement that the quadruplement of the machine's speed is now also exponentially distributed. Once again, conceptually, his perceptions are wrong and contain a level non-realism. If one were now able of making several realisations of both the analyst's model and the phenomenon in reality, then the stochastic features of the proposed model would in no way be distinguishable from reality. Hence, stochastic behaviour validity does not automatically invoke conceptual validity.

2.3. Validation versus verification

So far we agreed that, from a theoretical perspective, the conceptual model or its behaviour represents the primary subject of validation. But what about a practical validation scenario? Is it still the abstract, conceptual model and its behaviour that constitutes the subject of validation? In an attempt to answer this question, we get to the issue of verification. Verification is generally concerned with the amount of distortion that occurred when translating a conceptual model towards a formal representation. We use the term *formal model* for any concrete representation of a conceptual state of mind in a language of choice [Oral 1993]. In that respect, the formal model can be a set of differential equations for the natural physicist, an object oriented system specification for the computer scientist or a collection of macro-economic equations for the economist.

Definition 3: *(A formal model)*
A formal model of a phenomenon in reality is the concrete projection of a corresponding conceptual model on a language of choice.

Hence, one could claim that where validation addresses the issue of building the right model, verification focuses on building the model right [Balci 1998b]. Notice that, besides the terms formal and conceptual model, one uses sometimes the terminology of a computerised model. A *computerised* (or *computer*) *model* is in fact nothing else than the concrete implementation of a formal model on a digital computer. Taking a look again at the book of Kelton et al., *verification* is the process of determining whether a simulation computer program performs as intended. Verification is thus concerned about both the translation from a conceptual model into a formal representation (formal model verification) as well as the implementation of a formal model on a computer (computer model verification). Where validation is inspired by a philosophy of science, verification is instructed by the principles of the modelling framework applied. A collection of guidelines, programming principles, testing and debugging techniques, development recommendations, etc. make up a standard guide of computer model verification [Balci 1998a]. Computer model verification is in that way a matter of software testing and related to good programming practice and software engineering. Verification of the formal model is a matter of finding out whether in the articulation of a conceptual model, there has occurred a level of erroneous projection of conceptual beliefs on the chosen formalism.

In discussing the relation between verification and validation, it is often emphasised [Balci 1998a] that both processes should be executed along the continuum of a simulation study. Not only should they be jointly active in the development phase of a model, they should also be carried out during the entire time span the model is used in practice. This principle of carrying out V&V (verification and validation) along the life cycle of a model, albeit it is undoubtedly valuable in practice [Sargent 1998], is however unattainable in a formal reflection on validation. As a matter of fact, following the above definitions of validation and verification, it seems imperative to regard the conceptual model or its behaviour as the official working object of validation and to use the formal model or the computer model as the object of interest for verification. However, the inherent, intangible identity of the conceptual model makes it very unsuitable for any validation study in practice. As a matter of fact, any validation study will be working with the formal or the computer model and the behaviour that is generated by these models. This sentiment of the formal model being the practical subject of validation makes it highly unwanted of carrying out verification and validation in an intertwined fashion. Therefore, in the remainder of this writing, we will assume a model has been completely verified before it undergoes a verdict on validity. In other words, we will assume that the formal model is a faultless projection of ones conceptual convictions. In that respect, an assessment about structural (conceptual) and operational (behavioural) model validity can be carried out using the formal model without a confrontation with any level of an imperfect projection of ones conceptual convictions.

2.4. A summary

In an attempt to compose a visual overview of the terms that were introduced in this paragraph, Figure 1 gives a structural representation of all introduced concepts.

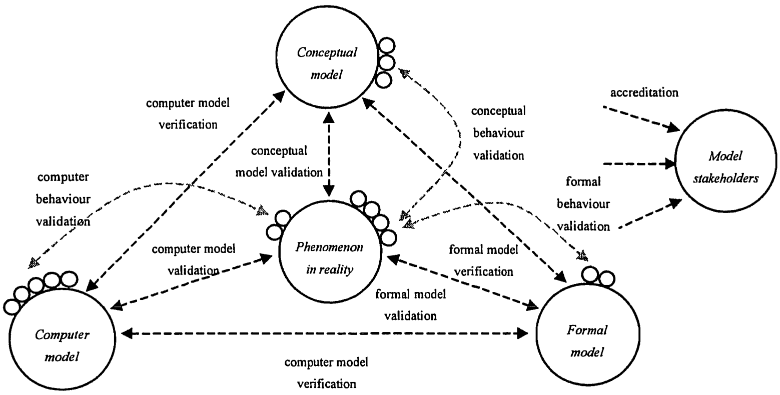

Figure 1: Validation, verification and accreditation in relation to different kinds of models

As can be read from the figure, validation involves a comparison with reality of either the model or its behaviour on a conceptual, formal or computerised level. Although from a pure scientific point of view, validation is defined on the conceptual level, it was yet briefly mentioned that for practical purposes it is the formal or the computerised model that makes up the subject of validation. Whether one investigates then a model's structure or its behaviour is in fact a matter of the goal of the simulation study. Naturally, models that ought to be used as forecasting tools need to undergo a profound behaviour validation study in advance of an implementation. On the other hand, models that are expected to give insight in a system's structure, perhaps in the context of a reengineering study, demand for a structural validation approach. In any case, validation in contrast with verification is a relation that involves reality. As can be seen from the figure, verification questions the correspondence of a system's conceptual, formal and computerised model. Verification is an answer to the question whether the model, as it has been built, conforms to original intentions. Do both the formal and the computer model give an adequate representation of our conceptual beliefs? Finally, the models that have been developed, together with their behaviour, need to receive a "green light" from the model's stakeholders before they can be fully operationalised. It is this process of building confidence in the simulation model as well as marking its value and its competitive advantage for practical decision making that constitutes the act of accreditation.

3. A topological view on modelling

In view of the popularity of a relativistic approach to modelling and validation, an elaboration on the formal aspects of both objective and relative model validity seems appealing. This paragraph is intended to represent the reader with a topological approach to structural modelling. In order to address structural validity through a topological perspective, it is however imperative we make an initial revisit to the primary players in validation, i.e. a conceptual and formal model of a phenomenon in reality. Since in revisiting these concepts we are ourselves bounded by our own imperfect impressions of reality, we will introduce them in a manner of postulates their proof remains an open debate on physical truth. Because of the particular difficulties that arise when introducing the notion of time in validation, we will treat time effects on conceptual models in a separate paragraph.

3.1. Fundamental postulates on conceptual models, phenomena and systems

In an earlier paragraph, we gave yet an informal definition of a conceptual model being a mental image of a phenomenon in reality. Also, we stated a conceptual model is in fact an intellectual collection of tangible elements or objects and driving causal interrelationships. Let us now formalise this notion of a conceptual model in a postulate and position a conceptual model of reality in a so-called conceptual universe:

***Postulate 1**: A conceptual model (ξ) of reality resides in a conceptual universe (Ξ) that is raised by a finite number of elements ($\{E_{ij}\}$) and interrelationships ($\{P_{jj}\}$).*[6]

In studying the collection of all real world phenomena (in studying complete reality), we will assume that at a particular moment in time, from a conceptual point of view, reality can be

[6] We will apply the convention of using upper case symbols for identifying a space (Ξ,E,P,Γ,etc...) and lower case symbols for representing a member of that space (ξ,ε,ρ,γ,etc...).

described by a collection of tangible elements or objects $\{E_i\}$ and causal interrelationships or dependencies $\{P_j\}$. A conceptual model of reality (ξ) is then in fact nothing else than a fine description of each of those reality components[7]. Notice that, from a topological standpoint, this collection of objects and relations induces a space that we shall refer to as the conceptual universe (Ξ). As a matter of fact, the conceptual universe is the collection of all possible reality descriptions using the same template of objects and relations. Examining the space of conceptual models at a particular moment in time is in essence inspecting a collection of possible reality realisations. Formally we write the space that is constructed by the set of reality components as:

$$\Xi \equiv \mathop{\times}_{i,j} \{E_i, P_j\} \qquad \textbf{(1)}$$

A conceptual model of reality can be designated by means of a point in this conceptual universe, describing all residing objects and relations. As a matter of fact, since the conceptual universe is assumed to be a finite product space, the conceptual model can be written as a finite (l+m)-tuple in this space:

$$\xi \equiv (\xi_{E_1}, \xi_{E_2}, \ldots, \xi_{E_l}, \xi_{P_1}, \xi_{P_2}, \ldots, \xi_{P_m}) \qquad \textbf{(2)}$$

We will refer to the co-ordinates ξ_{E_i} and ξ_{P_j} as the projection ($p_{\rightarrow}$) of ξ on the respective universes E_i and P_j. We define this projection function ($p_{\rightarrow}$) of conceptual models ξ as:

$$\begin{aligned} p_{\rightarrow E_i}(\xi) &= \varepsilon_i,\ \varepsilon_i \in E_i \wedge \varepsilon_i \equiv \xi_{E_i} \\ p_{\rightarrow P_j}(\xi) &= \rho_j,\ \rho_j \in P_j \wedge \rho_j \equiv \xi_{P_j} \end{aligned} \qquad \textbf{(3)}$$

Notice that spatial displacements of a conceptual model towards nearby locations are in fact fine alterations of functional dependencies or object descriptions within a universe that remains essentially unchanged. Hence, a particular conceptual universe allows for a collection of alternate portrayals of reality components using the same template of elements and relations.

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[7] We will use the term *reality components* to indicate both objects and relations.

Having introduced a conceptual model of all real world phenomena as a point in a conceptual universe, which is constructed by elements and relations, an interesting matter that poses itself considers the uniqueness of the conceptual universe Ξ. In other words, in thinking of models of the world, should we confine our analysis to a single conceptual universe Ξ and search for conceptual models that are elements of the latter? Since Ξ is nothing else than a product space of elements and relations, it is likely that in search of a conceptual model of reality, a multitude of universes will be passed. Hence, our second postulate:

***Postulate 2**: A conceptual universe (Ξ), representing a product space of reality, resides in a space of conceptual universes (Ω), that is defined as the universe of all product spaces of elements and relations.*

In the context of a collective study of reality, we might expect a collection of conceptual models would be constructed that reside in a collection of different conceptual universes. If, in defining reality, we assume it are objects $\{E_i\}$ and relations $\{P_j\}$ that constitute the building blocks of our environment, every finite product of these components makes up a candidate conceptual universe of reality. Let us denote the universe of all product spaces of elements and relations as Ω:

$$\Omega \equiv \{ \underset{I,J}{\times} \{ E_{i \in I}, P_{j \in J} \}, \text{all I and J} \} \quad \textbf{(4)}$$

Notice that the elements of Ω are in fact nothing else than conceptual spaces. Considering the power set of Ω, $P(\Omega)$, we have the collection of all subsets of conceptual spaces. Notice that an element of $P(\Omega)$ coincides in fact with a particular set of intellectual frameworks (a set of conceptual universes) to approach reality. Since the set of all subsets of the collection of reality frameworks (Ω) could be thought of as a set of all possible pathways to express the conglomeration of reality, the power set of Ω ($P(\Omega)$) constitutes of all possible intellectual windows on reality. An intellectual window on reality is thus nothing else than a grouping of intellectual frameworks or conceptual spaces through which a declaration of reality is looked for. In that respect, at a particular moment in time, an intellectual window represents a state of conceptual world views (a state of conceptual spaces). In an objective atmosphere, it is through the act of research and science that the intellectual window may be steered towards the disclosure of the foundation.

In light of a set of elements and relations that construct a conceptual space of reality, it is only natural to ask oneself how each of these reality components can be accurately described. Put differently: what are the necessary ingredients to make up a description of a particular element or relationship? A third postulate defines the description of reality components:

Postulate 3: Every element (E_i) and relationship (P_j) can be defined using a finite number of state variables.

It is assumed in this postulate that there exists a class of attributes to fine describe each reality component. We shall call these attributes description variables for elements and a mathematical specification for causal interrelationships. Description variables and mathematical expressions specify reality elements and relations, which in turn construct a product space that allows for a topological approach to reality and conceptual modelling. In accordance with the terminology that was yet introduced by Zeigler [Zeigler 1976], we will group these description variables and functional expressions in a collection that we shall refer to as *state variables*. Notice that the collection of state variables is defined as the collection of variables that, if having full knowledge about their values, enables us to derive any other output variable from reality. In other words, a full description of reality cannot be done without knowledge of the state variables. Put differently, the collection of state variables forms the most compact, indispensable and complete set of variables to define reality.[8] Taking a closer look at the description of a state variable, it turns out that a causal relationship has a maximum of only one state variable. As a matter of fact, the expression representing the causal influence forms the only possible state variable of the relation. Since in light of our further discussion, an adequate notion of the concept of a state variable is necessary, let us first clarify any left confusion on the difference between a state variable and an output variable.

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[8] Output variables are thus some complex function of state variables while state variables themselves cannot be derived from output variables.

<u>Example 2</u>: *State variables and output variables*

Consider the following phenomenon that occurs in reality. In a computernetwork, a relation (R1) has been defined that causes the activation of a second server (S2) as soon as network traffic exceeds the level of 10MB per second. As long as the total packet size stays below this critical value, a single server (S1) remains responsible for routing and traffic management. In other words, the relation R1 is a relation between network traffic per second on one hand and the activation of S2 ($S2_\alpha$) on the other hand. Is this functional dependency of the activation of S2 on traffic a state variable of R1? Applying the definition of a state variable, the answer to this question is *yes*. The functional dependency of $S2_\alpha$ on traffic is in no way deducable from other variables that make up the phenomenon. Suppose now however that a second relation (R2) is present, relating the treshold level of 10MB to the total number of computers logged on. The more computers that are logged on, the sooner S2 will become activated (the lower the treshold level). R2 is thus a relation between the number of computers logged on and the functional relationship representing R1. Is the mathematical expression representing R1 still a state variable? The answer is *no*. Since the expression for activation of S2 is now completely determined by other variables in the system (the state variable of R2), the functional relationship of R1 is no longer a state variable. We state that the functional relationship of R1 has become an output variable, a variable that is some complex function of other variables in the system. Now, let us take a look at the variables queue length (Q_λ) and maximum allowed queue length (Q_{max}) at S1. Is the length of the queue at S1 a state variable? Since the queue length at S1 is some complex result of other parameters (variables) in the system, the answer is again *no*. Queue length is not a variable that is not completely determined by other variables in the system. What about maximum allowed queue length? This is again a state variable. Since the maximum queue length is in no way influenced, deducable, determined or related to other variables, maximum queue length is a state variable of the object *queue*.

Now, for a particular object E_i, the collection of state variables $\{\Sigma_k^{E_i}\}$, k = 1...n, constitutes a space Γ_{E_i} that allows to approach the object on the basis of a point[9]:

$$\Gamma_{E_i} = \underset{k}{\times} \Sigma_k^{E_i} \qquad \textbf{(5)}$$

For a relationship P_j, we define a space of causal factors Φ and a single dependent variable Δ. On the product space $\Phi \times \Delta$, there exists a collection of functions from Φ into Δ that we shall denote as $\wp_{\Phi\to\Delta}$. Notice that each of these functions coincides with a particular value the single state variable of the relationship can attain. Hence, formally, the state space Γ_{P_j} for a relationship is written as:

[9] We implicitly assumed object E_i can be described by a total of n state variables.

$$\Gamma_{P_j} = \Sigma^{P_j} \equiv \wp_{\Phi \to \Delta} \qquad \textbf{(6)}$$

Considering the relation between the state variable spaces Γ_{E_i}, Γ_{P_j} and their corresponding universes E_i and P_j in the conceptual universe Ξ, we assume there exist one-on-one, bijective mappings f and g between respectively elements of Γ_{E_i} and object descriptions in Ξ_{E_i}[10] and elements of Γ_{P_j} and relationship specifications in Ξ_{P_j}. Hence we can write:

$$\begin{aligned} &\forall\, \gamma_{E_i} \in \Gamma_{E_i} : \exists! \varepsilon_i \in E_i \;\wedge\; f(\gamma_{E_i}) = \varepsilon_i \\ &\forall\, \gamma_{P_j} \in \Gamma_{P_j} : \exists! \rho_j \in P_j \;\wedge\; g(\gamma_{P_j}) = \rho_j \end{aligned} \qquad \textbf{(7)}$$

Irrespective of the topological nature of Γ_{E_i} or Γ_{P_j}, we will assume that f and g are continuous functions, i.e. that their image of open sets in the Γ_X-space[11] is open in the space Ξ_X and vice versa. This assumption is not that hard to defend since component descriptions in the Γ_X-space are nothing else than a refinement of their position in the Ξ_X-space. As a result of this assumed space consistency, we call the topological spaces Ξ_X and Γ_X homeomorphic or topological equivalent spaces.

Having launched the notion of a conceptual universe, model and state space, a natural question to ask is whether in a study of some real world phenomenon one is obliged to come up with a triplet $<\Xi,\xi,\{\Gamma_k\}>$ that constitutes a model of *complete* reality. From a modelling point of view, the answer is obvious: it is not a model of complete reality that is of concern, but a model that represents a particular *subspace* of the latter. In the context of modelling and simulation, our interest goes to a model of some phenomenon that takes place in our environment and that demands for a deeper investigation. Let us now formally define the notion of a conceptual model of a system or phenomenon in reality:

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[10] $\Xi_{E_i} \equiv E_i$

[11] Let us use the general notation of a component X instead of the component specific notations of E_i and P_j.

***Postulate 4:* A system (S) description is a conceptual model (ξ_S) that is defined in a subspace (Ξ_S) of a conceptual universe (Ξ).**

In order to fully comprehend the assertion of this postulate, let us first clarify the distinction between reality, a phenomenon in reality and a system. In a practical simulation study, a model builder will concentrate his efforts on a part of reality (a phenomenon) that is of interest for his particular analysis. In fact, a phenomenon is nothing else than a general observation, a sequence of events that occur and that demand for a further investigation.[12] In an attempt to explain, to analyse or to simulate the phenomenon, the analyst will strive to come up with relevant reality elements and relations that hopefully account for a satisfying part of the structural dynamics of the phenomenon. Certain reality components and state variables will have been unveiled by the analyst during his research, other elements, relations and variables will remain absent in his collection of relevant phenomenon components. We state that the analyst tries to explain the phenomenon through the introduction of a system S. This system is in fact nothing else than the collection of identified objects, relations and state variables. Since this system was retrieved in studying a single phenomenon, we can think of the system as being a subspace of the most ample phenomenon that can ever be studied, i.e. a subspace of reality.

Definition 4: *(A system)*
A system (S) is defined as being a collection of reality components and interrelations together with a set of state variables that comprises a subspace Ξ_S of a conceptual universe Ξ.

If a system equals a subspace of Ξ (Ξ_S), it is only natural to associate the concept of a system's description with a particular point in that subspace, ξ_S, where $\xi_S \in \Xi_S$. Notice that in studying a phenomenon, the analyst has a certain level of freedom to come up with a system (a subspace) and a description (a point) that, according to his beliefs, allows for a powerful investigation of the phenomenon. In other words, a phenomenon can be studied through the introduction of many systems, some of which will be more effective in explaining the structural dynamics of the phenomenon than others.

[12] Examples of phenomena may include a production process in a firm, network traffic in a computer environment, the world's industrial pollution, the movement of stars and planets, etc... In other words, any aspect of reality that one can be aware of constitutes a phenomenon.

3.2. From conceptual beliefs towards a formal model

Earlier on, we stated yet that structural validation is in theory defined on the conceptual level and that every conceptual model remains in fact an intangible, mental image of reality. Since in practice, it is a formal specification (or a computer model) that constitutes the subject of validation, a study of the formal modelling level and its relation to the area of conceptual models is required. Let us first of all postulate the existence of a universal language L:

***Postulate 5*: There exists a language (L) with a universe of discourse (Π) that allows us to describe a conceptual model of reality (ξ) in a formal model (π) up to any degree of precision.**

We assume that there exists a language (L) that is capable of describing any conceptual model up to any degree of precision. In other words, there exists a language with a universe of discourse (Π) that puts no restriction at all on the possibility of a model builder to accurately express his conceptual beliefs. The formal model (π) could then be interpreted as being a cognitive projection of ones beliefs onto a specific linguistic geometry Π. In any case, we will assume that the language one has chosen to explicate ones conceptual model is the language L, no matter ones philosophical scholarship. If this would not be the case, structural model validation would be impaired by a highly uncomfortable handicap in a way that it tries to prove the truthfulness of a (formal) model, knowing very well in advance that it might be the formal language itself that has curved adequate reality perceptions towards an invalid articulation. For preservation of validity of any construct proposed to measure the validity of a simulation model, it is therefore imperative that the language in which the formal model resides puts no restriction at all on the ability to express any theory of reality.

The existence of the language L and the possibility of projecting a true conceptual model onto the formal universe of L indicates the existence of a relation between elements of Ξ and Π, which is more profoundly expressed in the following postulate.

***Postulate 6*: There exists a one-to-many mapping (ϕ) from elements in Ξ to elements in Π.**

Having a conceptual universe and model on one hand (Ξ & ξ) and a formal universe and model on the other hand (Π & π), it is only natural to ask what kind of relation there exists between these conceptual and formal concepts. In this postulate, we presume the existence of such a relation (ϕ) that represents a one-to-many mapping[13] from a conceptual model (ξ) to a collection of formal models ($\{\pi_i\}$). Any conceptual state of mind can be expressed through usage of the language L in at least one formal model. In other words, judging the validity of the more abstract notion of a conceptual model or the concrete version of conceptual beliefs in the shape of a formal model is essentially indifferent. For a particular formal model description, there exists only one corresponding conceptual equivalent and investigating the validity of the latter is essentially in no way different from assessing the correctness and the truthfulness of the former.

$$\forall \pi \in \Pi : \exists! \xi \in \Xi \wedge \phi^{-1}(\pi) = \xi \qquad \textbf{(8)}$$

Further, we will assume that any model builder applies the relation ϕ to map his conceptual beliefs on a set of formal models. In that way, formal model errors can never be traced to the incapability of a system analyst to articulate his beliefs of reality. In other words, we assume that the model builder masters a language L with a skill of infinite precision. His final formal model of reality may contain errors, but these are never due to the application of an inferior language or the incompetence of the analyst to express his beliefs.

3.3. The introduction of time

Since in the context of simulating real world phenomena the notion of time will play an incontrovertible role, it is worthwhile to investigate the repercussion of the introduction of time on the concepts that were introduced earlier. More specific, how do we delineate a time varying reality environment within the notion of a conceptual universe and model? And, how do we account for reality alterations in a definition of structural validity?

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[13] The one-to-many mapping takes into account a level of semantic relativism: the ability to express a single, conceptual state of mind through a multitude of articulations.

3.3.1. Conceptual walks

In the next postulate, we define the cognitive projection of the process of reality alteration as a conceptual walk:

Postulate 7: A conceptual walk (ξ(t),Ξ(t)) is the process of reality component description alterations in a universe that is submit to conceptual expansion, contraction and curvature through time.

In the previous paragraphs, we introduced a collection of time invariant postulates. At a particular moment in time, we can speak of conceptual universes, conceptual models, phenomena, systems and their formal equivalents. Since however for most simulation studies, models are developed taking into account the existence of time, it remains to be seen what kind of effect the notion of time resorts on these concepts. As soon as one introduces the notion of time, a natural question that arises is whether a conceptual universe can be the subject of any modification. If one were just to follow the well known relationship between energy and matter, it requires only a minor effort to convince oneself that a conceptual universe is likely to expand in some directions and to contract in others. At the very moment that new matter is created, a new object comes to existence and an expansion has occurred. Similarly, adding relationships amongst existing objects implies conceptual expansion. At the same time, in light of a structural change, some variables that were output variables may become state variables and vice versa. In a later paragraph, we will introduce the notion of conceptual orthogonality and derive that the addition or deletion of state variables leads to a curvature of the conceptual universe.[14] Besides additions or deletions of components and state variables, notice that it may very well be a conceptual model itself that is the subject of a translation. Any altering of an object description or a relational specification[15] results in a translation of a conceptual model along one or more dimensions of the corresponding conceptual universe. This process of model translations and universe modifications we shall refer to as a conceptual walk (ξ(t),Ξ(t)).

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[14] Notice that in developing a real world simulation model, the pathway of expansion, contraction and curvature, unless one has the ability to make changes on the fly, is determined a priori before any execution of the model.

[15] In the assumption that one of the state variables of the component are changed.

Consider now at time t, a conceptual model $\xi(t)$ of reality. What can we say about the position of this model at time $t+\delta t$? Given the structural identity of $\xi(t)$ at t, should we expect the process of a conceptual walk from t onwards being deterministic or stochastic? It is in a last postulate that we suppose the stochastic character of conceptual walks of conceptual models of reality.

<u>Postulate 8</u>: The process of conceptual walks ($\xi(t),\Xi(t)$) is a stochastic process.

We assume that the expansion, contraction or curvature of the conceptual universe as it occurs is essentially a stochastic process. Let us first clear up the difference between a stochastic conceptual walk on one hand and a stochastic behaviour of conceptual models on the other hand. Assuming that the process of conceptual changes is stochastic equals stating that, given a conceptual model of reality at some moment in time, the precise location of this model in the future is essentially unknown. The assumption of a stochastic conceptual walk indicates there exists some kind of complex probability distribution that represents the likelihood of future positions for a particular model of reality at time t. In other words, the exact future identity of the model is unpredictable. Stochastic behaviour on the other hand indicates that, irrespective of the stochastic nature of conceptual alterations, observations that are made from reality are samples that, if given the possibility to collect several observations, constitute a probability distribution of reality realisations[16]. In other words, in the unlikely event that, from a structural viewpoint, reality would not be changing at all, its behaviour would still be a collection of stochastic processes. Observed behaviour processes from a changing reality are thus the result of both probable (stochastic) structural changes as well as the stochastic nature of reality itself. As a result, a stochastic reality implies automatically stochastic behaviour but the opposite is not necessarily true.

3.3.2. Time and validity

Labelling the occurrence of reality as stochastic has important consequences on any study of modelling phenomena in reality. As a matter of fact, the information a model builder retains when studying reality is in fact nothing else than a single observation of one particular realisation of some complex conceptual walk process. What is probably most important of all, once a model has been created, its stochastic future is determined in advance (unless one has the ability to "steer" the model on the fly). How should we account for this fact of determined stochastic conceptual alterations in a later study of structural validity of the evolved model? Let us address this interesting issue by means of the following example.

<u>Example 3</u>: Stochastic conceptual evolvement and validity

Suppose that a prominent model builder is given the opportunity to freeze reality and to study extensively the objects and relations that make up his environment. Let us suppose that after some time he comes up with the true (stochastic) conceptual model[17]. At that particular moment, the modeler is asked to unfreeze reality and at the same time to start his model on a digital (super)computer. After some time, reality is frozen again and a first independent model consultant is asked to verify the equality of the evolved proposed model by the analyst and the conceptual model of reality as it is true at that moment in time. Due to the stochastic nature of the process of conceptual evolvement in both reality and the proposed model, the proposed model is likely to be different from the true model. Not only will they be displaced from each other, it is likely the conceptual universes in which they reside are different. Based on these considerations, the consultant judges the model to be unequal to its true equivalent and hence labels it as invalid. After his analysis, a second consultant is invited to perform the same task of validation of the proposed model by our prominent model builder. Based on the stochastic nature of reality and model, he concludes that in order to found his assertion on validity he should be given a collection of possible true and simulated conceptual evolvements[18] so that he can construct a governing probability law of both real and simulated structural changes. He goes on that once these probability laws are discovered all that matters is that they are identical. Identical probability laws imply validity of the model, not only at the time of consulting but at any time along the trajectory the conceptual model has followed. Finally, a third consultant shares the opinion of his second colleague but marks that it would never be possible to find out whether a particular conceptual model (that differs from reality) could be a model that resides on a different trajectory of conceptual evolvement that might have been realized. Given a particular true conceptual model, releasing time induces a plethora of possible conceptual models any time later of which only one has been realised (reality). He continues his statement to tell that if one came up with a conceptual model that differs from its true equivalent at some point in time, it would be impossible to judge whether it fits on a different realisation track since simply one has no knowledge about the nature of these different tracks of realisation. As a conclusion, he states that one might come up with a collection of conceptual models that might have been realisations of reality and that labelling one them as *valid* is only a matter of convention. The verdict on validity should be made on a comparison of real and proposed conceptual model. The verdict on a possible valid realisation should be made on a comparison of distrubutions and stochastic laws and remains inconclusive.

For determining model validity, we state it is a comparison between model and reality that is of importance. Dissimilarities between the model and reality indicate a level of invalidity. But does

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[16] Notice that in practice one will never be able to collect additional realisations of a particular process in reality.

[17] The example is written in a foundational argument.

[18] In other words, he should be given a collection of replications of both reality and model.

it also mean the proposed model might never have been realised? The answer to this question remains essentially open. Since one simply has no knowledge about the possible trajectories of reality realisations through time, determining whether a model would have been a valid model of reality under a different occurrence of chance remains inconclusive. In any case, if it would, we would call the model not a (the) valid model of reality but a possible valid realisation that might have taken place.

4. Topological cornerstones of structural validity

Having met a collection of fundamental postulates that are indispensable for a further elaboration on structural validity, a natural question to ask concerns the topological structure of the conceptual universe Ξ. In light of the different philosophical schools in validation, a challenge remains to come up with a topological variant of the duality of objectivism versus relativism. In other words, can we express ones philosophical beliefs and ones attitude towards structural validation by the topological nature of Ξ? It lies within the aim of this paragraph to derive a natural topology for the conceptual universe that would represent ones proper philosophical attitude towards the notion of structural model validity.

4.1. A foundational topology

If a conceptual universe is defined as the Cartesian product of objects and relations, what is then the natural topological structure of this universe for a foundationalist? In the theory of general topology, topologies can be induced by a metric or distance function or they can be constructed irrespective of the notion of distance. In light of our discussion, it is the former type of topologies that is of particular interest. Let us first introduce some basic topological concepts. A topology (T) on a non-empty set X is generally defined as the class of subsets of X that satisfy the following properties [Lipschutz 1965]:

<u>*Definition 5:*</u> *(A topology T)*

A class T of subsets ($\tau_1, \tau_2, \ldots$) of a non-empty set X is a topology on X $\Leftrightarrow$

(1) $X \wedge \phi \in T$ *(X and the empty set belong to T)*

(2) $\bigcup_{i=l}^{k} \tau_i \in T$, *any l, k* $\wedge\ k > l$ *(The union of any members of T belongs to T)*

(3) $\tau_i \cap \tau_j \in T$, *any i, j* *(The intersection of any pair of members of T belongs to T)*

The elements τ_i of the topology T on X are called T-open sets and are subsets of X. Applying the notion of a topology to a component of a conceptual universe, the set X equals the collection of all component descriptions. Notice that it was yet assumed there is in fact a one-on-one relationship between these component descriptions and points in the state variable space. Since the component is completely described in the state variable space, the question of the topological nature of X parallels the issue of the topological nature of Γ_X. Let us have a look at a single state variable Σ_i^X within Γ_X. With the binary attitude of deciding on structural validity in mind, can we define a topology that would be the natural topology for Σ_i^X? If validation is generally a matter of a binary comparison, we might state that validation of a particular facet in a global description of a component is a matter of determining whether the description of that facet (state variable) coincides with its single true counterpart. In other words, the binary outlook on validation sounds very similar to an application of the so-called trivial metric in the Γ_X-space. Let us first formally define a metric or a distance function for a space X as follows [Lipschutz 1965]:

<u>*Definition 6:*</u> *(A metric or distance function δ)*

A real valued function δ from $X \times X$ into $\Re$ is called a metric or distance function on X $\Leftrightarrow$

$\forall \chi_1, \chi_2, \chi_3 \in X$:

(1) $\delta(\chi_1,\chi_2) \geq 0$ *and* $\delta(\chi_1,\chi_1) = 0$

(2) $\delta(\chi_1,\chi_2) = \delta(\chi_2,\chi_1)$ *(symmetry)*

(3) $\delta(\chi_1,\chi_3) \leq \delta(\chi_1,\chi_2) + \delta(\chi_2,\chi_3)$ *(triangle inequality)*

(4) *if* $\chi_1 \neq \chi_2$ *then* $\delta(\chi_1,\chi_2) > 0$

Having defined a metric or distance function, the definition of the trivial metric is uncomplicated:

Definition 7: *(The trivial metric)*

A well defined metric δ on X is called the trivial metric $\Leftrightarrow \forall \chi_1, \chi_2 \in X$:

(1) $\chi_1 = \chi_2 \Rightarrow \delta(\chi_1,\chi_2) = 0$

(2) $\chi_1 \neq \chi_2 \Rightarrow \delta(\chi_1,\chi_2) = 1$

In other words, the trivial metric assigns the value *null* or *one* to the distance between a pair of points, depending on their equality. As a matter of fact, applying the trivial metric in validation is the most undiluted expression of an objectivistic temperament. Facets (state variables) of a component are either perfectly well described or completely erroneous. An adequate description of a facet is indicated by its zero distance towards the single true description. Description errors invoke a displacement with respect to the true portrayal and induce a distance increase from absolute validity (zero) to absolute invalidity (one). As a matter of fact, *zero* and *one* are the only distance-oriented expressions an objectivist will apply in an assessment of description accuracy.

Having related this trivial metric to the yardstick of validation for a foundationalist, can we now determine a natural foundational topology? Let us first launch the notion of an open sphere in the space of a particular state variable Σ_i^X. The collection of points (state variable values) that are situated within a distance r from a point σ_i^X is called an open sphere around σ_i^X with radius r:

Definition 8: *(An open sphere)*

An open sphere $S(\sigma_i^X, r)$ around σ_i^X with radius r is the collection of all points that are situated within a distance r from σ_i^X.

$S(\sigma_i^X, r) = \{s : \delta(\sigma_i^X, s) < r\}$

In deriving a natural topology for the space Σ_i^X, the notion of a base and subbase needs to be defined first. A class of open sets is called a base for a topology if and only if:

Definition 9: *(A base for a topology T)*

A class B of subsets of T is called a base for T $\Leftrightarrow$ any open set of T can be written as the union of members of B.

A subbase is generally defined as a class of subsets of a topology for which taking the family of all finite intersections forms a base for that topology:

Definition 10: *(A subbase for a topology)*
A class S of subsets of T is called a subbase for T ⇔ the family of finite intersections of members of S forms a base for T.

Now, it can be proven [Lipschutz 1965] that the class of open spheres in a set Σ_i^X with metric δ constitutes a base for a topology on Σ_i^X. Further, consider an open sphere $S(\sigma_i^X, r)$ with radius r ∈]0,1] surrounding a point σ_i^X. This sphere contains the singleton σ_i^X: $S(\sigma_i^X, r) = \{\sigma_i^X\}$. Since every sphere is by definition open, every singleton set is open and since the union of open sets is open again, every set is open. As a result, the class of all singleton subsets of Σ_i^X forms a base for a topology on Σ_i^X. We call this topology, that constitutes of all subsets of Σ_i^X the discrete topology T_d on Σ_i^X and we call the space (Σ_i^X, T_d) a discrete topological space.

Having defined the natural topology of a single state variable space, can we from here induce the natural topology for the entire state space and finally for the conceptual universe? Notice that for a particular component X, the state space is a countable, finite product of several state variable spaces. Following general topology theory, we can define the so-called product topology on Γ_X as:

Definition 11: *(The product topology)*
The product topology T_p defined on a product of sets $\Gamma_X \equiv \times_i \Sigma_i^X$ is the topology generated by the inverse projections from the Σ_i^X's into Γ_X.

In search for a basis of this finite product topology T_p, one considers the collection of inverse projections from state variable spaces (Σ_i^X) into the state space Γ_X. Notice that an inverse projection of an open subset of Σ_i^X, $p^{-1}_{\to\Gamma_X}(\tau_j^{\Sigma_i^X})$ is defined as the collection of points whose projection falls within the open subset $\tau_j^{\Sigma_i^X}$. Now, it can be proven that the class of subsets, generated by taking the product of an open subset $\tau_j^{\Sigma_i^X}$ of a state variable space Σ_i^X with the collection of all other state variable spaces $\neg\Sigma_i^X$ - hence by looking at the inverse projection of $\tau_j^{\Sigma_i^X}$ into Γ_X - generates a subbase for the product topology T_p on Γ_X. This subbase is referred to as the defining subbase for the product topology. Following the definition of a subbase, the collection of finite intersections of members of this class of subsets forms the (defining) base for the product topology T_p on Γ_X. Now, since each of the state variable spaces is discrete, it can be proven the product topology on the product of discrete spaces is again the discrete topology. Further, since we stated the space Γ_X is homeomorph with Ξ_X, the natural topology on Ξ_X is the discrete topology. Finally, since Ξ is the Cartesian product of the individual spaces Ξ_X, the natural topology on Ξ is the discrete topology. As a result, the foundational attitude towards modelling and validation induces the notion of a trivial metric as the yardstick for validity assessment and naturally defines the discrete topology on Ξ.

Theorem 1: *(The natural foundational topology of Ξ)*
For a foundationalist, the natural topology on Ξ is the discrete product topology, induced by the trivial metric.

4.2. An anti-foundational topology

If for a foundationalist, the conceptual universe is naturally equipped with the discrete topology and the trivial metric, a natural question to ask is whether this particular metrisation still holds for the anti-foundationalist. According to the relativistic approach to validation, models (and hence component descriptions) are more or less valid depending on their usefulness, their practical value, their ability to survive tests and criticism or their social acceptance. In other words, the foundation to which the validity of theories or models should be verified is relative, context dependent and non-unique. Consider now in an objective, foundational context, for each component X the true description $\dot{\chi}$ in the true state space $\dot{\Gamma}_X$. Further, consider the true conceptual model $\dot{\xi}$ in the true conceptual universe $\dot{\Xi}$. Remember that for a foundationalist, the set of all true descriptions of component X (ς_X) is the singleton $\{\dot{\chi}\}$, the collection of untrue component descriptions (I_X) is the set $\neg\varsigma_X$ and that the distance between $\dot{\chi}$ and any other point in $\dot{\Xi}_X$ equals 1, no matter their closeness. In the context of relativism however, can we still apply the notion of this single true conceptual model of the world $\dot{\xi}$ and this single true conceptual universe $\dot{\Xi}$? First, since the foundation to validate theories is relative, we might have to give up the objective uniqueness of the true conceptual universe $\dot{\Xi}$. Further, since in deciding about model validity every participant has its own view of the world, its own perceptions of what the model should be capable of explaining and hence its own impressions about the usefulness of the model within the operational context it has been constructed, from a relativistic point of view, we might have to give up the consensus about the existence of a single, true model of reality $\dot{\xi}$ as well. Now, in light of this anti-foundational context, it is only natural to accept that the distance between a pair of models is not objectively or unanimously defined, but becomes a fuzzy or vague expression. In addition, a more powerful distance expression than the trivial, binary objective 0/1-metric is desired. Since both the position of the true model and universe as well as the validity of a proposed model is fuzzy and context dependent, it feels only natural to define structural model validity as a fuzzy distance towards a fuzzy set of possible valid models in a fuzzy set of true universes. (see [Fuller 1995, Zimmerman 1996] for an introduction to fuzzy set theory)

Having launched the notion of validity as a context dependent fuzzy distance number, what can we say about the repercussion on the topological nature of the conceptual universe for a relativist? As a matter of fact, we will derive that the notion of a relative, fuzzy distance metric on the unit interval will induce the so-called discrete fuzzy topology. Let us first formally define a fuzzy topology as follows [Wong 1975]:

Definition 12: *(A fuzzy topology)*

A class $\hat{T}$ of fuzzy subsets ($\hat{\tau}_1, \hat{\tau}_2, \ldots$) of a non-empty set X is a fuzzy topology on X $\Leftrightarrow$

(1)	$X \wedge \varnothing \in \hat{T}$	*(X and the empty set belong to $\hat{T}$)*
(2)	$\bigcup_{i=l}^{k} \hat{\tau}_i \in \hat{T}$, *any l, k* $\wedge$ $k > l$	*(The union of any members of $\hat{T}$ belongs to $\hat{T}$)*
(3)	$\hat{\tau}_i \cap \hat{\tau}_j \in \hat{T}$, *any i, j*	*(The intersection of any pair of members of $\hat{T}$ belongs to $\hat{T}$)*

The subsets $\hat{\tau}_1, \hat{\tau}_2, \ldots$ are called fuzzy open sets and the space $(X, \hat{T})$ is called a fuzzy topological space. Similarly to our foundational analysis, the one-on-one relationship between component descriptions in X with elements of the state variable space Γ_X translates the issue of the topological nature of X towards the topological nature of Γ_X. Let us consider again a single state variable Σ_i^X. Notice that the true description of the facet Σ_i^X is relative, unknown and the distance from alternative descriptions of Σ_i^X is a fuzzy number. In parallel to the classical definition of a metric, let us define a fuzzy metric or fuzzy distance function as follows [Kaleva 1984]:

<u>*Definition 13:*</u> *(A fuzzy metric δ)*

A fuzzy real valued function δ from $X \times X \rightarrow G$[19] is called a fuzzy metric or fuzzy distance function $\delta \Leftrightarrow \forall \chi_1, \chi_2, \chi_3 \in X$:

(1) $\delta(\chi_1,\chi_2) = \dot{0}$[20] $\Leftrightarrow \chi_1 = \chi_2$

(2) $\delta(\chi_1,\chi_2) = \delta(\chi_2,\chi_1)$

(3) $\delta(\chi_1,\chi_3) \leq \delta(\chi_1,\chi_2) \oplus \delta(\chi_2,\chi_3)$ *with $\oplus$ defined as the fuzzy addition operator*

In the context of relativism, it feels only natural to extend the 0/1 domain of the trivial metric to the whole range of distances situated within the interval [0,1]. In that respect, the distance between a pair of points becomes a number, ranging from strong validity (distance 0) towards strong invalidity (distance 1). Further, since validity or invalidity of a model is not unanimously decided upon and depends highly on the particular worldviews of the model stakeholders, the precise distance from a model towards its true equivalent remains fuzzy. In other words, we state that model validity becomes a fuzzy number on the interval [0,1]. In parallel to our foundational derivation, let us define a fuzzy trivial metric as a normal upper convex fuzzy real number on the unit interval [0,1]:

<u>*Definition 14:*</u> *(A fuzzy trivial metric)*

A well defined fuzzy metric δ is called a fuzzy trivial metric $\Leftrightarrow$

(1) $\chi_1 = \chi_2 \Rightarrow \delta(\chi_1,\chi_2) = \dot{0}$

(2) $\chi_1 \neq \chi_2 \Rightarrow \delta(\chi_1,\chi_2) \in F_{[0,1]}$ *with $F_{[0,1]}$ the set of all normal, upper convex fuzzy sets on the unit interval [0,1]*

In view of a fuzzy trivial metric as the yardstick for assessing model validity, can we derive the natural topology for an anti-foundationalist? Let us first introduce the notion of an α-open sphere $S_\alpha(\sigma_i^X, r)$ as follows [Das 1999]:

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[19] G represents the set of all upper semi continuous normal convex fuzzy real positive numbers.

[20] The upper dot is used to indicate the fuzzy set that assigns the membership value 1 at the proposed number and 0 elsewhere.

Definition 15: *(An α-open sphere)*

An α-open sphere $S_\alpha(\sigma_i^X, r)$, $\alpha \in]0,1]$ and $r > 0$, surrounding a point σ_i^X, is the fuzzy set of points with membership value α for which the supremum of the α-cut of their distance to $\sigma_i^X < r$.

$S_\alpha(\sigma_i^X, r) = \{s, \mu_S(s) = \alpha : \delta(\sigma_i^X, s)_2^\alpha < r\}$

Notice that an α-open sphere with radius r around a state variable value σ_i^X equals a fuzzy set of points for which taking the upper limit of the α-cut[21] of their distance to σ_i^X is smaller than r. Can we claim that the family of α-open spheres forms a base for the space Σ_i^X? Let us look at the definition of a fuzzy base and fuzzy subbase first:

Definition 16: *(A fuzzy base for a topology)*

A set $\hat{B}$ is a fuzzy base for a fuzzy topology $\hat{T}$ ⇔ every member of $\hat{T}$ can be written as the union of members of $\hat{B}$.

Definition 17: *(A fuzzy subbase for a topology)*

A set $\hat{S}$ is a fuzzy subbase for a fuzzy topology $\hat{T}$ ⇔ the family of finite intersections of $\hat{S}$ forms a fuzzy base for $\hat{T}$.

Now consider the class of all α-open spheres with radius r ∈]0,1] and α ∈]0,1] surrounding a point σ_i^X in Σ_i^X. Since the distance between σ_i^X and any other point of Σ_i^X is an upper convex fuzzy normal number on [0,1], we can always find, for any α and for any subspace of Σ_i^X including σ_i^X, an α-open sphere with radius r that contains the fuzzy singleton $\{\sigma_i^X\}$ with membership value α. Further, letting the subspace of Σ_i^X approach Σ_i^X, there remains always a collection of radii for which holds that open spheres with a radius an element of that collection contain the fuzzy singleton σ_i^X. In other words, the class of all α-open spheres covers at least the family of all α-open fuzzy singletons in Σ_i^X. Notice that by taking the union of members of the class of α-open fuzzy singletons, we can construct any fuzzy open subset of the space Σ_i^X. As a

[21] An α-cut of a normal, upper convex fuzzy number is defined as the set of points whose membership value to the fuzzy set is at least α.

result, we state that the class of all α-open spheres and a fuzzy trivial metric being a fuzzy upper convex normal number on [0,1] generates the family of all fuzzy open subsets of Σ_i^X, which is referred to as the fuzzy discrete topology on Σ_i^X. The space Σ_i^X together with the fuzzy discrete topology $\hat{T}_d$ is called the fuzzy discrete topological space $(\Sigma_i^X, \hat{T}_d)$. In parallel to our foundational analysis, we can now derive the natural topology for the entire state space of the component X. We define the product fuzzy topology as:

<u>*Definition 18*</u>: *(The product fuzzy topology)*

The fuzzy product topology $\hat{T}_p$ on a product of sets $\Gamma_X \equiv \times_i \Sigma_i^X$, each equipped with a fuzzy topology, is the fuzzy topology generated by the inverse fuzzy projections from the Σ_i^X 's into Γ_X.

In a derivation similar to our foundational analysis, a defining fuzzy base and subbase for the product fuzzy topology can be defined in the following manner. Consider the family of inverse projections of fuzzy sets $\hat{\tau}_j^{\Sigma_i^X}$, all j and i, $\{ p_{\rightarrow}^{-1}(\hat{\tau}_j^{\Sigma_i^X})\}$. Consider the class of all finite intersections of this family. Now it can be proven [Wong 1975] the class of finite intersections and the family of inverse projections form respectively a defining fuzzy base and fuzzy subbase for a topology. This topology is referred to as the product fuzzy topology, generated by a family of inverse projections. In order to derive the topological nature of this product fuzzy topology, consider the family of inverse projections of fuzzy base sets. Since for each space in the product, fuzzy base sets form a family of fuzzy singletons, the inverse projection of each base set into Γ_X is a family of cylindrically extended fuzzy singletons. Finite intersections of fuzzy singletons are again fuzzy singletons. As a result, defining the discrete fuzzy topology on each space of the product induces the discrete product fuzzy topology on Γ_X. Since it is assumed Γ_X is fuzzy homeomorph with Ξ_X, Ξ_X forms a discrete fuzzy topological space. Finally, it is easy to see the product space Ξ is equipped with the discrete fuzzy topology. As a result, moving from an objective approach towards a relative attitude to validation constitutes a fuzzification of the conceptual universe. The natural topology for the relativist is the fuzzy analogue of the discrete product topology for the objectivist.

Theorem 2: *(The natural anti-foundational topology of Ξ)*
For an anti-foundationalist, the natural topology on Ξ is the fuzzy discrete product topology, induced by a fuzzy trivial metric.

5. Sources of conceptual errors

Before we address ourselves to a formal definition of structural model validity for both objectivists and relativists, let us first have a look at possible conceptual modelling errors that might occur and discuss their effect on the conceptual universe, the conceptual model and the state space. In modelling real world phenomena, we might expect the model builder to be burdened with an enterprise of immense complexity. Armed with a careful collection of observational data, it is essentially a matter of his rational capabilities how well he can infer from his observations accurate descriptions of system components. Since the information base for the analyst is a mixture of effects, caused by a plethora of reality elements and relations, it will be a gigantic challenge to decompose the information in an accurate set of elements, description variables, causal factors and dependency relations. Therefore, it is likely the model builder will commit errors. It is presumable that the mass of data he receives from the system under study will imply a level of rational confounding where the analyst comes up with misspecified functional dependencies, non-existing causal influences, erroneous descriptions, invalid state variables, etc... Ruling out, overlooking or ignoring certain reality components or state variables is likely to bring about a cascade of fault perceptions that shifts a conceptual model miles away from its true location. Before we make an attempt to come up with a classification of possible modelling errors, let us first have a look at the conceptual repercussion of ignoring certain reality components and state variables.

5.1. A model in a subspace of reality

In any practical simulation study, an analyst will focus his modelling efforts on some fragment of reality, making abstraction of a large pool of other reality objects and interrelationships. Hence, the constructed model will contain only a subset of existing elements and relations that, hopefully, together account for an acceptable part of the structural dynamics of the system of interest. Moreover, one will probably not attempt to come up with the full collection of state variables for every particular reality component. In summary, the model builder will come up with a perceived conceptual universe $\tilde{\Xi}_S$, a perceived conceptual model $\tilde{\xi}_S$ and a collection of state variable spaces for every included component $\{\tilde{\Gamma}_k\}$[22]. Having introduced a metric approach to the notion of validity, how do we measure the distance from a system description in $\tilde{\Xi}_S$ to the true model of reality? Notice that, for an objectivist, the collection $(\dot{\varsigma}_\xi)$ of true models is the singleton $\{\dot{\xi}\}$, residing in a unique space $\dot{\Xi}$ that forms the only member of the collection of valid conceptual spaces $(\dot{\varsigma}_\Xi)$. Conversely, for a relativist, the collection of valid conceptual spaces is a fuzzy set $\hat{\varsigma}_\Xi$ and for every fuzzy space $\hat{\Xi}$, there exists a fuzzy collection $\hat{\varsigma}_\xi$ of possible true conceptual models $\hat{\xi}$. In order to ease notation, let us stick to the fuzzy notation of $\hat{\varsigma}_\Xi$ to denote the collection of true conceptual universes. In case of an objectivist, the membership function of $\hat{\varsigma}_\Xi$ can be thought of as a special function that assigns the value 1 for a single universe and 0 elsewhere. Similarly, let us use $\hat{\varsigma}_\xi(\hat{\Xi})$ to denote the fuzzy collection of true models for a particular conceptual universe. Note that for an objectivist, $\hat{\varsigma}_\Xi \equiv \{\dot{\Xi}\}$ and $\hat{\varsigma}_\xi(\dot{\Xi}) \equiv \{\dot{\xi}\}$. In other words, depending on the particular form of the membership functions, the notation of $\hat{\varsigma}_\Xi$ and $\hat{\varsigma}_\xi(\hat{\Xi})$ can be used for both objectivists and relativists without loss of generality.

Returning to our prime issue of measuring the distance between a point in a proposed space $\tilde{\Xi}_S$ and the collection of true points $\{\hat{\varsigma}_\xi(\hat{\Xi})$, all $\hat{\Xi} \in \hat{\varsigma}_\Xi\}$, let us for the moment assume that $\tilde{\Xi}_S$ is a subspace of every $\hat{\Xi}$ for which $\mu_{\hat{\varsigma}_\Xi}(\hat{\Xi}) > 0$[23], and that for each component X in $\tilde{\Xi}_S$, $\tilde{\Gamma}_X = \hat{\Gamma}_X$. Now, in portraying a conceptual model, defined in a space $\tilde{\Xi}_S$, in a true universe $\hat{\Xi}$, we construct the inverse projection of the conceptual model in $\tilde{\Xi}_S$ into the space $\hat{\Xi}$. This leaves us with a well-defined open set $p^{-1}_{\rightarrow\hat{\Xi}}(\tilde{\xi}_S)$ in $\hat{\Xi}$. Let us denote the open set that equals the inverse projection as the set $\hat{\tau}_{\hat{\Xi}}$ in the space $\hat{\Xi}$. We call the portrayal of $\tilde{\xi}_S$ in $\hat{\Xi}$ $(\hat{\tau}_{\hat{\Xi}})$ the natural extension of $\tilde{\xi}_S$:

<u>*Definition 19:*</u> *(The natural extension of a model)*

The natural extension of a model $\tilde{\xi}_S$ that resides in a space $\tilde{\Xi}_S$ into a space $\hat{\Xi}$ is the open set $\hat{\tau}_{\hat{\Xi}}$ formed by taking the inverse projection of $\tilde{\xi}_S$ into $\hat{\Xi}$.

[22] The tilde embellishment is used to stress the fact the conceptual model and universe are particular perceptions of the model builder.

[23] The symbol μ is used to denote the membership function.

Since we assumed $\tilde{\Xi}_S$ is a subspace of every $\hat{\Xi}$ for which $\mu_{\hat{\varsigma}_\Xi}(\hat{\Xi}) > 0$, and that for each component X, $\tilde{\Gamma}_X = \hat{\Gamma}_X$, measuring the distance from the extended model $p^{-1}_{\rightarrow\hat{\Xi}}(\tilde{\xi}_S)$ to its true equivalent $\hat{\varsigma}_\xi(\hat{\Xi})$ coincides with measuring the distance between an inverse projected open set $p^{-1}_{\rightarrow\hat{\Xi}}(\tilde{\xi}_S)$ and a resident open set $\hat{\varsigma}_\xi(\hat{\Xi})$ of $\hat{\Xi}$. For an objectivist, this procedure is easy and the distance equals the value 0 if the set $p^{-1}_{\rightarrow\hat{\Xi}}(\tilde{\xi}_S)$ covers the true model $\dot{\xi}_{\dot{\Xi}}$. On the other hand, a relativist is confronted with a collection of possible natural extensions. As a matter of fact, for each $\hat{\Xi}$ for which $\mu_{\hat{\varsigma}_\Xi}(\hat{\Xi}) > 0$, we can think of a natural extension of $\tilde{\xi}_S$ into $\hat{\Xi}$. Notice that for a particular $\hat{\Xi}$, the fuzzy set of valid models is given by $\hat{\varsigma}_\xi(\hat{\Xi})$. Hence, the distance between an inverse projected set $p^{-1}_{\rightarrow\hat{\Xi}}(\tilde{\xi}_S)$ into a particular $\hat{\Xi}$ and the resident fuzzy set of valid models $\hat{\varsigma}_\xi(\hat{\Xi})$ becomes a distance between a pair of fuzzy sets. The final measure of validity of $\tilde{\xi}_S$ becomes some aggregate of all individual fuzzy distances, where the validity of each conceptual universe is combined with the distance of $p^{-1}_{\rightarrow\hat{\Xi}}(\tilde{\xi}_S)$ to the fuzzy set of valid models within that universe.

What happens in case some state spaces for components in $\tilde{\Xi}_S$ are not identical to their real equivalents but are subspaces of the latter? More specifically, what happens if one has not only made abstraction of certain reality components but also confined the description of some components to a subset of their state variables? In that case, some $\tilde{\Gamma}_X$ would be a subspace of $\hat{\Gamma}_X$, i.e. some $\tilde{\Gamma}_X \subset \hat{\Gamma}_X$. Let us denote the collection of true state spaces for a particular component X as $\hat{\varsigma}_{\Gamma_X}$ and let us assume that $\tilde{\Gamma}_X$ is a subspace of every $\hat{\Gamma}_X$ for which $\mu_{\hat{\varsigma}_{\Gamma_X}}(\hat{\Gamma}_X) >$ 0. Applying an analogue of the natural extension in the Γ space, we associate with each component description in $\tilde{\Gamma}_X$ a collection of descriptions in each $\hat{\Gamma}_X$ for which holds that $\mu_{\hat{\varsigma}_{\Gamma_X}}(\hat{\Gamma}_X) > 0$. In other words, we construct the inverse projection of elements of $\tilde{\Gamma}_X$ into $\hat{\Gamma}_X$, i.e. we consider $p^{-1}_{\rightarrow\hat{\Gamma}_X}(\tilde{\gamma}_X)$ for each $\hat{\Gamma}_X$. Since in general there exists a fuzzy homeomorphism between the spaces Ξ_X and Γ_X, every element of $\tilde{\Xi}_X$ is matched with a(n) (fuzzy) open subset $\hat{\tau}_{\hat{\Xi}_X}$ of every possible space $\hat{\Xi}_X$. We will refer to this process of matching as the natural extension of a model component:

Definition 20: *(The natural extension of a model component)*
The natural extension of a description of a model component $\tilde{\Xi}_X$, that is described in a space $\tilde{\Gamma}_X$, is formed by taking the inverse projection of the description $\tilde{\xi}_X$ into the space $\hat{\Gamma}_X$.

In the assumption that $\tilde{\Xi}_S$ is still a subspace of every $\hat{\Xi}$ for which $\mu_{\hat{\varsigma}_\Xi}(\hat{\Xi}) > 0$, but that there exists now a component X in $\tilde{\Xi}_S$ for which $\tilde{\Gamma}_X \subset \hat{\Gamma}_X$ and that $\tilde{\Gamma}_X$ is a subspace of every $\hat{\Gamma}_X$ for which $\mu_{\hat{\varsigma}_{\Gamma X}}(\hat{\Gamma}_X) > 0$, what can we tell about the distance of a conceptual model in $\tilde{\Xi}_S$ towards its collection of real equivalents? Similarly to the natural extension of a model, we consider the inverse projections of mapped elements of $\hat{\Xi}_X$ into each possible $\hat{\Xi}$. Logically, the removal of a state variable in addition to an entire component, induces a superset of inverse projections in each $\hat{\Xi}$ in relation to the set that was generated by the sole removal of the component. Once again, for an objectivist, the construct "each $\hat{\Xi}$" is read as the singleton $\dot{\Xi}$. Only if the set of inverse projections covers the single true model in $\dot{\Xi}$, the proposed model $\tilde{\xi}_S$ is called valid. For a relativist, inverse projections can be made for each possible $\hat{\Xi}$. The distance between the set of inverse projections $\{ p^{-1}_{\rightarrow\hat{\Xi}}(\tilde{\xi}_S)\}$ and its family of true equivalents in a particular $\hat{\Xi}$ is a distance between a set of inverse projections and a fuzzy resident set of true models in $\hat{\Xi}$. Again, an aggregation operator is needed to combine the validity of conceptual universes $\hat{\Xi}$ and the fuzzy distance from the set of inverse projections to its family of true equivalents in $\hat{\Xi}$.

5.2. Type Ξ, type ξ and type Γ errors

In the previous paragraph, we deliberately restrained our analysis to conceptual models and universes that are completely contained in every of their possible true equivalents. As a matter of fact, we indicated how the distance concept should be applied when comparing models that reside in a subspace of one another. It lies within the aim of this paragraph to extend our discussion of the distance concept when models may reside in spaces that are completely different, i.e. that do not necessarily form subspaces of one another. In an attempt to compose an overview of modelling errors, let us clarify the concept of an error first. Consider at a particular moment in time the collection of true conceptual universes $\hat{\varsigma}_\Xi$ and true conceptual models of reality $\{ \hat{\varsigma}_\xi(\hat{\Xi})$, all $\hat{\Xi} \in \hat{\varsigma}_\Xi\}$. For a foundationalist, we state an error occurs at the very moment a proposed description of a reality component deviates from its true counterpart, at the moment the proposed conceptual universe contains non-existing objects or relations (the universe becomes partially complex[24]) or at the moment the collection of state variables that were defined for a particular component is a superset of the true set of state variables (the state space is partially complex). As a matter of fact, we call these types of errors respectively type ξ, type Ξ and type Γ errors.

Complex conceptual universes or state spaces and translated conceptual models form the topological representatives of *foundational* errors. But what about an *anti-foundationalist*? We claim that the same types of errors may occur (complexity and translation) but that their relation towards validity is different. Notice carefully that the occurrence of errors and their relation to validity are highly unpredictable for an anti-foundationalist. In some cases, it may be that models containing a vast amount of errors are more useful, acceptable or robust than others. So, in the context of relativism, their degree of validity may be higher than models that would contain fewer errors. Notice also that, in some operational context, a model containing only a few errors may be less useful or acceptable (valid) than in another operational environment. The amount of errors is in that way no indication at all of the adherence to the set of valid models. Additionally, the adherence to the set of valid models is a relative concept, highly depending on the operational context of the model. So, as a conclusion, foundationalists and anti-foundationalists will agree on the existence and the interpretation of the concept of an error, but in contrast to the former, the latter will see the accumulation of errors as a modification of a model's validity, represented by its fuzzy distance membership function.

Type	Source of error
Ξ	inclusion of non-existing elements or relations
ξ	mathematical misspecification of relations or inclusion of entity description errors
Γ	inclusion of non-driving causal factors or non-existing state variables

Table 1: Possible sources of modelling errors

In Table 1, we listed possible sources of errors for both foundationalists and anti-foundationalists, which we shall discuss below.

[24] The notion of complexity of a conceptual universe resembles the notion of complex numbers in standard algebra.

5.2.1. Type Ξ error

An analyst commits a type Ξ error if he includes objects, entities or relations in his conceptual model that do not reside in any part of reality. More specifically, a type Ξ error occurs if there exists at least one possible conceptual universe that does not include the proposed component by the model builder. Formally:

<u>*Definition 21*</u>: *(A type Ξ error)*

In proposing a conceptual universe $\tilde{\Xi}_S$*, we state an error of type Ξ occurs* $\Leftrightarrow \exists\ \hat{\Xi}, \mu_{\hat{\varsigma}_{\Xi}}(\hat{\Xi}) > 0$*:* $\tilde{\Xi}_S \not\subset \hat{\Xi}$.

From a collection of observational data, the analyst may have derived a causal influence among a set of variables that essentially does not exist[25]. Likewise, the model builder may erroneously have concluded the presence of a particular system component from inspected behaviour. The frequency of type Ξ errors, caused by induction of absent relationships, heavily depends on the cognitive robustness of the model builder. How strong will the analyst's conceptual determination resist not to induce non-existing relationships from apparent correlational behaviour? It is likely that, approaching a phenomenon of reality by a subset of elements and relations, the combined influence of both included and omitted reality elements and relationships will prevent the analyst from coming up with a justified collection of causal dependencies.

<u>*Example 4a*</u>: *A foundational illustration of a type Ξ error*

Suppose that in a fictive queuing system, there exists a relation between the length of the queue (Q_λ), the activation of a second server ($S2_\alpha$) and the average working rate of the primary servant ($S1_\omega$). The relation, as it resides in reality is depicted in the left side of the figure below.

[25] Taking a queuing process as an example model, the analyst might incorrectly claim that the average number of customer arrivals per hour is functionally dependent of the number of sunshine hours during the morning. Although reality may utter a level of correlation between sunshine hours and average customer arrival rate, in case a true causal relation is absent, the inclusion of any such kind of relationship leads to a type Ξ error.

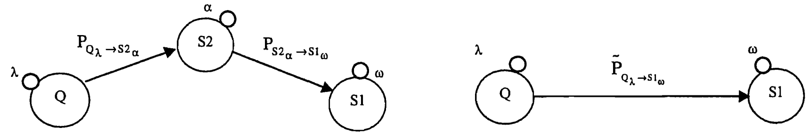

Figure 2: (Left) Present relation between Q_λ, $S2_\alpha$ and $S1_\omega$ (Right) Perceived relation between Q_λ and $S1_\omega$.

Suppose now that a system analyst is given the opportunity to construct a model of the above queuing system and that he comes up with a model in which the existence of the second server is excluded. Either he decided the server plays a minor role in the dynamics of the queuing system, either he is completely ignorant regarding the presence of the second servant. In any case, exclusion of the server will inevitably lead to an exclusion of both relationships $P_{Q_\lambda \to S2_\alpha}$ and $P_{S2_\alpha \to S1_\omega}$. Moreover, as a result of the residing causal influences, there will probably be an apparent correlation between the queue length (Q_λ) and the working rate of server 1 ($S1_\omega$). Consequently, it is not unthinkable the analyst will come up with a causal dependency of working rate on queue length, i.e. he will infer a relation $\tilde{P}_{Q_\lambda \to S1_\omega}$. As a result of his simplified look on the queuing phenomenon, an error of type Ξ has occurred. Whether the effect of ignoring server 2 will stop at the sole, erroneous inference of the relationship $\tilde{P}_{Q_\lambda \to S1_\omega}$ remains however to be seen. It is not improbable that the analyst doesn't recognize the correlation between Q_λ and $S1_\omega$ as being causal and instead concludes there must be some kind of operator that controls the speed of server 1. In that case, the inclusion of an operator involves the inclusion of a non-existing object and hence a type Ξ error will once again occur.

In the previous example, we indicated that the act of leaving out certain objects (or relations) in a model is largely responsible for inducing other, non-existing objects or relations. From a foundational point of view, as a result of this inclusion of essentially absent components, the retained conceptual universe $\tilde{\Xi}_S$ is partially a subspace of $\dot{\Xi}$ and partially complex (not existing in $\dot{\Xi}$). What about the foundational validity of a model $\tilde{\xi}_S$ that resides in the complex space $\tilde{\Xi}_S$? Attempting to make the natural extension of $\tilde{\xi}_S$ into $\dot{\Xi}$ results in a degeneration. As an unfortunate consequence of the inclusion of a non-existing component, we state that, for a foundationalist, the model $\tilde{\xi}_S$ as a whole is invalid. As a matter of fact, there exists no real natural extension of $\tilde{\xi}_S$ that is completely contained in $\dot{\Xi}$. For analysing the repercussion of a complex space on anti-foundational validity, let us rephrase our example as follows:

Example 4b: An anti-foundational illustration of a type Ξ error

Consider the same collection of objects as in the previous example: Q (queue), S1 (server 1) and S2 (server 2). However, let us imagine the situation where it is unclear whether the speed of server 1 is influenced by the activation of server 2 or by the queue length. In contrast to our foundational example, we thus state that the precise set of components that define this phenomenon in reality remains a fuzzy picture. Where the relationships $P_{Q_\lambda \to S2_\alpha}$ and $P_{S2_\alpha \to S1_\omega}$ determined reality in a foundational context, we are now confronted with a triplet of possible relations that might individually, combined or all together constitute the causal interdependencies within our phenomenon. In the figure below, we indicated the undeterminance about the existence of these relations by an additional arc on each relationship.

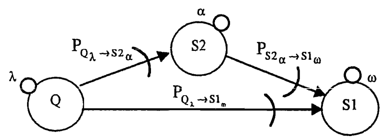

Figure 3: Undeterminance about the true nature of a queuing phenomenon

Suppose now additionaly that, in analyzing the phenomenon, one has agreed that it is either the direct influence of queue length or the indirect activation of server 2 that accounts for the speed modification of server 1, but certainly not both. In other words, there exists a pair of conceptual universes (one containing the single relation $P_{Q_\lambda \to S1_\omega}$, the other containing the relations $P_{Q_\lambda \to S2_\alpha}$ and $P_{S2_\alpha \to S1_\omega}$) that are believed to be possible universes explaining the phenomenon. In light of these considerations, when will a type Ξ error then occur? As a matter of fact, choosing a particular influence path (direct or indirect) automatically induces an error since there exists a possibility that the retained choice is incorrect. In other words, there exists no real natural extension of models that follow the direct path into the universe of indirect path models and vice versa. So choosing a particular influence scenario invokes an error in the context of a particular world view. Notice carefully since both possible conceptual world views do not share any relation, there exists no method to accurately describe the phenomenon in all possible shades of reality awareness. What happens now if the coexistence of both direct and indirect influence would become possible? In that case, choosing for a double causal dependency of service rate (on both queue length and activation) does not invoke any error in light of the world view that accepts this coexistence. Notice that in light of the other views, incorporating both direct and indirect relationships will invoke a type Ξ error. And still, there exists no conceptual model that would fit all shades of reality awareness. What happens if only the coexistence of both types of influences would constitute the single valid world view? In that case, we can construct for every model the natural extension into the space, representing the coexistence of the relations. In that respect, no type Ξ error will occur when modelling either the direct, the indirect or the combined path.

From an anti-foundational point of view, the validity repercussion of including particular objects or relations becomes a world view dependent issue. At the very moment that there exists a possible worldview that does not allow for the existence of an included component, we state an error of type Ξ has occurred. Notice that within that worldview, the inverse projection of the proposed model results in a degeneration.

5.2.2. Type ξ error

A type ξ error occurs when an analyst failed to correctly position an object with respect to a particular description variable, or when he felt short in an accurate revelation of the true expression that governs a specific relationship. Of course, the interpretation of the constructs "accurate" and "true" are scholar dependent. For an objectivist, the position of the conceptual model of the analyst is simply shifted or translated somehow along the particular dimension that represents the misspecified relation or erroneous object description. As a result, the distance between true and proposed description raises from absolute 0 to 1 and the model is called invalid due to the occurrence of a type ξ error. For a relativist however, altering a component description will still translate a conceptual model but the effect of this translation on validity is different. Since the collection of true models in a particular conceptual universe is a fuzzy set, it is likely a conceptual shift will induce an alteration of the membership value to the set of valid models.

<u>*Definition 22*</u>: *(A type ξ error)*

In proposing a conceptual model $\tilde{\xi}_S$, we state an error of type ξ occurs $\Leftrightarrow \exists\ \hat{\xi}, \hat{\Xi}, \mu_{\hat{\varsigma}_{\Xi}}(\hat{\Xi}) > 0 \wedge$

$\mu_{\hat{\varsigma}_{\xi}(\hat{\Xi})}(\hat{\xi}) > 0 : \overset{-1}{p}_{\rightarrow\hat{\Xi}}(\tilde{\xi}_S) \cap \hat{\varsigma}_{\xi}(\hat{\Xi}) = \varnothing.$

The following example is an extension of the fictitious queuing system launched previously to illustrate the occurrence of a type ξ error.

<u>*Example 5a*</u>: *A foundational illustration of a type ξ error*

Let us extend the example of the previous section and assume the presence of an operator who permantly controls the activation of the second server in addition to the causal relation $P_{Q_\lambda \rightarrow S2_\alpha}$. If, for some reason, the operator or his relation in regard to the activation of server 2 is not recognized in the model of an analyst, what will be the repercussion of this omission on the mathematical specification of the relations $P_{Q_\lambda \rightarrow S2_\alpha}$ and $P_{S2_\alpha \rightarrow S1_\omega}$? Since the activation behaviour of server 2 is not completely explainable by looking at the queue length factor alone, the mathematical formula that represents $P_{Q_\lambda \to S2_\alpha}$ will probably become confounded with the operator's relation $P_{O_\chi \to S2_\alpha}$ into a perceived dependency $\tilde{P}_{Q_\lambda \to S2_\alpha}$. A similar reasoning leads to an incorrect representation of $P_{S2_\alpha \to S1_\omega}$ ($\tilde{P}_{S2_\alpha \to S1_\omega}$). In any case, the conceptual model as inferred by the model builder will probably be displaced from its true equivalent. The figure below illustrates the occurrence of a misspecification of the relations between the queue and server 2, and server 1 and server 2. In addition to the notation that was introduced previously, a condition variable χ is defined that, if true, fires an "entity" along the relationship $P_{O_\chi \to S2_\alpha}$, causing the activation of server 2 irrespective of the queue length. In the right portion of the figure, the relation $P_{O_\chi \to S2_\alpha}$ is not recognized and, as a probable consequence, neither will be the condition variable χ. Moving from left to right in Figure 4, the occurrence of a type ξ error becomes apparent in the altered functional specifications of the residual causal relationships.

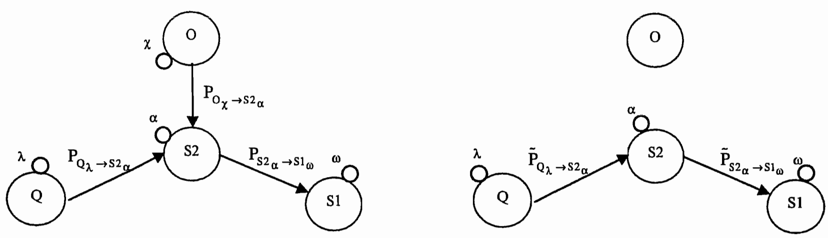

Figure 4: Omission of a relationship leads to incorrect specifications of causal dependencies

Example 5b: An anti-foundational illustration of a type ξ error

In an anti-foundational context, let us assume that every possible conceptual universe contains the components O, Q, S1 and S2, the relations $P_{O_\chi \to S2_\alpha}$, $P_{Q_\lambda \to S2_\alpha}$ and $P_{S2_\alpha \to S1_\omega}$ and the state variables χ, λ, ω, α, ρ_1, ρ_2 and ρ_3[26]. In other words, universes that would not contain at least one of these elements are believed to be impossible (they have no support in the set of valid universes). Now, differing from our foundational discussion, suppose the true mathematical expression representing a relationship remains a fuzzy set on a domain of possible functional dependecy relations.[27] In the figure below, we indicated this level of fuzziness surrounding the true model by means of a fuzzy set of valid functional dependency forms for each causal relation. In light of this "world view" scenario, when will an error of type ξ occur? As a matter of fact, in the event one would construct a model with particular expressions for the residing causal relations for which there is no support in the fuzzy set of valid models for a particular conceptual universe, an error of type ξ takes place. So, in search of an occurrence of a type ξ error, one should look at every possible conceptual universe individually and decide on the support of valid models for the

[26] The state variables ρ_1, ρ_2 and ρ_3 represent the state variables of the relations $P_{Q_\lambda \to S2_\alpha}$, $P_{S2_\alpha \to S1_\omega}$ and $P_{O_\chi \to S2_\alpha}$.

[27] Let us assume there is no argument about the true description of the objects in the model.

proposed relations within that universe. Of course, the decision on occurrence of type ξ errors is not limited to an investigation of the validity of relationships alone. As a matter of fact, one should commit the same validity study for every object description as well.

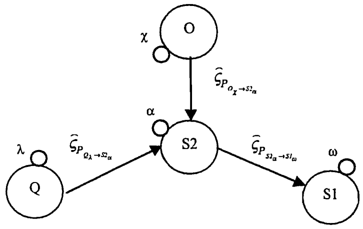

Figure 5: Fuzziness concerning the true expressions for relations

Comparing the foundational with the anti-foundational attitude towards type ξ errors, we state that for an objectivist, deciding on the occurrence of a type ξ error is essentially an objective, straightforward task to perform (if one is in possession of the foundation in relation to which the validity of theories can be verified). Having knowledge about the single true conceptual universe and model, all that matters is that the proposed model fits its true equivalent within the system's universe that was retained for studying the phenomenon. For a relativist however, the decision is more complicated. Being confronted with a collection of possible true universes and models, he should verify the support of the proposed model within this entire collection. If there exists support in at least one universe, the model is (more or less) valid in at least one view of the world. However, if there exists a universe for which there is no support at all for the proposed model, a type ξ error occurs (within the context of that universe). Notice that, for a given model that does not contain any type ξ error, modifying a single component description may result in a series of type ξ errors depending on the validity of models in the collection of possible conceptual universes.

5.2.3. Type Γ error

Where a model builder can commit errors with respect to the set of retained system components and component descriptions, it is only natural to extend the error concept to the state space as well. We state an error of type Γ occurs if one has essentially come up with a state space for a component (object or relation) that does not have any support in at least one space of the collection of true state spaces. Formally:

Definition 23: *(A type Γ error)*

In proposing a state space $\tilde{\Gamma}_X$ to describe a particular component X, we state an error of type Γ occurs $\Leftrightarrow \exists\ \hat{\Gamma}_X,\ \mu_{\hat{\varsigma}_{\Gamma_X}}(\hat{\Gamma}_X) > 0 : \tilde{\Gamma}_X \not\subset \hat{\Gamma}_X$.

In other words, a type Γ error occurs if a description of a component is defined on a domain for which there is no support in at least one member of the set of all valid domains. For relations, this means one may have included a non-driving factor in a multivariate relationship[28] or one may have erroneously retained the mathematical expression for the relationship as a state variable. For objects, type Γ errors indicate one has extended the state space with one or more variables that essentially do not constitute state variables[29]. Analogue to the theory of complex numbers, we claim that the modelled reality component consists both of a real part and an imaginary part. Any fine positioning of one's conceptual model along the universe of the perceived component can be decomposed into a real and an imaginary part. The degree of realism of the retained component is nothing else than the amount of curvature that occurred from the true component towards the imaginary universe. In other words, picturing the conceptual universe of the analyst in a true conceptual universe for which there exists no support for the retained state space, leaves us with a non-orthogonality. Let us illustrate the occurrence of type Γ errors using an extension of our small factory example.

Example 6: *A foundational illustration of a type Γ error*

Let's extend once again our factory example and suppose in addition there is a repair worker (R) who inspects articles coming from server 1, repairs them if necessary and sends them to server 2. Furthermore, let's assume the working rate of server 1 is directly determined by two causal factors. First, as soon as the error rate of goods produced by server 1 surpasses a critical level, the server's speed is reduced. Second, the activation of the repair

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[28] Suppose that in a queuing system the analyst has determined that not only the length of the queue but also the number of sunshine hours during the morning influences the service rate of the single active server. If however in reality, the number of sunshine hours has nothing to do with the service rate, the analyst has come up with a non-driving factor in the dependency of service rate.

[29] In the same queuing system, the length of the single queue is some complex result of a collection of state variables of the system. Queue length is thus not a state variable and retaining it as such would involve a conceptual error. Similarly, in an earlier example on network traffic, the relation between network traffic and server activation had no state variable. Retaining the functional form between traffic and activation as a state variable would involve once again a conceptual error.

worker will slightly increase the server's speed since in the case of errors, the repair worker (instead of the server itself) will handle the production fault. Figure 6 contains a visual overview of the reality components and relations that comprise our small factory system.

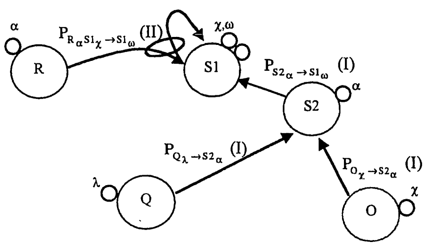

Figure 6: Conceptual model of a small manufacturing system

For clarification, we indicated the number of *causal factors* (I or II) that define the domain of each relationship in reality. In summary, the following causal relationships are thus present among the system components queue (Q), repair worker (R), server 1 and 2 (S1, S2) and the operator (O):

$P_{R_\alpha S1_\chi \to S1_\omega}$ (II):	the working rate of S1 is adjusted ($S1_\omega$) in function of the activation of R (R_α) and a condition variable χ of S1 ($S1_\chi$)
$P_{O_\chi \to S2_\alpha}$ (I):	S2 is activated ($S2_\alpha$) as soon as a condition variable of the operator becomes true (O_χ)
$P_{S2_\alpha \to S1_\omega}$ (I):	activation of S2 ($S2_\alpha$) automatically leads to a decrease of the working rate of S1 ($S1_\omega$)
$P_{Q_\lambda \to S2_\alpha}$ (I):	queue length at S1 (Q_λ) triggers the activation of S2 ($S2_\alpha$)

Now, since the speed of server 1 will, unavoidably, be related to the queue length, it is not unthinkable a model builder would infer an additional causal factor (queue length) in the relation $P_{R_\alpha S1_\chi \to S1_\omega}$ and come up with a causal dependency $\tilde{P}_{Q_\lambda R_\alpha S1_\chi \to S1_\omega}$ that is defined on the domain $Q_\lambda \times R_\alpha \times S1_\chi$. Since the function space $\tilde{\wp}_{\tilde{\Phi}\to\Delta}$, where $\tilde{\Phi} \equiv Q_\lambda \times R_\alpha \times S1_\chi$, is essentially an imaginary extension of the true function space $\wp_{\Phi\to\Delta}$, $\Phi \equiv R_\alpha \times S1_\chi$, there exists no *real* image of any relation defined in $\tilde{\wp}_{\tilde{\Phi}\to\Delta}$. (Attempting to project elements of $\tilde{\wp}_{\tilde{\Phi}\to\Delta}$ into $\wp_{\Phi\to\Delta}$ results in a degeneration.) However, we might create a complex extension of functions defined on Φ by taking the inverse projection of any function in $\wp_{\Phi\to\Delta}$ towards $\tilde{\wp}_{\tilde{\Phi}\to\Delta}$. Hence, the collection of all extended functions forms a subset of $\tilde{\wp}_{\tilde{\Phi}\to\Delta}$. Similar to the theory of complex numbers, we state that the class of "complex" functions $\tilde{\wp}_{\tilde{\Phi}\to\Delta}$ are the result of a confounding between an imaginary part $\tilde{\wp}_{(\tilde{\Phi}\setminus\Phi)\to\Delta}$ and a real part $\wp_{\Phi\to\Delta}$. To use the geometric terminology, functions that have a non-nill complex part are rotated at some non-zero angle with respect to their real part. The amount of rotation indicates the amount of complex presence and confounding that has occurred.

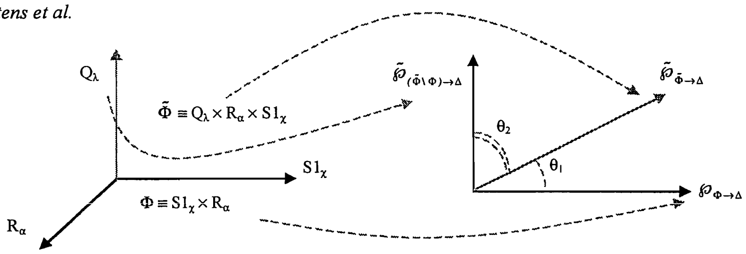

Figure 7: (Left) Causal relationship domains (Right) Real, complex and imaginary causal relation

Notice that in the figure above, we did not draw the concrete perceived, true and imaginary relations but instead the templates (the dimensions) of which these concrete relationships could be called an instantation. Notice also that the angle between perceived and imaginary relationship (θ_2) determines the amount of influence from the included causal factor. The smaller the angle, the more powerful is the influence of the additional causal factor and the more "leakage" will occur from the imaginary influence towards the perceived influence. In other words, the smaller the angle, the more a level of rational confounding will have occurred. It should be clear from the figure that the perceived conceptual universe $\tilde{\Xi}_S$ is non orthogonal in Ξ_S. The amount of non-orthogonality (space curvature) indicates the amount of leakage that has occurred from non-existing, but included causal influences to perceived relationships. Of course, in practice, leakage can occur towards many relationships and it is likely the space will be curved in many directions.

6. Structural validity revisited

In the advance of our argument, we defined a topological resemblance of the philosophical scholarships of objectivism and relativism in the context of modelling and validation. We considered models as points in a conceptual space and introduced a metric approach to validation. We indicated the yardstick for measuring validity is determined by ones philosophical adherence and discussed the repercussion on validity of several types of modelling errors that might occur for both objectivists and relativists. At this moment, an engaging obstacle remains how we can finally express the structural validity of a proposed model/universe pair ($\tilde{\xi}_S, \tilde{\Xi}_S$)?

6.1. Foundational structural validity

Remember that for a foundationalist, modelling errors come down to a complexity (Ξ) or curvature (Γ) of the conceptual universe or a displacement (ξ) of the conceptual model. How do we measure the distance then between a proposed model $\tilde{\xi}_S$ that might have been inflicted with any of those error types and the true model $\dot{\xi}$? First, we should attempt to make the projection of $\dot{\xi}$ onto the retained system (S) universe $\tilde{\Xi}_S$. If however $\tilde{\Xi}_S$ is not a subspace of $\dot{\Xi}$, the attempt to project $\dot{\xi}$ onto $\tilde{\Xi}_S$ results in a degeneration. Within the system universe $\tilde{\Xi}_S$ retained, there simply does not exist a unique image of the true model of reality $\dot{\xi}$. As a matter of fact, it is the conceptual space $\tilde{\Xi}_S$ that is in fact invalid. In a foundational context, a complex conceptual space automatically induces the invalidity of any model defined in that space. Besides the complexity of $\tilde{\Xi}_S$, the space may also be curved. Similarly, $\dot{\xi}$ cannot be uniquely projected on the curved space $\tilde{\Xi}_S$. Indeed, the position of $\dot{\xi}$ regarding the included, non-existing state variables or causal factors remains inconclusive. Therefore, in a foundational context, curvature of a proposed conceptual space automatically induces invalidity of any residing model. Second, if the attempt to project $\dot{\xi}$ on $\tilde{\Xi}_S$ does not fail, it remains to be seen whether the proposed model coincides with our projection. Only if the model and the projection are identical, the model/universe pair $(\tilde{\xi}_S, \tilde{\Xi}_S)$ is called structurally valid. Formally:

<u>*Definition 24*</u>: *(Foundational structural validity)*
The structural validity of a proposed model $\tilde{\xi}_S$, that resides in a conceptual system universe $\tilde{\Xi}_S$, is given by the truth value of the expression:

$$\tilde{\Xi}_S \subset \dot{\Xi},\ \tilde{\Xi}_S \text{ is } \perp \text{ and } \delta(p_{\rightarrow\tilde{\Xi}_S}(\dot{\xi}), \tilde{\xi}_S) = 0, \text{ with } \delta \text{ the trivial metric.}$$

Notice that the formal definition of objective, foundational structural validity involves a satisfaction of a triplet of validity conditions. The retained system universe must be an orthogonal subspace of the true conceptual universe of reality (the foundation) and the distance, measured by means of the trivial metric, between proposed model and projected truth on the retained universe must equal the value zero. If any of these conditions is violated, the proposed model/universe pair $(\tilde{\xi}_S, \tilde{\Xi}_S)$ is called invalid. If all conditions are satisfied, the truth-value of the expression equals 1 and the model is believed to be structurally valid. The alternative scenario is that one or more conditions are violated which invokes a truth-value of 0, or a verdict of structural invalidity.

6.2. Anti-foundational structural validity

For an anti-foundationalist, modelling errors came down to a complexity (Ξ) or curvature (Γ) of the retained conceptual universe, or a displacement (ξ) of proposed conceptual model in at least one possible conceptual universe. How do we express now the validity of the same proposed model/universe pair ($\tilde{\xi}_S, \tilde{\Xi}_S$) in a relative, anti-foundational context? As a matter of fact, the true foundation ($\dot{\Xi}$) becomes a fuzzy set $\hat{\varsigma}_\Xi$ of true conceptual universes ($\hat{\Xi}$). The structural validity of a model is then defined in regard to each of those conceptual universes for which their membership value to the set $\hat{\varsigma}_\Xi$, $\mu_{\hat{\varsigma}_\Xi}(\hat{\Xi})$, is strictly higher than null. Notice that the contours of the fuzzy set $\hat{\varsigma}_\Xi$ (the expression for the membership function $\mu_{\hat{\varsigma}_\Xi}$) are context dependent and are tuned by an aggregate of different world views of the model stakeholders, by the particular goal of the modelling study, by the operational environment, etc... So, for a particular conceptual universe $\hat{\Xi}$, $\mu_{\hat{\varsigma}_\Xi}(\hat{\Xi}) > 0$, we define the structural validity of a proposed model/universe pair ($\tilde{\xi}_S, \tilde{\Xi}_S$) formally as follows:

<u>*Definition 25*</u>: *(Anti-foundational structural validity)*
The structural validity of a proposed model $\tilde{\xi}_S$, that resides in a conceptual system universe $\tilde{\Xi}_S$, with respect to a conceptual universe $\hat{\Xi}$, $\mu_{\hat{\varsigma}_\Xi}(\hat{\Xi}) > 0$, is given by δ^* with a fuzzy truth value determined by the expression:

$$\tilde{\Xi}_S \subset \hat{\Xi},\ \tilde{\Xi}_S \text{ is } \perp \text{ and } \delta(p_{\rightarrow\tilde{\Xi}_S}(\hat{\xi}), \tilde{\xi}_S) = \delta^*, \text{ with } \delta \text{ a fuzzy trivial metric.}$$

Notice the distinction between objective and relative, anti-foundational structural validity. The outcome of a relative validation study is non-binary, but a fuzzy validity indication on a zero-one scale. For every δ^*, $\delta^* \in [0,1]$, we evaluate, in addition to the orthogonal subspace conditions, the degree to which the distance between proposed model and projected truth equals δ^*. The structural validity of a model is then believed to be δ^* with a truth-value of the expression as given in the above definition. So, in contrast to objectivism, there is now a continuum of validity states a model can take on. The binary zero/one distance concept is extended to the entire unit interval [0,1]. Further, instead of attaining a crisp, objective outcome of a validation study, the structural validity is believed to be a fuzzy number on the unit interval. There exists now a continuum of structural validity expressions[30] that are evaluated by the satisfaction of the condition of an orthogonal subspace and the membership value to the calculated fuzzy distance.

7. Conclusion and future research

In this paper, we reflected on the issue of model validity in the context of simulation. In an early section, we discussed different kinds of validation and explained their relation with verification and accreditation. For a more profound investigation into the issue of model validity, it seemed inescapable to have a look at the classic philosophical duality of objectivism versus relativism, which plays a central role in the validation of any scientific theory. In an attempt to consolidate this polarity in our reasoning of validity, we formulated a topological interpretation of both a foundational and an anti-foundational attitude towards structural validation. As we have demonstrated, in an objective context, there exists a unique triplet of conceptual universe, model and state spaces that constitutes the foundation to which the validity of proposed simulation models can be revealed. In this objective context, we derived the natural topology on the conceptual universe is the discrete topology, induced by the trivial metric. The natural metrisation of the conceptual universe, together with the objective uniqueness of truth allowed us to define objective, structural validity as the truth value of an expression, embodying the orthogonality of the retained system universe, its containment in the true universe of reality as well as the equality of the proposed model with projected truth. In a relativistic context however, we indicated the foundation to validate models becomes relative, context dependent and related to the particular worldviews of the model stakeholders. The objective uniqueness of the true triplet of universe, model and state spaces, was questioned. In this relative context, we derived the natural topology on the conceptual universe is the fuzzy discrete topology, induced by a fuzzy trivial metric. This metrisation of the conceptual universe allowed us to define a fuzzy set of valid models with respect to a particular worldview. We defined relative, structural validity as a fuzzy number on the unit interval, portraying a fuzzy perception concerning the validity of the proposed model.

By approaching the matter of validation through the pathway of a fuzzy measure of model validity, a challenge for future research remains to come up with a technique that allows for such a fuzzy appraisal of validity. Moreover, we restricted our analysis to the matter of structural validation only. It is again an issue of additional investigation to examine the topological version

[30] These expressions take on the form *"structural validity equals $\tilde{\delta}$ with truth-value x"*.

of behaviour validity and to study the relation between structural and behavioural validity. In summary, it seems most fruitful to direct future research towards the development of a practical technique that enables a fuzzified assessment of both structural and behavioural validity.

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