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Discrete element simulations of the influence of fertiliser physical properties on the spread pattern from spinning disc spreaders

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This paper describes a sensitivity study of the flow of granular fertiliser particles on a spinning disc using a discrete element model. The aim was to get a qualitative insight in the influences of individual physical properties of the particles (such as friction coefficient, restitution, and shape) as well as their bulk behaviour on the resulting spread pattern. The results show that certain particle properties, particularly friction coefficients, have a large influence on the spread pattern, and hence should be examined carefully in the process of producing granular fertilisers, perhaps by particle coating, or taken into account when applying the fertiliser in the field. Other properties, such as particle stiffness hardly affect the results. Furthermore, it was shown that the friction coefficient and shape of a particle strongly interfere in their particular influence on the spread pattern. Overall, the discrete element model could provide a powerful instrument for the manufacturers in the development of new kinds of spreaders and fertilisers.

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1. Introduction

Manufactures of fertiliser spreaders have, over many years, built up experience in obtaining the optimal machine settings for various types of fertilisers and application conditions. However, expensive calibration methods for the spread pattern remain essential for this purpose. Modelling, to improve the operation and control of centrifugal fertiliser spreaders in agriculture, dates back to the pioneering work of Patterson and Reece (1962); Inns and Reece (1962); Mennel and Reece (1963); Dobler and Flatow (1968) and later Pitt *et al.* (1982). Renewed interest in the 1990s resulted in a series of Ph.D. studies on the subject (Adjroudi, 1993; Tissot, 1995; Olieslagers, 1997) and research papers (Hofstee, 1995; Olieslagers *et al.*, 1996; Reumers *et al.*, 2003; Dintwa *et al.*, 2004, and

Villette *et al.*, 2005). These studies showed machine properties and settings, such as the radius or the rotational speed of the disc, were not the only factors that influence the behaviour of fertiliser grains during spreading. It is commonly known by farmers, and been reported by authors (Hofstee, 1995) that modifying the particle friction coefficient or application rate influences the spread pattern. However, as all the spreader models noted above are based on the accumulation of single particle trajectories with small changes in initial conditions, they are unable to provide understanding of the influence of particle interactions on particle flow (Olieslagers, 1997).

The modelling of an interactive particle flow on a centrifugal spreader was introduced by Van Liedekerke *et al.* (2006, 2008) using discrete element modelling (DEM) simulations. Validation of the model was carried out for using single

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Nomenclature

A	(cross) sectional area [m^2]
c_N	damping coefficient in Kono–Kuwabara model (Kuwabara and Kono, 1987) [$\text{N s m}^{-3/2}$]
d	distance between the two particles of a cluster [m]
e	restitution coefficient
I	inertia tensor [kg m^2]
$k_{N,T}$	normal or tangential particle stiffness [$\text{N m}^{-3/2}$]
m	mass [kg]
m_c	mass of cluster particle generated by overlapping spheres [kg]
N	normal contact force [N]
n	normal direction vector
p	model output
R	particle radius [mm]
$\bar{R}$	average particle radius [mm]
R_s	particle radius [mm]
t	tangential direction vector
T	tangential contact force [N]
v	particle velocity [m s^{-1}]
$\bar{v}$	average particle velocity [m s^{-1}]
v_0	initial downward velocity (IDV) of the particles in the bin [m s^{-1}]
v_{12}	relative velocity of two colliding particles [m s^{-1}]

X	model parameter
S	model sensitivity
α	compartment angle in CSP [$^\circ$]
$\bar{\alpha}$	average compartment angle in CSP [$^\circ$]
β	width of cylindrical spread pattern [$^\circ$]
$\delta_{N,T}$	normal or tangential virtual overlap component [m]
χ	cluster eccentricity
μ	friction coefficient
μ_{wp}	wall-particle friction coefficient
μ_{pp}	particle-particle (internal) friction coefficient
ρ	particle density [kg m^{-3}]
ρ_b	bulk density [kg m^{-3}]
Ψ	bulk flow [kg s^{-1}]
σ_v	standard variation on the particle velocities [m s^{-1}]
σ_R	standard variation on the particle radius [mm]
ω	angular velocity vector of the particle [rad s^{-1}]
Ω_d	disc rotational speed [rpm] or [rad s^{-1}]

Abbreviations

CSP	cylindrical spread pattern
DEM	Discrete Element Method
IDV	initial downward velocity

particle and particle flow experiments. It was found that, despite fertiliser particle properties being difficult to measure, and exact experiment conditions (e.g. initial conditions and machine geometries) almost impossible to simulate, generally acceptable agreement can be found between experimental and simulated spread patterns. In this paper, DEM simulations will be used to explore the sensitivity of the spread pattern to the physical properties of fertiliser (e.g. particle friction coefficients, stiffness, diameter, and shape). It should borne in mind that the main goal is to develop trends and not exact predictions concerning the influence of these properties on particle flow on the disc and their effect on the particle distribution at the edge of the disc.

2. DEM computational procedure

Current fertiliser spreader hoppers commonly hold from 10^4 to 10^8 particles that are non-spherical and whose initial conditions are only approximately known to (i.e. they are located in a bin prior to being dropped onto the spinning disc). As a result, for computational reasons, the DEM procedure has the following restrictions:

1. Spheres are used as basic particles with a radius normal distribution $N(\bar{R}, \sigma_R)$ with $\bar{R} = 1.26 \text{ mm}$, and $\sigma_R = 0.15 \text{ mm}$.
2. The spreader geometry is constructed in three-dimensions using simple surfaces, such as cylinders, rectangles, and discs.
3. Each simulation uses 7000 particles. Particles are kept in a cylindrical “bin”. Particles crossing the disc edge are recycled to the cylinder where they randomly (by gravity)

fall back into the particle stack (see Fig. 1). In this way, the number of particles in the simulation remains constant. Once the particles reach the bin orifice (approximately 10 mm above the orifice edge), their initial downward velocity (IDV) is fixed to a predetermined value. However, from the orifice edge downwards, particles are further accelerated by gravity. In this way, the bulk flow of the particles through the bin can be controlled by the IDV. This mechanism was described more in detail in Van Liedekerke et al. (in press).

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In DEM simulations, the interaction force model has normal n and tangential directions t . The normal interaction force is based on Hertz–Kono–Kuwabara model (Kuwabara and Kono, 1987), while the tangential force is a general “slip-stick” model (Schäfer et al., 1996). Thus

$$\begin{cases} T = -\text{sign}(v_{12})\min(k_T\delta_T, \mu|N|)t & \text{(normal force)} \\ N = -\min\left(0, |\delta_N|^{1/2}(k_N\delta_N + c_N\dot{\delta}_N)\right)n & \text{(tangential force)} \end{cases} \quad (1)$$

where T is the tangential contact force and N is the normal force. The quantity δ_N , termed the “virtual overlap”, is computed as the difference of the distance of the two particles and the sum of their radii. The model parameter list is determined by $k_{N,T}$ ($k_N = k_T$ for the simplicity) and c_N , the non-linear contact stiffness and damping parameter respectively, and μ is the sliding friction coefficient, which plays a crucial role in the rolling-sliding regime of the particles (see Section 4). A graphical representation of two bodies in contact according to DEM is shown in Fig. 2. We refer to Van Liedekerke et al. (2006, in press) and Schäfer et al. (1996) for more details on the principles of a DEM simulation.

Fig. 1 – (a) Dimensions of the standard spinning disc (plan view) and vanes used in the DEM simulations. (b) Particle recycling to the bin after they leave the disc.

For irregular particle shapes, instead of using a rolling friction coefficient (Brilliantov and Pöschel, 1998), a two-sphere cluster is proposed. Using a rolling friction coefficient in the equations of motion only adds a proportional correction to the moment of forces acting on the particle. Therefore this approach has less physical meaning. However, even a perfect sphere has a rolling resistance (which largely depends on the Young modulus of the particle), but this was not incorporated in the model. Sphere clusters have been investigated by various researchers to account for irregularly shaped particles in granular flow or to model cohesive forces among particles. Favier et al. (1999), Vu-quoc et al. (2000) studied rigid sphere clusters, and Bucholtz and Pöschel (1994) bonded (or cohesive)

spheres. In this paper rigid clusters, containing two spheres of equal radius R_s , as represented in Fig. 3 were considered. Although irregularly shaped particles lose their freedom of free rolling, two-sphere clusters still have a degree of freedom for rolling, because of their axial symmetry. Introducing a third sphere, not lying on this axis, could eliminate this degree of freedom, but it was not used here because computationally it is too expensive. The mass of the clusters was assumed to equal to that of the initially used spheres. In all cases, we restricted the distance d between the spheres along to the X-axis to $d \leq 2R_s$, where R_s is the particle radius. We define χ as:

$$\chi = d/2 \quad (2)$$

Generally, one calculates the mass of two intersecting spheres m_c by volume integration of each in X direction, limited by χ :

Fig. 2 – Definition of the normal and tangential contact force on two bodies in contact according to DEM.

Fig. 3 – (a) A two-sphere cluster; (b) type B cluster; (c) type A cluster.

$$m_c = 2\rho \int_{-R_s}^x \int_{-\sqrt{R_s^2-x^2}}^{\sqrt{R_s^2-x^2}} \int_{-\sqrt{R_s^2-x^2-y^2}}^{\sqrt{R_s^2-x^2-y^2}} dz dy dx \quad (3)$$

with ρ particle density. For an overlapping sphere cluster, the inertia tensor with respect to the XYZ frame is obtained applying Eq. (3). If the X-boundary for each sphere part ranges from $-R_s$ to χ , then:

$$I = 2\rho \int_{-R_s}^x \int_{-\sqrt{R_s^2-x^2}}^{\sqrt{R_s^2-x^2}} \int_{-\sqrt{R_s^2-x^2-y^2}}^{\sqrt{R_s^2-x^2-y^2}} \begin{pmatrix} y^2 + z^2 & -yz & -(x-\chi)z \\ -yz & (x-\chi)^2 + z^2 & -(x-\chi)z \\ -(x-\chi)z & -(x-\chi)z & y^2 + (x-\chi)^2 \end{pmatrix} dz dy dx \quad (4)$$

Two types of clusters are distinguished in the simulations. The first group is half-overlapping spheres (type A). The mass of a half-overlapping spheres cluster is obtained by setting $\chi = R_s/2$ in Eq. (3). By imposing the restriction on equal mass to a spherical particle ($m = \rho 4/3\pi R_s^3$), we obtain following radius for each sphere:

$$R_s = \sqrt[3]{4/9}R \quad (5)$$

The second group consists of adjoined spheres (type B). The mass of a non-overlapping spheres cluster is obtained by setting $\chi = R_s$ in Eq. (3). Hence, we obtain:

$$R_s = \frac{R}{\sqrt[3]{2}} \quad (6)$$

3. Sensitivity study

For the parameter study, a distributor disc (0.3 m radius) was simulated by a flat circular plate with two vanes (each of 30 mm height) spinning at 52.4 rad s^{-1} (500 rpm). The vanes have horizontal flaps of approximately 10–20 mm to minimise particle overspill. All vane edges had 2 mm thick cylindrical chamfers to avoid discontinuous contacts. A 20 mm radius cylindrical bin was located 70 mm from the disc centre and 100 mm above the disc plane (see Fig. 1).

All particle properties X shown in Table 1 are within a range $X \in [X_{\text{low}}, X_{\text{high}}]$. These values were obtained from measurements (Van Liedekerke et al., 2006, in press). They are

Table 1 – Input model parameter names and their default, low and high values

Parameter	Default value	Low value	High value
P-P friction coefficient μ_{pp}	0.5	0.1	0.7
W-P friction coefficient μ_{wp}	0.35	0.1	0.7
W-P normal stiffness $k_N(\text{N m}^{-3/2})$	1.3E7	1.3E6	1.3E8
Restitution coefficient e	0.5	0.25	0.8
Particle average radius $\bar{R}$ (mm)	1.26 ($\sigma_R=0.15$)	1	2
Bulk flow ψ (kg s^{-1})	0.5	0.25	0.75
Particle shape χ (m)	0	$R_s/2$	R_s

similar to other values found in literature (Hofstee, 1992; Aphale et al., 2003; Grift et al., 2006). For all model parameters, default values are defined, which are used as reference when this specific parameter is kept constant, and others are varying. They could be the average of $[X_{\text{low}}, X_{\text{high}}]$ but this is not particularly necessary. For some parameters, this range is not easy to determine. In particular, for particle-particle (P-P) friction, no accurate data is available and as a result the range could be quite large. However, we can assume that friction coefficients generally are in the range 0.1–0.7. A choice of the low, high, and default input values for the DEM simulations can be seen in Table 1. The sensitivity $S_{p,x}$, applied to this parameter study is defined as the proportion of the model output difference to the relative difference in model input:

$$S_{p,x} = \frac{\Delta p \bar{X}}{|X_{\text{high}} - X_{\text{low}}|} \quad (7)$$

where Δp is the variation on the model output variable p , and $\bar{X} = (X_{\text{low}} + X_{\text{high}})/2$. A more rigorous approach to sensitivity analysis would suggest the method proposed by Saltelli et al. (2004) based on the calculation of the variances of the model input and output should be used. However, despite the usefulness of this approach, many simulations are required (typically 1000) to compute these variances so this method would not be feasible here.

In each simulation, the individual particle coordinates were recorded at the edge of the disc, from which a particle distribution can be derived (called the cylindrical spread pattern or CSP). In these spread pattern measurements, the particles are collected in segmental compartments along the edge of the disc (Reumers et al., 2003). Each compartment is characterised by an angle α , starting from zero (see Fig. 1). The resolution of the segments is usually 10° . In the simulations, after determining in which compartment the particles are

Table 2 – Influence of particle friction coefficients, stiffness, restitution coefficient, flow rate, radius, and shape on the CSP

Particle property	Value	k_N ($\text{N m}^{-3/2}$)	$\bar{\alpha}$ ($^\circ$)	β ($^\circ$)	$\bar{v}$ (m s^{-1})	$\sigma_{\bar{v}}$
W-P friction coefficient, μ_{wp}	0.1		134.9	32.2	20.7	0.75
	0.3		154.1	34.1	19.2	0.79
	0.5		162.9	35.3	18.7	1.04
	0.7		166.1	36	18.6	1.1
P-P friction coefficient, μ_{pp}	0.1		149.4	35.6	19.6	0.71
	0.3		156.0	34.6	19.2	0.81
	0.5		157.9	34.7	19.0	0.86
	0.7		158.7	35.1	18.9	0.89
Restitution/stiffness k_N ($\text{N m}^{-3/2}$)	0.2	4.10^6	158.6	34.1	19.0	0.77
		4.10^8	157.8	34.5	19.0	0.83
	0.8	4.10^6	157.7	35.8	19.0	0.93
		4.10^8	157.2	35.2	19.0	0.96
Flow rate, ψ (kg s^{-1})	0.25		155.8	28.0	19.2	0.73
	0.75		158.0	36.2	19.0	0.91
Particle average radius, $\bar{R}$ (m)	0.001		159.6	35.1	19.0	0.88
	0.002		150.1	38.8	19.2	0.98
Particle shape, χ (m)	Sphere		157.8	34.7	19.0	0.87
	Type A		159.8	32.8	18.9	0.80
	Type B		160.2	31.4	18.8	0.90

located, the model output variables are calculated by using the following quantities:

- The average mass distribution angle along the disc edge, $\bar{\alpha} = \sum f_i \alpha_i$
- The width of the pattern, $\beta = \sqrt{\sum f_i (\alpha_i - \bar{\alpha})^2}$

where α_i is the angular position of each compartment, and f_i are the particle mass fractions in this compartment. From the simulations, one can further extract average particle discharge speed $\bar{v}$.

Before describing the quantitative work using Eq. (7), more insight into the underlying mechanisms between model output and model parameters is given.

In Table 2, the results of the model output are summarised, giving the average angle $\bar{\alpha}$; the width of the cylindrical pattern, β ; the average outlet speed (not counting the particles which bounce with the vane edge), and $\bar{v}$ with standard deviation $\sigma_{\bar{v}}$.

4. Influence of fertiliser particle properties

4.1. Wall-particle (W-P) friction coefficient

Four simulations were run to investigate the influence of the W-P friction coefficient, using values of 0.1, 0.3, 0.5, and 0.7. Meanwhile, the (P-P) friction was kept at 0.5. To characterize the particle distribution at the edge of the disc, the angular positions α and outlet speeds v of the individual particles are studied. In Fig. 4, the particle fractions are plotted as a function of the angular position (i.e. CSP), while in Fig. 5, the speeds $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ are given. From Fig. 4, we deduced that the particle distribution along the disc edge shifted to the right with higher friction. However, this distinction gradually reduced, and there was almost no difference for a friction coefficient of 0.5 compared to one of 0.7. Also, the pattern got slightly broader for higher coefficients.

The graph of particle speed (Fig. 5) showed that low particle speeds occurred at compartment angles close to the location of the bin orifice ($\alpha=0$). Here particles gain little speed because: (1) they are not fully accelerated by the vanes and (2) they bounce on the vane edges in random directions (as seen in the simulations and experiments). Particles trapped in the compartments $\alpha > 120^\circ$ reach full speed ($\sim 20 \text{ m s}^{-1}$) when they leave the disc. Interestingly, this speed does not appear to change much in the further compartments. The increase in speed from $\alpha = 120^\circ$ to 250° was only 0.2 m s^{-1} . To give an idea of the variation in speed, a deviation bar of $\pm 2\sigma_v$ was plotted for the speed curve of $\mu_{wp} = 0.1$. This showed a large variation where $\alpha < 70^\circ$ but very low variations when $\alpha > 150^\circ$. It is most likely, that this is due to the particle scattering caused by bouncing on the edges of the vanes in the region where the particles fall from the bin. In the compartments with high values of α , only particles that were intercepted by the vertical plane of the vanes are collected.

Specifying the influence of friction coefficient, the lowest value results in the highest outlet speeds (20.6 m s^{-1}), but the relationship did not appear to be linear; for higher friction, the decline became less and there was very little difference between a value of 0.5 and one of 0.7. The most likely explanation for this is that an increasing fraction of the spheres starts to roll against the vanes. This behaviour was previously observed in the single particle simulations (Van Liedekerke et al., 2006).

The angular speed was also investigated. In Fig. 6, the mean of rotational speeds $\omega = \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2}$ of the particles are plotted. Despite the high variability in rotation, it is possible that higher wall-particle (W-P) friction coefficients induce higher W-P rotation. However, particles in the middle compartments, where the mass flow density is the highest, have significantly lower rotational speeds than those in the higher angle compartments, where the particle density is lower. This means that the denser the particle flow, the more variability there is in rotational direction. Note also that all the curves appear to converge in the last compartment, to

Fig. 4 – Simulated CSP for particles with different W-P friction coefficients: $\mu_{wp} = 0.1$ (○); $\mu_{wp} = 0.3$ (□); $\mu_{wp} = 0.5$ (★); $\mu_{wp} = 0.7$ (✱).

Fig. 5 – Simulated outlet speeds at the edge of the disc, as a function of the compartment angle for various μ_{wp} : $\mu_{wp} = 0.1$ (○); $\mu_{wp} = 0.3$ (□); $\mu_{wp} = 0.5$ (★); $\mu_{wp} = 0.7$ (*). The deviation bars are given for $\mu_{wp} = 0.1$.

a unique value of $\pm 10,000 \text{ rad s}^{-1}$, which is equivalent to full rolling ($v = \omega R_s$) against the vane at the edge of the disc.

Table 2 shows another interesting fact: the higher friction coefficient, the more dispersion is observed on the outlet speeds. Apparently, when W-P friction decreases, the particle flow appears to behave less erratically, i.e. the apparent spread on the output variables is less. This is probably due to a lower momentum transfer from the particles in contact with the vanes to the rest of the particle flow.

4.2. Particle-particle (P-P) friction

As in Section 4.1, four simulations were run, with P-P friction values ranging from 0.1 to 0.7. The W-P friction was fixed at 0.35. A decrease of the outlet speed, shown in Fig. 7, with

increasing friction was noted, although the effect was less distinct compared to W-P friction.

The same applies to the average outlet angles, given in Table 2. This is probably because the normal forces between particles are generally lower than those between particles and vanes. The normal forces induced by contact with the vanes (the Coriolis force in analytical models) acts in a unique direction, perpendicular to the vane, and consequently the friction force acts uniquely towards the disc centre. For internal particle contact, normal forces act more randomly (although still for a major part towards the vane), and they partially cancel each other out.

Regarding the angular speeds, as shown in Fig. 8 and contrary to what was found in Fig. 6, a decrease in angular speed with increasing friction is observed. In addition, the dispersion appears to be much higher for low friction in the

Fig. 6 – Simulated outlet angular speeds at the edge of the disc, as a function of the compartment angle for various μ_{wp} . $\mu_{wp} = 0.1$ (○); $\mu_{wp} = 0.3$ (□); $\mu_{wp} = 0.5$ (★); $\mu_{wp} = 0.7$ (*). The deviation bars are given for $\mu_{wp} = 0.7$.

Fig. 7 – Simulated outlet speeds at the edge of the disc, as a function of the compartment angle for various μ_{pp} ; $\mu_{wp} = 0.1$ ($\circ$); $\mu_{pp} = 0.3$ ($\square$); $\mu_{pp} = 0.5$ ($\star$); $\mu_{pp} = 0.7$ ($\ast$). The deviation bars are given for $\mu_{pp} = 0.1$.

middle compartments. The explanation for this behaviour can probably be found in the fact that lower friction transfers less angular momentum between the particles, allowing them to spin more freely (and faster) but with a high variation. When the friction is higher the particles transfer more momentum to each other, and neighbouring particles can influence each other because of differences in angular direction, hence lowering the angular speed.

4.3. Particle stiffness and damping

In this section, simulations were run to determine the influence of particle restitution (defined as the ration of the impact velocity to the rebound velocity) and the collision times. Generally, normal particle stiffness k_N prevails at the contact time of the collision, while the particle damping coefficient c_N

mainly influences the restitution coefficient (i.e. energy dissipation). A 2×2 design was constructed with simulations of “stiff particles” with high and low restitution and “soft particles” with high and low restitution coefficients. High and low restitution particles were defined as having $e = 0.8$ and $e = 0.2$, respectively. The elasticity modulus was considered over a range of 100 (i.e. $4 \cdot 10^6$ – $4 \cdot 10^8$) for the stiff particles compared to soft particles. The simulations showed that for low stiffness, slightly higher speeds are obtained in the lower compartments for particles with a higher restitution coefficient. In addition, they appeared to have a greater dispersion. In the later compartments however, the difference in outlet speed was almost negligible. A similar observation, not shown here, can be made for particles with high stiffness.

This behaviour is caused by the fact that elastic particles gain more speed after bouncing with the disc and vane

Fig. 8 – Simulated outlet angular speeds at the edge of the disc, as a function of the compartment angle for various μ_{pp} : $\mu_{wp} = 0.1$ ($\circ$); $\mu_{pp} = 0.3$ ($\square$); $\mu_{pp} = 0.5$ ($\star$); $\mu_{pp} = 0.7$ ($\ast$). The deviation bars are given for $\mu_{pp} = 0.1$ (dotted line) and $\mu_{pp} = 0.7$ (solid line).

(edges). Particles stiffness appears to have very little influence on the spread pattern. Overall, the influence of particle stiffness and elasticity on the particle output speed is low.

For the CSP plots, all curves showed no greater relative difference than 1.5%. Average outlet angles were not affected by stiffness or elasticity. Concerning the pattern width β ; slightly higher values were found for the more elastic particles. The most likely cause is the larger initial area on the disc for the more elastic particles, which lose less energy when colliding with the disc and therefore spread more. Because of the small differences observed, the results are not shown in plots but summarised in Table 2.

4.4. Influence of bulk flow

The mass flow rate is modelled by giving the particles an Δv v_0 in m s^{-1} . The flow rate ψ is calculated by:

$$\psi = \rho_b A v_0 \quad (8)$$

where A is the cross-sectional area of the orifice, and ρ_b is the bulk density of the particle stack in the bin. Two simulations were run, one with a flow of 0.25 kg s^{-1} , and one with 0.75 kg s^{-1} . In Fig. 9, the CSP plot is shown for both simulations. It is evident that the effect of flow on the spread pattern was significant. The most important trend appears to be that the pattern becomes wider in case of higher flow. Olieslagers (1997) observed the same trends in experiments. Most it is likely that the effect is caused by a larger dispersion area on the disc, in the case of high particle flow both in the direction of the centre and the disc edge. Concerning the average angle, the curve for high flow was slightly shifted to the right, although there was only a 1% difference. From the outlet speed plot, not shown here, one can conclude that the curves are hardly influenced by flow rate, which is in agreement with high speed camera measurements (Olieslagers, 1997), but in contrast with the observations of Hofstee (1995), using a Doppler velocity metre, where there was a significant increase in outlet speed. It should be noted that the approach of changing velocity in Eq. (8) is somewhat artificially. In

reality, the mass flow is changed by making the orifice area larger. Nevertheless, one could argue that a common effect is created: the initial area of the particle flow on the disc is increased. In case of larger orifice area, it is evident that more particles will be restricted in their vertical movement and will be forced to spread out horizontally. Consequently, one should interpret these results with some caution. Both methods will probably not yield the same quantitative results. However, a detailed study about mass flow is beyond the scope of this paper.

4.5. Influence of particle size and segregation

4.5.1. Particle size

To investigate the influence of particle size on the spread pattern, DEM simulations were conducted with a 12,000 particle sample with “small” average radius (1 mm, $\sigma_R = 0.15 \text{ mm}$), and a 4000 particle sample with “large” average radius (2 mm). Because the bulk mass density for the large particles is less than that for small particles, a small proportional correction $\rho_{\text{bulk-2mm}}/\rho_{\text{bulk-1mm}} = 0.95$ for the bin particle velocity was made in Eq. (8) in order to maintain the same mass flow rate. From Fig. 10, it can be observed that the CSP curve for coarse particles is shifted to lower α by 9° . And, for larger particle samples, the pattern gets wider.

The plot given in Fig. 11 shows that the speeds of the larger particles are somewhat higher, especially in the first compartment (because of bouncing with the vane edges), and generally show less dispersion than the small particles. It is likely that this is because large particles are fewer in number and experience fewer collisions in the normal contact direction and hence less energy is dissipated per unit of fertiliser mass. A similar phenomenon appears to occur in the tangential direction when the particles are moving along the vane or disc. The effect can be seen clearly in Fig. 12, where a snapshot is given of the simulations with small and large particles. The regions denoted by an ellipse show that for the large particles, more mass is concentrated outwards the disc centre. This is in agreement with Reumers et al. (2003) and

Fig. 9 – Simulated CSP for different particle flows: 0.25 kg s^{-1} (○) and 0.75 kg s^{-1} (*).

Fig. 10 – Simulated CSP for different particle radii: 2 mm (○) and 1 mm (*).

Pettersen *et al.* (1991) who studied experimental spread patterns conducted with different particle sizes.

4.5.2. Segregation

To investigate particle size segregation, a standard simulation was run with a particle distribution uniformly spread over [0.8 2.2] mm. To exclude the particles which are impacted by the vane edges (bouncing of particles), only the particles that are accelerated by the vanes were taken into consideration. At the edge of the disc, the size fractions of the particles were recorded.

In Fig. 13, histograms are given of the fractions $R < 1.1$ mm (grey) and $R > 1.9$ mm (black), for each compartment. Fig. 13 indicates that larger particles tend to be present in the earlier compartments. In the middle compartments, they more or less equalize, while in the later compartments, the smaller

particles outnumber the larger. This result agrees with the segregation measurements, carried out by Olieslagers (1997) with a different particle size distribution, and disc and bin geometry, but comparable rotational disc speed and particle flow.

The explanation for this is probably to be found in the fact that larger particles respond less to other particles. Consequently, when the particle flow impacts the disc, larger particles might break out of the flow more easily than small particles. Particles located at the outer regions reach the disc edge earlier, and are captured in the earlier compartments; while those located near the disc centre agglomerate with the other particles. On the other hand, segregation may start in the bin. This is especially true for the large bins where the particles have time to rearrange. In the simulations, the particles are recycled to the (smooth) cylindrical bin more or

Fig. 11 – Simulated outlet speeds at the edge of the disc for different particle radii: 2 mm (○), deviation bars are solid lines indicated by arrows, and 1 mm (*), deviation bars are dotted lines.

Fig. 12 – Snapshot of DEM simulations with particles having radius 2 mm (a) and particle with radius 1 mm (b).

less randomly. Additionally, because the number of particles (~ 1000) as well as their retention time (~ 0.3 s) in the bin was relatively low, segregation in the bin was likely to be of minor importance in the simulations.

4.6. Particle shape

In Fig. 14, the CSP plot is shown for both type A and type B clusters, in comparison with the pattern for spheres. The major finding from this plot is that the pattern gets narrower as the asphericity increases. This is because of the lower dispersion of non-spherical particles as they hit a flat surface. On the disc, this implies a more coherent particle flow and hence less dispersion of the outlet angle.

Concerning the outlet speeds, shown in Fig. 15, the cluster types have lower speeds in the low angle compartments, but are almost equal to that of the spheres at the higher angle compartments. However, they do not fully roll, as the spheres do, for $\alpha > 240$.

For the outlet angular speeds, as can be seen in Fig. 16, there was generally a decrease as particles became more aspherical. This was expected, because irregular shaped particles oppose each other more in the angular motion compared to spherical ones. However, further investigation revealed that the influence of internal friction diminishes

when using type B clusters. This probably indicates a (partial) reduction in their rotational degrees of freedom because of inter-locking.

4.7. Interaction between fertiliser friction coefficients and shape

Whilst it was initially assumed that the physical properties of the particles are independent, they can reinforce or diminish each other in the model output. Theoretically, numerous combinations of interaction can be obtained. However, here we have only concentrated on the relationship between the W-P friction coefficient and particle shape, because both singly have an important influence on the spread pattern, and they are closely related through the rolling/sliding phenomena. Other interactions between model parameters might be important and could form part of a separate study.

The default W-P friction coefficients are obtained from experiments conducted by Van Liedekerke et al. (2006). Due to the variability in particle texture, the changing moisture content, and the intrinsic measurement errors of the method, a high variability of the friction coefficients can be expected. This is different from the approach used by Grift et al. (2006) or Kweon et al. (2007), where the friction coefficient was determined by measuring the outlet velocity, and was primarily related to single particle trajectories. In that approach, friction coefficients can also incorporate other factors such as particle bouncing effects, particle shape and diameter, and aerodynamic drag. We refer to the former papers for a more detailed discussion.

As the friction force is caused by sliding, and rolling is inherently more difficult for non-spherical particles, it may be expected that the particle shape will interact with the friction coefficient. The results of the effect of W-P friction coefficient and particle shape on the outlet speeds $\bar{v}$ are shown in Fig. 17. Also, the results for the sphere simulations and a single particle motion are shown. From Fig. 17 we conclude that, compared to a single particle trajectory; friction has

Fig. 13 – Histogram of simulated particle fractions by compartment angle, assuming an initial uniform distribution over [0.8 2.2] mm; black: particle radius > 1.9 mm, grey: radius < 1.1 mm.

Fig. 14 – Simulated CSP for spherical particles (○); type A clusters (□), and type B clusters (★) with $\mu_{wp} = 0.35$.

a significantly higher effect on the outlet speed for a particle flow. At $\mu_{wp} = 0.2$ (dashed vertical line 1), the single particle starts rolling and loses no more energy, whereas for particle flow, for all particle types, the sliding regimes persists with the same (linear) effect on the particle speed. At $\mu_{wp} = 0.35$ (dashed vertical line 2) a distinction is formed between spheres, type A, and type B clusters. Spheres begin to roll, whereas the type B mostly continue on sliding. The type A clusters lie somewhat in between these two regimes. From this we may conclude that the influence of shape on the average outlet speeds could be minimal if the friction coefficient is not too large.

A comparison of single particle simulations and multi-particle simulations leads to the major observation that the outlet speeds are lower in case of a particle flow. Furthermore, it appears that a single particle trajectory is only sensitive to friction at low friction coefficients; while for particle flow, it persists longer.

4.8. General discussion

In the previous sections, it was shown that some particle properties play an important role in the CSP distribution, while the effect of others is minimal. In order to obtain a quantitative comparison, the definition of sensitivity $S_{p,x}$ of a parameter X , to a spread pattern characteristic p was defined by Eq. (7), based on “low” and “high” values. For the particle shape, the low and high values are defined by $\chi = 0$ and $\chi = R_s$ (respectively a sphere and an adjoined sphere cluster). The average values are computed from $\bar{X} = (X_{low} + X_{high})/2$. In Table 3 all the calculated sensitivities of the particle properties are given.

Table 3 shows that friction plays a dominant role in the spreading process, especially on the outlet angles and speeds; although it is mainly due to the $W-R$ interaction. The particle radius also affects the outlet angle, and particularly the

Fig. 15 – Simulated outlet speed as a function of the outlet angle for spherical particles (○); type A clusters (□), and type B clusters (★) with $\mu_{wp} = 0.35$.

Fig. 16 – Simulated outlet angular speed as a function of the outlet angle for spherical particles (○), type A clusters (□), and type B clusters (★) with $\mu_{wp} = 0.35$.

Fig. 17 – Simulated outlet speed as functions of the friction coefficient; spherical particles (○), type A clusters (□), and type B clusters (★) with $\mu_{wp} = 0.35$.

dispersion of the distribution, but has no effect on the average outlet speeds. Concerning the dispersion on outlet angle; the latter seems to be very sensitive to the bulk flow rate of the particle, but other parameters are less influenced by this. The restitution coefficient, as well as the particle stiffness seems have little effect on the CSP.

In conclusion, our simulations show that all particle properties differ in their relative importance to the spreading process. Therefore they should be treated with specific care during the quality control of fertiliser production. Also, as some fertiliser properties are difficult to measure and are highly variable (e.g. particle shape and friction coefficients), the accuracy of the model will be highly restricted with a strong relation to their sensitivity. However, using a DEM model it could be possible to compute quantitative limits.

Table 3 – Simulated sensitivities of the CSP characteristics to fertiliser properties

	$S_{\alpha,x}$	$S_{\beta,x}$	$S_{\bar{v},x}$
μ_{wp}	18.4	2.3	1.2
μ_{wp}	5.2	0.4	0.4
$k_N(\text{N m}^{-3/2})$	0	0	0
e	0	0.2	0.8
$\psi (\text{kg s}^{-1})$	2.8	8.2	0.2
$\bar{R} (\text{m})$	11.7	4.4	0
$\chi (\text{m})$	1.2	1.65	0.1

5. Conclusions

In this paper, DEM simulations were used to investigate the model sensitivity to parameters that influence the spread pattern. These parameters concern the properties of individual fertiliser particles as well as their bulk flow (rate). Some of the results of this study were compared with tendencies found by other authors.

Several authors (Patterson and Reece, 1962; Hofstee, 1995; Olieslagers, 1997; Aphale et al., 2004) have concluded that friction is one of the main fertiliser properties that influence the spread pattern. The DEM simulations agree with these conclusions, but show that some nuances:

- The more spherical the particles are the less sensitive the particle flow is for higher μ_{W-P} friction.
- Internal particle friction has little influence on the spread pattern compared to μ_{P-P} friction.
- For higher flow rates, μ_{W-P} friction becomes more important.
- Higher μ_{W-P} friction causes particles to spread further in the field.

A variation in the coefficient of restitution showed that it has very little influence on the CSP, and produces a wider pattern with more elastic particles. However, no variation in the outlet speeds was observed. Furthermore, the stiffness of the particles does appear to have any significant effect on the spread pattern. It is likely that the normal interaction forces are less important and the energy loss due to these interactions is low compared to the tangential contact force, which is largely influenced by friction.

Another important parameter for the spread patterns is the particle size. It was shown by simulations that larger particle samples tend to spread out further along the disc edge, while they are also forced to leave the disc at lower outlet angles. In agreement with experiments, segregation occurs in the simulations, as bigger particles tend to leave the disc earlier than the smaller fractions.

It was clearly shown that an increase in flow rate results in a broader mass distribution along the disc edge, but outlet speeds remain almost unaffected if the disc rotational speed of the disc is not reduced by the increased mass load.

The effect of particle shape was investigated by the use of two-sphere clusters with a distinction made between half-overlapping and adjoined spheres. It was found that the more irregular the particles, the more coherent the particle flow (i.e. less variable in speed and position). Furthermore, it was shown that outlet speeds are only influenced when the friction coefficient exceeds approximately 0.35, because all types of particles slide until this point. When friction increases, a bifurcation develops among all particle types and from this point irregular particles persist in sliding, whereas more spherical particles start rolling.

This research shows that all particle properties differ in their relative importance for the spreading process and therefore some should be treated with specific care in the fertiliser production quality control, or in the spreading application itself. Although the results of the simulations exhibit the same tendencies as found in experiments, we

stress that the results were obtained with a simplified model and hence should be treated with caution. More specifically, following areas need investigation:

- Comparing the particle flow obtained from the artificial orifice with a real orifice and bin.
- Studying of the cluster representation of irregular particles (particle shape).
- P-P friction coefficients, despite their importance in the model, are difficult to measure.
- Validation en search for alternative contact force models as presented.

Further particle flow and single particle experiments validating the model are therefore necessary. Therefore, ongoing research with recent techniques such as presented by Villette et al. (2008) and Kweon et al. (2007) could assist in this.

However, it is clear that DEM simulations could be potentially helpful tool in the design and optimisation of granular fertiliser application.

Uncited references

Dintwa, 2006; Mohsenin, 1986.

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REFERENCES

- Adjroudi R (1993). Comportement d'un flux de particules solides hétérogènes sous l'action d'un lanceur rotatif. [Behaviour of a flux of heterogeneous solid particles during treatment by a centrifugal spreader]. Ph.D. thesis, Institut National Agronomique, Paris-Grignon, France.
- Aphale A; Bolander N; Park J; Shaw L; Svec J; Wassgren C (2003). Granular fertiliser particle dynamics on and off a spinner spreader. *Biosystems Engineering*, **85**(3), 319–329.
- Brilliantov N V; Pöschel T (1998). Rolling friction of a viscous sphere on a hard plane. *Europhysics Letters*, **42**, 511–516.
- Bucholtz V; Pöschel T (1994). Numerical investigation of the evolution of sandpiles. *Physical A*, **202**, 390.
- Dintwa E; Van Liedekerke P; Tijssens E; Ramon H (2004). Model for simulation of particle flow on a centrifugal fertilizer spreader. *Biosystems Engineering*, **84**(4), 407–415.
- Dintwa E (2006). Development of accurate contact force models for use with Discrete Element Method (DEM) The Discrete element Method (DEM) modelling of bulk fruit handling processes. Ph.D. thesis, Faculteit Landbouwkundige en Toegepaste Biologische Wetenschappen, K.U.Leuven, Belgium.
- Dobler K; Flatow J (1968). Berechnung der würfgänge beim schleuderdüngerstreuen. [Calculation of particle trajectories, forced by a centrifugal spreader]. *Grundlagen der Landtechnik*, **18**(4), 129–164.

- Favier J F; Abbaspour-Fard M H; Kremmer M; Raji A O (1999). Shape representation of (axisymmetrical) non spherical particles in DEM using multi-element model particles. *Engineering Computations*, **16**(4), 467–480.
- Grift T E; Kweon G; Hofstee J W; Piron E; Villette S (2006). Dynamic friction coefficient measurement of granular fertiliser particles. *Biosystems Engineering*, **95**(4), 507–515.
- Hofstee J W (1995). Handling and spreading of fertilizers part 5: the spinning disc type fertilizer spreader. *Journal of Agricultural Engineering Research*, **62**, 143–162.
- Hofstee J W (1992). Handling and spreading of fertilizers part 2: the physical properties of fertilizer, measuring methods and data. *Journal of Agricultural Engineering Research*, **53**, 141–162.
- Inns F M; Reece A R (1962). The theory of the centrifugal distributor II: motion on the disc, off-centre feed. *Journal of Agricultural Engineering Research*, **7**(4), 345–353.
- Kuwabara G; Kono K (1987). Restitution coefficient in a collision between 2 spheres. *Japanese Journal of Applied Physics*, **26**(8), 1230–1233.
- Kweon G; Grift T E; Miclet D (2007). A spinning-tube device for dynamic friction coefficient measurement of granular fertiliser particles. *Biosystems Engineering*, **97**(2), 131–143.
- Mennel R M; Reece A R (1963). The theory of the centrifugal distributor III: particle trajectories. *Journal of Agricultural Engineering Research*, **8**(1), 78–84.
- Mohsenin N (1986). *Physical Properties of Plant and Animal Materials*. Gordon and Breach.
- Olieslagers R; Ramon H; De Baerdemaeker J (1996). Calculation of fertilizer distribution patterns from a spinning disc spreader by means of a simulation model. *Journal of Agricultural Engineering Research*, **63**, 137–152.
- Olieslagers R (1997). Fertilizer distribution modelling for centrifugal spreader design. Ph.D. thesis, Faculteit Landbouwkundige en Toegepaste Biologische Wetenschappen, K.U.Leuven, Belgium.
- Patterson D E; Reece A R (1962). The theory of the centrifugal distributor I: motion on the disc, near-centre feed. *Journal of Agricultural Engineering Research*, **7**(3), 232–240.
- Petterssen J M; Svendsen J A; Ovland S A (1991). Method of studying the influence of fertilizer particle size on the distribution from a twin-disc spreader. *Journal of Agricultural Engineering Research*, **50**(4), 291–303.
- Pitt R E; Farmer G S; Walker L P (1982). Approximating equations for rotary distributor spread patterns. *Transactions of the ASAE*, **25**(6), 1544–1552.
- Reumers J; Tijssens E; Ramon H; De Baerdemaeker J (2003). Experimental characterisation of dynamical properties of fertiliser grains in a centrifugal disk spreader: towards an alternative for spreading hall measurements. *Biosystems Engineering*, **86**(3), 327–337.
- Saltelli A; Tarantola S; Campolongo F; Ratto M (2004). *Sensitivity Analysis in Practice. A Guide to Assessing Scientific Models*. John Wiley & Sons Publishers.
- Schäfer J; Dippel S; Wolf D E (1996). Force schemes in simulations of granular materials. *Journal de Physique (France)*, **6**, 5–20.
- Tissot S (1995). Analyse de l'homogénéité de la repartition transversale des engrais épanchés par des distributeurs centrifuges. [Analysis of the homogeneity of the transverse distribution of fertiliser distributed by centrifugal spreaders.] Ph.D. thesis, Faculté des Sciences Agronomiques de Gembloux, Belgium.
- Van Liedekerke P; Tijssens E; Dintwa E; Anthonis J; Ramon H (2006). A discrete element model for simulation of a spinning disc fertilizer spreader I: single particle simulations. *Powder Technology*, **170**(2), 71–85.
- Van Liedekerke P; Tijssens E; Dintwa E; Rioual F; Vangeyte J; Ramon H. DEM simulations of the particle flow on a centrifugal fertilizer spreader. *Powder Technology* (in press).
- Villette S; Cointault F; Piron E; Chopinet B (2005). Centrifugal spreading: an analytical model for the motion of fertiliser particles on a spinning disc. *Biosystems Engineering*, **92**(2), 157–164.
- Vu-Quoc L; Zhang X; Walton O R (2000). A 3-D discrete-element method for dry granular flows of ellipsoidal particles. *Computer Methods in Applied Mechanics and Engineering*, **187**, 483–528.