
Cindy Bastiaensen, Student Member, IEEE, Wim Deprez, Student Member, IEEE, Wim Symens, and Johan Driesen, Member, IEEE

Abstract—This paper studies measurement uncertainty propagation and parameter sensitivity based on a torque-estimation model for induction machines. The model is based on the equation that describes the interaction of rotor flux and rotor currents. Contrary to classical schemes for induction motor control, this is an open-loop scheme; however, the model still requires different machine parameters. Therefore, the parameter sensitivity of the model is performed. For validation, the model is implemented in the real-time environment dSPACE, and a test induction machine is subjected to different combinations of speed and torque profiles. The identified model can be used to replace mechanical torque-measurement devices or as a backup for a low-cost torque sensor.

Index Terms—Drives, induction machines, measurement uncertainty propagation, open-loop schemes, parameter sensitivity, torque-estimation.

I. INTRODUCTION

Reliable torque-measurement equipment is expensive, and often, calibration and maintenance are also required. In some applications, for a variety of reasons, the overhead of torque-measurement devices is unwanted or not optimal, and sometimes, there is even no space to equip the application with such systems. In most cases, it is sufficient to make an accurate estimation of the torque. Many expressions for the electromechanical torque of an induction machine can be found [1]–[6]. Based on basic formulas that require electrical quantities and, sometimes, mechanical speed, the mechanical torque can be estimated. In this paper, it is illustrated that such torque-estimation expressions could be used to replace mechanical torque-measurement devices. Moreover, the result of the estimation model can be used as a backup for a less-accurate but low-cost torque sensor. The model, which has been previously presented in [17], is based on the equation that describes the interaction of rotor flux and rotor currents; therefore, contrary to classical schemes for induction motor control, this is an open-loop scheme. However, as in all observers, control schemes, or drives for induction machines, different (machine) parameters are required [1]–[8]. Therefore, the parameter sensitivity of the used torque-estimation model is studied. The determination of the required motor parameters is based on the classical T-equivalent circuit and the corresponding no-load and locked-rotor tests [7]–[11].

The paper is structured as follows: Section II describes the developed torque-estimation model. This model requires motor parameters. How these parameters are determined is elucidated in Section III. The model validation is performed in the first part of Section IV. The second part of this section studies the parameter sensitivity. Finally, Section V of this paper describes how the propagation of measurement uncertainties through the model influences the accuracy of the estimated torque.

II. TORQUE-ESTIMATION MODEL

There are several expressions that describe the developed electromechanical torque of an induction machine [1]–[6]. However, they all originate from the same general description of three-phase ac machines, namely, the stator and rotor voltage equations, with \( R_s \) and \( R_r \) representing the resistance of the stator and the resistance of the rotor windings (stator referred), respectively

\[
\begin{align*}
\left( \begin{array}{c} u_{sa} \\ u_{sb} \\ u_{sc} \end{array} \right) &= R_s \left( \begin{array}{c} i_{sa} \\ i_{sb} \\ i_{sc} \end{array} \right) + \frac{d}{dt} \left( \begin{array}{c} \psi_{sa} \\ \psi_{sb} \\ \psi_{sc} \end{array} \right) \quad (1) \\
\left( \begin{array}{c} u_{ra} \\ u_{rb} \\ u_{rc} \end{array} \right) &= R_s \left( \begin{array}{c} i_{ra} \\ i_{rb} \\ i_{rc} \end{array} \right) + \frac{d}{dt} \left( \begin{array}{c} \psi_{ra} \\ \psi_{rb} \\ \psi_{rc} \end{array} \right) \quad (2)
\end{align*}
\]

with \( i_{ra} \), \( i_{rb} \), and \( i_{rc} \) being the phase currents in the rotor, \( i_{sa} \), \( i_{sb} \), and \( i_{sc} \) being the phase currents in the stator, \( u_{ra} \), \( u_{rb} \), and \( u_{rc} \) being the rotor phase voltages, \( u_{sa} \), \( u_{sb} \), and \( u_{sc} \) being the stator phase voltages, \( \psi_{ra} \), \( \psi_{rb} \), and \( \psi_{rc} \) being the rotor flux, and \( \psi_{sa} \), \( \psi_{sb} \), and \( \psi_{sc} \) being the stator flux. In case of squirrel-cage induction machines, the phase voltages of the rotor are zero. These equations can be simplified according to quite-established transformations in electrical engineering [1], namely, the transformation of rotor and stator variables to the \( \alpha/\beta \) reference frame (Clarke transformation) [12] and the transformation to the \( dq \) reference frame (Park transformation) [13]. Depending on the chosen reference frame, this can lead to different expressions for the torque. The expression for torque,
which has been obtained according to a reference frame rotating with the rotor flux, becomes

\[ T_{el} = p(\psi_{rq}i_r - \psi_{rd}i_q) = p\frac{L_h}{L_r} \psi_{rd}i_q = p\frac{L^2_h}{L_r}i_{\mu}i_q \quad (3) \]

with \( i_{\mu} \) being the magnetizing current, \( p \) being the number of pole pairs, \( L_h \) being the magnetizing inductance, and \( L_r \) being the rotor leakage inductance. The torque-estimation model, which was built in Simulink, is based on this expression; see Fig. 1. Note that this is an open-loop scheme, although there is an internal feedback. The input for the model consists of two currents \( i_a \) and \( i_b \) and the mechanical rotor speed. The block “phase_rotorflux” performs the Clarke and Park transformations. The block “angle_rotor” integrates the mechanical speed to obtain the rotor angle in radians. Equation (3) is concretized by the block “torque,” and in “fluxmodel,” the angular velocity of the rotor flux and the magnetizing current are calculated from the rotor equations

\[ L_r \frac{di_{\mu}}{dt} + i_{\mu} = i_d \quad (4) \]

\[ \omega_{\mu} = \omega_r + \frac{R_r}{L_r} \frac{i_q}{i_{\mu}}. \quad (5) \]

**III. PARAMETER DETERMINATION**

As can be noticed from (3)–(5), the torque-estimation model requires three important machine parameters: the magnetizing inductance \( L_h \), the rotor resistance \( R_r \), and the rotor leakage inductance \( L_r \) from the single-phase \( T \)-equivalent circuit representation of induction machines. The elements of this equivalent scheme can be obtained by basic straightforward calculations based on a short-circuit and a no-load test. However, to obtain correct torque-estimates, the skin effect in the rotor bars and, to a minor extent, temperature corrections should be taken into account. There are several ways to consider this skin effect [14], [15]. Anyhow, owing to the nature of this effect, the size (and shape) of the conductors is of importance; therefore, at least the height of the rotor bars should be known to be able to correct the rotor impedance [14], [15]. Such a correction is provided in the proposal for an International Electrotechnical Commission standard [11] for the determination of the quantities of equivalent circuit diagrams. In the first proposal, the rotor bar height was estimated as being 20% of the motor frame size in millimeters. The second proposal estimates the height as

\[ h = \left(0.21 - \frac{2p}{100}\right) \times \text{motor frame size}. \]

This is an understandable practice since this way, no manufacturer data are required. However, in this application, both proposals are too inaccurate. In the aspiration of obtaining precise measurements, the design data are necessary to bring into account the skin effect. Table I gives an overview of the errors made by using one of the proposals. The first column shows the used formula for the rotor bar height, the second column shows the corresponding value for \( h \), the third column shows the calculated torque for rated currents and speed, and the fourth column shows, in terms of percentage, the deviation of the rated torque.

<table>
<thead>
<tr>
<th>formula ( h )</th>
<th>( h ) [m]</th>
<th>( T_c ) [Nm]</th>
<th>% ( T_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 * motor frame size</td>
<td>0.0264</td>
<td>23.27</td>
<td>-4.94</td>
</tr>
<tr>
<td>((0.21 - \frac{2p}{100}) \times \text{motor frame size})</td>
<td>0.0251</td>
<td>23.57</td>
<td>-3.73</td>
</tr>
<tr>
<td>design data</td>
<td>0.0219</td>
<td>24.18</td>
<td>-1.21</td>
</tr>
</tbody>
</table>

In the standard determination of the parameters, temperature corrections can be also taken into account; however, 25°C is used as reference temperature. Small deviations from this reference temperature will hardly influence the determined motor parameters, and therefore, the environment parameters can be neglected in the torque-estimation model.
IV. VALIDATION AND PARAMETER SENSITIVITY

A. Model Verification

Once a set of parameters is determined for a particular induction machine, the torque-estimation model can be implemented in a real-time environment so that the torque can be estimated online. The model was validated by using the real-time tool dSPACE and by an in-house DSP-based measurement and rapid-prototyping platform [16]. For the validation, a test machine was subjected to different combinations of speed and torque profiles. The currents are measured and read in, and the speed is measured by a tachometer, but it could also be estimated or measured by other means. For the case where a tachometer is used, the angular position can be directly determined, and so, the block “angle_rotor” can be omitted. The torque is measured by a torque transducer.

In Fig. 2, the result of such a validation experiment is depicted. The machine under test for this particular case was an off-the-shelf 7.5-kW premium-efficiency two-pole induction machine with a rated speed of 2925 r/min. The rated current is 13.7 A, and the rated torque is 24.48 Nm. To the machine under test, a sinusoidally changing load at low frequency was applied. When this load change has completed one period, the speed is changed stepwise. During each period, the speed is kept constant. In Fig. 2, only part of the complete experiment is plotted. It is the part where there is a speed reversal. The speed is depicted in Fig. 2 in the subplot above, and the measured torque (black) and the estimated torque (gray) are plotted in the subplot below. There is quite some noise on both torque signals, but nevertheless, it can be found that the estimate perfectly coincides with the measurements. Other test profiles confirmed these findings, although they also revealed some problems at very low speeds. This will be investigated in the future; however, since this is not relevant for this paper, it will not be discussed.

B. Parameter Sensitivity

Now, the sensitivity of the torque estimation model to inaccuracy or changes of the different parameters is investigated. In this analysis of the parameters, the influence of the different parameters on the estimated torque is simulated, first, by changing in turn one of the parameters and, second, by holding in turn one of the parameters. The first set of simulations is carried out for rated conditions. From the first approach, varying only one parameter, it can be concluded that the order of the parameters according to the sensitivity on the calculated torque is $R_r > L_h > L_r$. Figs. 3–5 show the results of the second approach. On the vertical axis, the calculated torque is plotted in newton meters, whereas on the two axes composing the horizontal plane, the deviations of the respective parameters are plotted in percentage of their rated values. These figures show that $L_h$ and $L_r$ are opposing parameters, that $L_h$ and $R_r$ strengthen each other, and that the influence of $L_r$ is negligible compared with the influence of $R_r$. In this context, it should be mentioned that the measurement accuracy of the tests described in Section III is very important for the accuracy of the parameter determination. In this paper, the measurement accuracy was assessed based on the “per-point” residual of the least squares fitting of the
supplementary losses. The supplementary losses are calculated according to [10].

Until now, the susceptibility of the model to parameter inaccuracy was only studied for rated conditions. However, since the model should equally perform for nonrated conditions, simulations for parameter sensitivity are also performed at different speeds. Table II gives an overview of the sensitivity of the motor parameters as a function of speed. The first column shows the used percentage of the speed, and the second, third, and fourth columns show, in terms of percentage, the deviation of the torque (calculated at each speed without deviation on the parameters) for a deviation of, respectively, $+10\%L_r$, $+10\%L_h$, and $-10\%R_r$. The last column shows the combination of the third and fourth columns. Only this combination is made with a view to the expression for torque (1).

From these simulations, some important conclusions can be drawn. First, the influence of an inaccuracy on $L_r$ on the estimated torque is negligible and even decreases with the speed. For $L_h$ and $R_r$, the sensitivity is not negligible. An overestimation of $L_h$ causes the torque to be overestimated. The overestimation becomes more serious with decreasing speed. Evidently, the torque is underestimated for a negative deviation of this parameter; see Fig. 6. The behavior for an overestimation or underestimation of $R_r$ is more complicated. This is manifested in the last two columns of Table II and Fig. 6. When the deviation on $R_r$ is negative, the torque is underestimated at rated speed, but there is a transition, and at low speeds, the torque is overestimated.

From the table, it is clear that errors or deviations on the parameters of such a model can lead to serious overestimations or underestimations of the torque, particularly at low speeds. In fact, at low speeds, the errors are amplified. A simultaneous deviation of 10% on two parameters can lead to an overestimation of 33% at low speeds.

V. PROPAGATION OF MEASUREMENT UNCERTAINTIES

The influence of measurement uncertainties on the behavior of the model is handled in this section. This is necessary to check if the model does not amplify the measurement uncertainties as they propagate through it. To assess this influence, a different approach from that in Section IV is chosen. The model requires two currents and the mechanical rotor speed as input. The values of the currents are obtained from measurements. Since the measurement devices are calibrated, the confidence interval is known and can be found in the manual. This also counts for the speed measurement. For the current measurement, the width of the confidence interval $a$ is given as $a = 2\%$ of reading. For the speed measurement, the width of the confidence interval $b$ is expressed as $b = \pm 1\, \text{r/min}$. The uncertainty propagation is investigated for rated conditions; for the currents, this also means that the amplitudes of $i_a$ and $i_b$ have to be 120 electrical degrees out of phase. Since the probability distributions of the inputs are known, a Monte Carlo simulation can be performed. Fig. 7 depicts the resulting torque distribution for the conditions described above. To obtain the plot, 10,000 samples are taken. The samples are independently taken in each confidence interval, and as for each current measurement, a separate current clamp was used. The phase angle between the currents was kept constant. An important remark is that an additional effect on the estimated torque occurs when there is an uncertainty on the values of $i_a$ and $i_b$. The fact is that the calculated torque oscillates with a frequency of 100 Hz (with 50 Hz being the rated electrical frequency). The amplitude of the torque oscillation depends on the amplitude difference of $i_a$ and $i_b$; this is not visible in Fig. 7 since the RMS value of the calculated torque is plotted. A similar oscillation occurs when
the phase shift between $i_a$ and $i_b$ is not exactly 120 electrical degrees. The amplitude of the 100-Hz torque oscillation linearly varies with the phase shift. The current measurement devices used in the laboratory have a rather low accuracy. Therefore, the simulations are repeated for more accurate current devices, which results in a smaller confidence interval width. The width applied in this new simulations corresponds to $a = 0.2\%$ of reading. The plot is shown in Fig. 8. From the figure, it is clear that it is proper to invest in an accurate current measurement device, since the measurement uncertainty influence on the torque significantly decreases.

VI. CONCLUSION

The susceptibility of a torque estimation model to parameter deviations and measurement uncertainties has been studied. The model has been briefly discussed, and it has been illustrated that in general, it gives a good estimation of the actual torque. However, several simulations revealed that, particularly at low speeds, the sensitivity of the model for parameter faults is not negligible.

REFERENCES


Cindy Bastiaensen (S’04) was born in Belgium in 1981. She received the M.Sc. degree in mathematics, with an option in mathematical physics and astronomy, from the Katholieke Universiteit Leuven, Leuven, Belgium. She is currently working toward the Ph.D. degree with the research group ELECTA, Department of Electrical Engineering (ESAT), Katholieke Universiteit Leuven.

Her research topics include stochastic simulation in electrical energy techniques. Currently, she is working on probabilistic analysis of induction machines.

Wim Deprez (S’02) received the M.Eng. degree in electromechanical engineering and material sciences from the Group T, University College Leuven, Leuven, Belgium, and the M.Sc.Eng. degree in electrical engineering from the Katholieke Universiteit Leuven (K.U. Leuven), in 2002. He is currently working toward the Ph.D. degree with the research group ELECTA, Department of Electrical Engineering (ESAT), K.U. Leuven.

He is currently a Research Assistant with ELECTA. His research interests include energy-efficient motor technologies, motor design, motor testing, and motor standards. In that context, he is a member of the Standards for Energy Efficiency of the Electrical Motor Systems (SEEEM) International Harmonization Initiative.
Wim Symens was born in 1976. He received the B.Eng. degree in mechanical engineering, with an option in mechatronics, and the Ph.D. degree in mechanical engineering from the Katholieke Universiteit Leuven, Leuven, Belgium, in 1999 and 2004, respectively. His Ph.D. research dealt with “motion and vibration control of mechatronic systems with variable configuration and local nonlinear friction.” After that, he joined Flanders’ Mechatronics Technology Centre (FMTC), Leuven, where he was responsible for FMTC’s “high-productivity machines” research program. His research interests include identification and control of mechatronic systems, noise and vibration control, and energy efficiency of electromechanical drive systems.

Johan Driesen (S’93–M’97) was born in Belgium in 1973. He received the M.Sc. degree in electrotechnical engineering and the Ph.D. degree in electrical engineering from the Katholieke Universiteit Leuven (K.U. Leuven), Leuven, Belgium, in 1996 and in 2000, respectively. His Ph.D. thesis was on the finite-element solution of coupled thermal-electromagnetic problems and related applications in electrical machines and drives, microsystems, and power quality issues.

From 2000 to 2001, he was a Visiting Researcher with the Imperial College of Science, Technology, and Medicine, London, U.K. In 2002, he was with the University of California, Berkeley. He is currently an Associate Professor with the Department of Electrical Engineering (ESAT), K.U. Leuven, where he teaches power electronics and drives. Currently, he conducts research on distributed generation, including renewable energy systems and power electronics and its applications, for instance, in drives and power quality.