Citation	G. Pipeleers, B. Demeulenaere, F. Al-Bender, J. De Schutter, and J. Swevers, (2009), Optimal performance trade-offs in repetitive control: experimental validation on an active air bearing setup IEEE Transactions on Control Systems Technology, 17(4), 970-979.
Archived version	Author manuscript: the content is identical to the content of the published paper, but without the final typesetting by the publisher
Published version	http://dx.doi.org/10.1109/TCST.2009.2014358
Journal homepage	http://www.ieeecss.org/publications/tcst
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IR	https://lirias.kuleuven.be/handle/123456789/211798

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Optimal Performance Trade-offs in Repetitive Control: Experimental Validation on an Active Air Bearing Setup

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Abstract

In repetitive control, the Bode Sensitivity Integral dictates a trade-off between improved suppression of periodic disturbances and degraded performance for non-periodic inputs. This paper experimentally demonstrates the implications of this trade-off by applying a recently developed repetitive controller design approach to reduce the error motion of the spindle's axis of rotation on an active air bearing setup. This design methodology translates the performance trade-off into trade-off curves between a non-periodic and periodic performance index, of which the practical relevance is illustrated by the obtained experimental results.

Second, the relation is investigated between these two performance indices and the adaptive performance of the repetitive controller during large variations of the spindle's rotational speed setpoint. The experiments suggest that, although defined for steady state, the two performance indices also relate to the adaptive performance of the repetitive controller.

Index Terms

repetitive control, active air bearing, active vibration control, periodic disturbances, variable-speed rotor.

I. INTRODUCTION

Including a repetitive controller in the feedback loop constitutes a common way to improve the attenuation of periodic disturbances. However, this improved periodic performance comes at the price of degraded performance for non-periodic inputs, since pushing the sensitivity down to zero at the periodic input's harmonics increases the sensitivity at intermediate frequencies, as dictated by the Bode Sensitivity Integral [7]. Theoretically, this non-periodic performance degradation can be rendered infinitely small [21], but this comes at the expense of a very sluggish transient response¹ and relies on very accurate knowledge on the periodic input's period. In practice, however, the latter assumption is jeopardized by clock error drift, jitter, measurement noise, ..., while sluggish transient responses are unacceptable. Taking into account these practical aspects, the Bode Sensitivity Integral dictates a trade-off between closed-loop periodic and non-periodic performance.

To allow for a systematic analysis of this performance trade-off in repetitive control, Pipeleers et al. [16] introduced two performance indices quantifying the repetitive controller's effect on the closed-loop non-periodic and periodic performance, hereby explicitly accounting for period-time uncertainty. The repetitive controller is designed to yield an optimal trade-off between both performance indices and by application of the (generalized) KYP lemma [12], [17], the resulting optimization problem is translated into a semi-definite program (SDP). The convexity of this optimization problem guarantees the reliable and efficient computation of the global optimum, and facilitates the generation of trade-off curves between the two conflicting performance indices.

The purpose of this paper is twofold. First, the value of the trade-off curves [16] in dealing with the performance trade-off in repetitive control is experimentally demonstrated on an active air bearing setup. The control objective is to reduce the error motion of the spindle's axis of rotation by appropriate actuation of the active journal bearing. This error motion, being due to mass unbalance and profile errors of the bearing parts, is periodic with the spindle rotation, leaving measurement noise as the sole non-periodic input to the control problem. Comparison of various Pareto optimal repetitive controller designs reveals that superior reduction of the periodic error motion comes at too high a price of measurement noise amplification, which deteriorates the overall closed-loop performance.

Second, the relation is investigated between the steady-state performance indices and the adaptive performance of the repetitive controller during large variations of the spindle's rotational speed setpoint, where the adaptive implementation is based on [6], [22]. The experiments indicate that good robust periodic performance translates into good adaptive performance during low variations of the spindle's rotational speed. Large variations, on the other hand, are no longer encompassed in the period-time uncertainty and cause performance degradation.

As periodic disturbances are characteristic for spindle applications, repetitive control is not new in the field of active bearing control. In active air bearing applications, repetitive control has been applied to overcome the low stiffness and damping of the air film [1], [3], [9], [10], whereas in magnetic bearings, repetitive control is essential for obtaining sufficient rotational accuracy

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¹This holds for a linear time-invariant repetitive controller design, as is considered in this paper, but can be circumvented by using time-varying repetitive control [14].

Fig. 1. Active air bearing setup: (a) top view of the front part of the experimental setup and (b) illustration of the active air bearing layout (not to scale), where the dashed line illustrates the bearing surface deformation (not to scale) due to actuation of the top piezo-actuator.

[23], [24]. While in these applications the repetitive controller is restricted to a basic, first-order design, more advanced repetitive controller designs have been experimentally validated for disk drive servo systems [19], [20], a major industrial application of repetitive control. However, all these repetitive controllers enforce perfect suppression of the periodic disturbances without investigating the consequences of the performance trade-off in repetitive control on the overall performance.

This paper is organized as follows. Section II describes the experimental setup and the corresponding control scheme. Section III elaborates on the repetitive controller design, while Section IV experimentally validates the selected controllers. Section V concludes the paper. The notation used in this paper is standard. To alleviate notation, the frequency response function (FRF) of a discrete-time system $H(z)$ is denoted by $H(\omega)$ instead of $H(e^{j\omega T_s})$. Vectors and matrices are indicated in bold while plain characters are used for scalars.

II. EXPERIMENTAL SETUP

The experimental setup comprises the active air bearing prototype presented in [2] and is described in Sec. II-A. Section II-B details the corresponding control scheme, while Sec. II-C discusses the parametric identification of the system.

A. Description

The experimental setup is depicted in Fig. 1, where Fig. 1(a) shows a top view of the front part of the setup and Fig. 1(b) illustrates the active air bearing layout.

At the rear end of the setup an asynchronous motor drives the spindle, where the rotational speed is controlled in open loop by a frequency converter. The spindle is supported by aerostatic bearings: the axial (thrust) and rear radial (journal) bearings are passive whereas the front radial bearing is active. The active bearing comprises a compliant bearing surface composed of four lands, which are each supported on a row of two piezo-actuators. Those actuators deform the bearing surface in a controlled manner so as to induce a radial force on the shaft via the air film. In the case of the radial spindle bearing, the shaft has two degrees of freedom normal to its axis. Consequently, two displacement sensors are needed, one for each of these two principal directions. Capacitive sensors are employed (Lion Precision) and a reference target ring is machined in situ on the spindle nose. The two measurement directions are aligned horizontally and vertically, and coincide with the working directions of the piezo-actuators. When driving the diametrically opposite piezo-actuators in anti-phase, both “pushing” and “pulling” in the two principal directions is possible. As a result, only two scalar control inputs, denoted u_h [V] and u_v [V], are needed to determine the amplifier inputs for the horizontal and vertical piezo-actuators, respectively. These signals are grouped into the vector

$$\mathbf{u} = \begin{bmatrix} u_h \\ u_v \end{bmatrix}, \quad (1)$$

and in the same way, the outputs of the displacement sensors, denoted $y_{m,h}$ [m] and $y_{m,v}$ [m], are grouped into $\mathbf{y}_m$ ².

At the nose of the spindle, a disk is clamped, which generates a 46 g mm mass unbalance and connects the spindle with an encoder (500 counts per rotation).

²Throughout this paper, the first and second element of a two-dimensional vector relate to the horizontal and vertical direction, respectively.

Fig. 2. Control scheme used to suppress the error motion e of the spindle's axis of rotation by repetitive control.

B. Control scheme

The objective of the controller is to suppress the error motion e [m] of the spindle's axis of rotation, i.e. the variation in position of the spindle's axis of rotation observed at the measurement plane [4], by controlling the piezo-actuators of the active air bearing based on the measured displacements y_m [m]. Since e is periodic with the spindle rotation, repetitive control is an appropriate control strategy. The control problem is handled in discrete time, where the sample frequency f_s equals 10 kHz and the corresponding control scheme is shown in Fig. 2. The experimental validation of the repetitive controllers is confined to spindle speeds between 900 rpm and 1200 rpm (mainly motivated by practical aspects).

The error motion e is the sum of two contributions: (i) e_d [m] related to the mass unbalance and profile errors of the bearing parts and (ii) e_u [m] caused by actuation of the active air bearing. The displacement sensors generate the measurement y_m , which corresponds to the actual error motion e supplemented with (i) (stochastic) measurement noise n [m] and (ii) a (periodic) systematic error r [m] due to the roundness error of the reference target ring for the displacement sensors.

As suppressing the error motion e constitutes the control objective, the repetitive controller $K_{RC}(z)$ should not respond to the systematic measurement error r . To this end, the measured error motion e_m [m] is constructed from the measurements y_m by subtracting an estimate $\hat{r}$ [m] of the roundness error r . To compute $\hat{r}$ at each time instant, the roundness error is determined a priori as a function of the spindle angle θ [rad] using the method of master reversal proposed by Donaldson [5]. During rotation, θ is measured by the encoder, which yields $\hat{r}$ as a function of time.

C. Identification

For the considered control problem, the plant $G(z)$ corresponds to the two-by-two system with input u and output e_u :

$$\underbrace{\begin{bmatrix} e_{u,h} \\ e_{u,v} \end{bmatrix}}_{e_u} = \underbrace{\begin{bmatrix} G_h(z) & G_{hv}(z) \\ G_{vh}(z) & G_v(z) \end{bmatrix}}_{G(z)} \underbrace{\begin{bmatrix} u_h \\ u_v \end{bmatrix}}_u. \quad (2)$$

This system is identified in open loop with separate excitation of the horizontal and vertical piezo-actuators, and the identification is performed at three spindle speeds: 900 rpm, 1050 rpm and 1200 rpm. The system is excited between 1 Hz and 2500 Hz, where the fundament of the excitation signals is a random phase multi-sine [15] with a frequency resolution of 1 Hz and flat amplitude spectrum. Excitation frequencies coinciding with harmonics of the spindle's rotational speed are eliminated to prevent that the error motion e_d (due to mass unbalance and profile errors of the bearing parts) hampers the identification.

Figure 3 shows the results of the identification. In the first step, non-parametric FRF estimates are obtained for the four plant components and at the three considered spindle speeds. As these FRF estimates reveal only a limited effect of the spindle's rotational speed (between 900 and 1200 rpm) on the plant dynamics, Fig. 3 shows all obtained non-parametric FRF estimates in grey (three spindle speeds and various experiments at each speed). This figure reveals that up to 600 Hz the off-diagonal gains $|G_{hv}(\omega)|$ and $|G_{vh}(\omega)|$ are on average 20 dB less than the diagonal gains $|G_h(\omega)|$ and $|G_v(\omega)|$.

In the second step, a parametric model $\hat{G}(z)$ is identified for $G(z)$ based on the obtained non-parametric FRF estimates. This model should be accurate up to 600 Hz, as extensive simulations reveal that the active frequency range of the controller is restricted to this frequency by the uncertainty on the antiresonance of $G_h(z)$ around 780 Hz. Based on the dominance of the diagonal plant components, a decoupled plant model is proposed

$$\hat{G}(z) = \begin{bmatrix} \hat{G}_h(z) & 0 \\ 0 & \hat{G}_v(z) \end{bmatrix}, \quad (3)$$

where the single-input single-output (SISO) models $\hat{G}_h(z)$ (9th order) and $\hat{G}_v(z)$ (17th order) are identified using the `fdident` toolbox [13]. Figure 3 evaluates these models by comparing their FRFs with the non-parametric FRF estimates for the diagonal plant components. Implied by the application of model $\hat{G}(z)$, the plant is assumed decoupled for the controller design and the simulations.

Fig. 3. Amplitudes of all the non-parametric FRF estimates obtained for the four plant components (grey); and validation of the parametric models $\hat{G}_h(z)$ and $\hat{G}_v(z)$ (black) identified for the diagonal plant components.

III. REPETITIVE CONTROLLER DESIGN

This section elaborates on the design of the repetitive controller $\mathbf{K}_{RC}(z)$. Relying on the decoupled plant model $\hat{\mathbf{G}}(z)$, a decoupled design of $\mathbf{K}_{RC}(z)$ suffices:

$$\mathbf{K}_{RC}(z) = \begin{bmatrix} K_{RC,h}(z) & 0 \\ 0 & K_{RC,v}(z) \end{bmatrix}, \quad (4)$$

where the SISO controllers $K_{RC,h}(z)$ and $K_{RC,v}(z)$ are designed according to the strategy proposed in [16]. Section III-A summarizes this design approach, while Sec. III-B discusses the selected repetitive controllers for the control problem at hand.

Concerning the rotational speed of the spindle, the following notation is used: The desired rotational speed used as input for the frequency converter that drives the asynchronous motor, is indicated by $f_{0,\text{des}}$ [Hz]. However, as explained below, a repetitive controller requires the period to contain an integer number N of samples. Hence, assuming that f_s cannot be changed, the best the repetitive controller can do is to account for $f_0 = f_s/N$ [Hz], where

$$N = \text{round}(f_s/f_{0,\text{des}}), \quad (5)$$

and $\omega_0 = 2\pi f_0$ [rad/s]. The actual rotational speed of the spindle is denoted by $f_{0,\delta}$ [Hz] ($\omega_{0,\delta} = 2\pi f_{0,\delta}$ [rad/s]) and may deviate from both $f_{0,\text{des}}$ and f_0 , where δ corresponds to the relative deviation from f_0

$$f_{0,\delta} = f_0(1 + \delta), \quad (6)$$

and is bounded by Δ :

$$|\delta| < \Delta. \quad (7)$$

A. Repetitive controller design strategy

The SISO repetitive controllers $K_{RC,h}(z)$ and $K_{RC,v}(z)$ feature the structure of Fig. 4, where only the design of filter $L(z)$ depends on the control direction. The horizontal repetitive controller equals

$$K_{RC,h}(z) = L_h(z) \frac{W(z)Q(z)}{1 - W(z)Q(z)}, \quad (8)$$

where

$$W(z) = \sum_{m=1}^M W_m z^{-mN}, \quad (9)$$

Fig. 4. Structure of the horizontal SISO repetitive controller, where the vertical equivalent is obtained by replacing subscript $\cdot_h$ by $\cdot_v$.

N is given by (5) and M is the order of the repetitive controller. Under the assumption of a decoupled plant, the decoupled controller $\mathbf{K}_{RC}(z)$ (4) gives rise to a decoupled closed-loop sensitivity function, of which the horizontal diagonal element is given by

$$S_h(z) = \frac{1 - W(z)Q(z)}{1 - W(z)Q(z)[1 - L_h(z)G_h(z)]}. \quad (10)$$

The vertical equivalents of (8) and (10) are obtained by replacing subscript $\cdot_h$ by $\cdot_v$.

Filters $Q(z)$ and $L(z)$ guarantee robust stability of the closed-loop system, whereas closed-loop performance is optimized through the design parameters W_m .

1) *Stability*: To achieve robust closed-loop stability, filters $L(z)$ and $Q(z)$ are designed in accordance with the common procedure in repetitive control [8], [11], [19]:

- $L(z)$ is designed as the series connection of the inverse plant model and a low-pass filter $F(z)$:

$$L_h(z) = \hat{G}_h(z)^{-1}F(z), \quad L_v(z) = \hat{G}_v(z)^{-1}F(z), \quad (11)$$

where the relative degree of the minimum-phase but strictly proper identified plant models is accounted for as explained in [20]. $F(z)$ is designed as a low-pass linear-phase finite-impulse-response (FIR) filter³ of order 300 with cut-off frequency 640 Hz.

- $Q(z)$ is designed as a low-pass linear-phase FIR filter of order 200 with cut-off frequency 620 Hz.

Including the inverse plant model in $L(z)$ guarantees nominal stability, since in case of perfect plant models $\hat{G}_h(z)$ and $\hat{G}_v(z)$ and $F(z) = Q(z) = 1$, $S_h(z)$ and $S_v(z)$ are both equal to:

$$\tilde{S}(z) = 1 - W(z). \quad (12)$$

However, the identified decoupled plant model $\hat{\mathbf{G}}(z)$ (3) is only accurate up to 600 Hz, raising the issue of robust stability. To this end, the low-pass filters $Q(z)$ and $F(z)$ turn off the repetitive controller from 600 Hz. Their orders and cut-off frequencies are tuned such that the following sufficient multiple-input multiple output (MIMO) stability criterion (derived from the small gain theorem, similar to [8]) is satisfied for the three designs of $W(z)$ selected in Sec. III-B:

$$\sup_{\omega \in \mathbb{R}} \left\{ \max_i \{ |\lambda_i(H(\omega))| \} \right\} < \frac{1}{\|W(z)\|_\infty}, \quad (13a)$$

where

$$H(z) = \begin{bmatrix} \left(1 - G_h(z)\hat{G}_h^{-1}(z)F(z)\right)Q(z) & -G_{hv}(z)\hat{G}_v^{-1}(z)F(z)Q(z) \\ -G_{vh}(z)\hat{G}_h^{-1}(z)F(z)Q(z) & \left(1 - G_v(z)\hat{G}_v^{-1}(z)F(z)\right)Q(z) \end{bmatrix} \quad (13b)$$

and $\lambda_i(X)$ denote the eigenvalues of X . The left-hand side of (13a) is computed from the non-parametric FRF estimates of $\mathbf{G}(z)$.

2) *Performance*: According to [16], performance is specified as an optimal trade-off between two performance indices that quantify the closed-loop steady-state performance with respect to periodic and non-periodic inputs, respectively. The definition of these performance indices relies on the assumptions that in the pass band of the filters $Q(z)$ and $F(z)$ (i) the filters equal their dc-gain and (ii) the identified plant models are perfect. These assumptions imply that $S_h(\omega) = S_v(\omega) = \tilde{S}(\omega)$ holds up to 600 Hz.

The *periodic performance index* $\gamma_{p,\Delta}$ corresponds to the smallest reduction $|\tilde{S}(l\omega_{0,\delta})|$ over all harmonics $l \leq \Lambda$ and over all potential $\omega_{0,\delta}$ -values:

$$\gamma_{p,\Delta} \equiv \max_{l \leq \Lambda} \left\{ \max_{\omega \in \Omega_l} \{ |\tilde{S}(\omega)| \} \right\}, \quad (14)$$

where Ω_l corresponds to the uncertainty interval on the l th harmonic:

$$\Omega_l = [l\omega_0(1 - \Delta), l\omega_0(1 + \Delta)]. \quad (15)$$

³Linear-phase FIR filters are preferred for $F(z)$ and $Q(z)$ since their linear phase behavior can be compensated for, as detailed in [20].

Fig. 5. Trade-off curve between $\gamma_{p,2\%}$ and γ_{np} for fifth-order repetitive controllers, where the selected controller designs are indicated. $\mathbf{K}_{RC2}(z)$ and $\mathbf{K}_{RC3}(z)$ correspond to Pareto optimal fifth-order repetitive controllers, whereas $\mathbf{K}_{RC1}(z)$ is a classical first-order controller.

Λ is set equal to the highest harmonic in the pass-band of $Q(z)$ that should be suppressed. Attenuation of the periodic disturbances corresponds to $\gamma_{p,\Delta} < 1$, where $\gamma_{p,\Delta} = 0$ indicates perfect rejection.

The *non-periodic performance index* γ_{np} is defined as the highest amplification $|\tilde{S}(\omega)|$ over all frequencies ω :

$$\gamma_{np} \equiv \|\tilde{S}(\omega)\|_{\infty}. \quad (16)$$

Due to the Bode Sensitivity Integral, repetitive controllers are bound to amplify the non-periodic disturbances, yielding $\gamma_{np} > 1$, where values closer to one are preferred.

The repetitive controller is designed to yield an optimal trade-off between the conflicting performance indices $\gamma_{p,\Delta}$ and γ_{np} . To this end, parameters W_m are computed as the solution of the following optimization problem

$$\text{minimize} \quad \gamma_{p,\Delta} + \alpha \gamma_{np} \quad (17a)$$

$$\text{subject to} \quad |\tilde{S}(\omega)| \leq \gamma_{np}, \quad \forall \omega \in \mathbb{R} \quad (17b)$$

$$|\tilde{S}(\omega)| \leq \gamma_{p,\Delta}, \quad \forall \omega \in \tilde{\Omega}_l; \quad \forall l \in \{1, \dots, \Lambda\} \quad (17c)$$

where the weight α quantifies the relative importance of $\gamma_{p,\Delta}$ and γ_{np} . As explained in [16], application of the (generalized) KYP lemma converts this optimization problem into an SDP. The convexity of the obtained optimization problem guarantees the reliable and efficient computation of the global optimum and facilitates the generation of trade-off curves between $\gamma_{p,\Delta}$ and γ_{np} .

B. Selected repetitive controllers

As explained in [16], given $W(z)$, $\gamma_{p,\Delta}$ depends only on the product $\Lambda\Delta$ and not on f_0 or on the individual values of Λ and Δ . In the repetitive controller design $\Lambda\Delta$ is set equal to 2% and with an abuse of notation the periodic performance index is indicated by $\gamma_{p,2\%}$ instead of $\gamma_{p,\Delta}$.

Three repetitive controllers, characterized by a different design of $W(z)$, are experimentally validated. The first repetitive controller, denoted by $\mathbf{K}_{RC1}(z)$, corresponds to the more frequently used basic first-order repetitive controller $W(z) = z^{-N}$. This controller yields $\gamma_{np} = 2$ and, although this controller perfectly rejects periodic disturbances at f_0 , its robust periodic performance is moderate: $\gamma_{p,2\%} = 0.13$.

The Pareto optimal repetitive controllers $\mathbf{K}_{RC2}(z)$ and $\mathbf{K}_{RC3}(z)$, on the other hand, are fifth-order repetitive controllers ($M = 5$), optimized according to (17) and corresponding to different trade-offs between $\gamma_{p,2\%}$ and γ_{np} (that is, different weights α). Figure 5 indicates both designs on the trade-off curve between $\gamma_{p,2\%}$ and γ_{np} for fifth-order repetitive controllers, which is computed by solving (17) with $M = 5$ for a range of α -values. For a given level of robust periodic performance $\gamma_{p,2\%}$, the trade-off curve indicates the minimal level of non-periodic performance degradation γ_{np} that has to be tolerated. Or,

Fig. 6. Comparison of $\tilde{S}(z)$, defined as the closed-loop sensitivity in case of a perfect plant model and $F(z) = Q(z) = 1$, for the selected repetitive controller designs. Since these FRFs are periodic with ω_0 they are only shown for the frequency range $[0, \omega_0]$. The shaded bands indicate the $\Delta\Delta = 2\%$ uncertainty interval.

vice-versa, for a fixed level of non-periodic performance, the trade-off curve indicates the best robust periodic performance that can be achieved. The steep slope between $\mathbf{K}_{RC2}(z)$ and $\mathbf{K}_{RC3}(z)$ indicates that improving the periodic performance below $\gamma_{p,2\%} = 0.022$ comes at the price of very high amplification of non-periodic disturbances: compared to $\mathbf{K}_{RC2}(z)$, $\mathbf{K}_{RC3}(z)$ improves the periodic performance from $\gamma_{p,2\%} = 0.022$ to $\gamma_{p,2\%} = 0.0013$, but degrades the non-periodic performance from $\gamma_{np} = 1.8$ to $\gamma_{np} = 3.3$. The first-order repetitive controller $\mathbf{K}_{RC1}(z)$ is also indicated in Fig. 5, but as it does not correspond to an optimal fifth-order design, it is located off the trade-off curve.

Figure 6 compares, for the considered repetitive controller designs, the FRF $\tilde{S}(\omega)$, which is a good approximation of both $S_h(\omega)$ and $S_v(\omega)$ up to 600 Hz. γ_{np} is found as the peak value of $|\tilde{S}(\omega)|$ over the entire frequency range, whereas $\gamma_{p,2\%}$ corresponds to the maximum of $|\tilde{S}(\omega)|$ over the 2% uncertainty interval, indicated by the shaded band.

IV. EXPERIMENTAL RESULTS

The three selected repetitive controllers are experimentally validated on the active air bearing setup. First, the controllers are validated for a fixed rotational speed of 1200 rpm, yielding $f_0 = f_{0,\text{des}} = 20$ Hz and $N = 500$. Sections IV-A and IV-B respectively assess the corresponding transient and steady-state performance of the controllers.

Second, the adaptive implementation of the repetitive controllers is experimentally validated (Sec. IV-C), which allows dealing with large variations of the spindle's rotational speed setpoint, as occurring during run-ups and run-downs. Following [6], [22], N , the only parameter that depends on the spindle's rotational speed, is adapted according to its estimate obtained from the index pulse (one pulse per revolution) of the encoder.

A. Transient response (1200 rpm)

To evaluate the transient response of the controllers, Fig. 7 shows the measured horizontal error motion $e_{m,h}$ of the spindle's axis of rotation, where the repetitive controllers are switched on at $t = 0.5$ s. In contrast to the following sections, which show results for the decoupled controllers (4), Fig. 7 relates to the situation where only the horizontal SISO controllers $K_{RC1,h}(z)$, $K_{RC2,h}(z)$ and $K_{RC3,h}(z)$ are switched on, while the vertical control loop is left open.

Whereas the first-order repetitive controller $K_{RC1,h}(z)$ needs only one period for reaching steady state, the transient response of the fifth-order repetitive controllers $K_{RC2,h}(z)$ and $K_{RC3,h}(z)$ lasts for five periods. Compared to $K_{RC2,h}(z)$, $K_{RC3,h}(z)$ yields a more gruff transient response, which is attributed to its worse non-periodic performance (higher γ_{np} -value) in combination with actuator saturation: only for $K_{RC3,h}(z)$ the transient control signal hits the ± 2 V input bounds of the piezo-actuator amplifier.

B. Steady-state response (1200 rpm)

Due to the Bode Sensitivity Integral, a repetitive controller's steady-state performance is bound by a trade-off between the suppression of periodic disturbances and the amplification of non-periodic inputs. This section experimentally investigates the implications of this performance trade-off on the test setup by comparing the selected repetitive controllers.

Fig. 7. Transient response (1200 rpm): measured horizontal error motion $e_{m,h}$ of the spindle's axis of rotation, where the repetitive controllers are switched on at $t = 0.5$ s. Results for $K_{RC1,h}(z)$ (a), $K_{RC2,h}(z)$ (b) and $K_{RC3,h}(z)$ (c).

	horizontal		vertical	
	per.	non-per.	per.	non-per.
no RC	95.2 nm	2.4 nm	113.4 nm	2.9 nm
$\mathbf{K}_{RC1}(z)$	4.3 nm	2.1 nm	3.9 nm	3.9 nm
$\mathbf{K}_{RC2}(z)$	2.7 nm	2.1 nm	2.7 nm	4.2 nm
$\mathbf{K}_{RC3}(z)$	0.7 nm	4.4 nm	0.9 nm	8.6 nm

TABLE I

STEADY-STATE RESPONSE (1200 RPM): ROOT-MEAN-SQUARE VALUES OF THE PERIODIC AND NON-PERIODIC CONTRIBUTION TO THE MEASURED ERROR MOTION OF THE SPINDLE'S AXIS OF ROTATION IN THE ACTIVE FREQUENCY RANGE OF THE REPETITIVE CONTROLLERS.

To this end, the measured steady-state error motion $\mathbf{e}_m$ is split up into its part that is periodic with the spindle rotation, and its non-periodic content. Table I summarizes the root-mean-square (rms) values of the periodic and non-periodic part of $\mathbf{e}_m$, where only the frequency content up to 600 Hz, the working range of the repetitive controllers, is accounted for. As explained below, Table I can to a large extent be explained based on the performance indices of the repetitive controller. Hence, the experiments support the accuracy of the identified decoupled plant model $\hat{\mathbf{G}}(z)$.

1) *Periodic performance*: To allow for a detailed evaluation of the repetitive controllers' periodic performance, Fig. 8 shows the amplitude spectrum of the periodic part of the measured horizontal error motion $e_{m,h}$ up to the 30th harmonic, as well as the corresponding reduction achieved by the repetitive controllers. The actual period of the spindle rotation is estimated a posteriori using the approach [18], yielding $f_{0,\delta} \cong 19.968$ Hz and hence, $\Delta = \delta = 0.16\%$.

Converted to dB, the $\gamma_{p,2\%}$ -values imply that all harmonics up to $\Lambda = \text{int}(2\%/\Delta) = 12$ are reduced by 18 dB by $\mathbf{K}_{RC1}(z)$, 33 dB by $\mathbf{K}_{RC2}(z)$, and 58 dB by $\mathbf{K}_{RC3}(z)$; which is confirmed by Fig. 8. Furthermore, both Fig. 8 and Table I indicate that a lower $\gamma_{p,2\%}$ -value gives rise to a smaller periodic contribution to $\mathbf{e}_m$, even if 30 harmonics are taken into account. From 600 Hz (30th harmonic), the experimentally observed performance deviates from the prediction based on $\tilde{S}(\omega)$ (Fig. 6) due to modeling errors and the decreasing amplitude of the filters $Q(z)$ and $F(z)$.

The results for $\mathbf{K}_{RC1}(z)$ reveal the necessity to account for uncertainty on f_0 . Whereas this controller would perfectly eliminate the periodic contribution to $\mathbf{e}_m$ if $\delta = 0\%$, the small deviation $\delta = 0.16\%$ causes a significant periodic performance degradation for $\mathbf{K}_{RC1}(z)$.

2) *Non-periodic performance*: For the control problem at hand, the measurement noise $\mathbf{n}$ constitutes the sole non-periodic input to the closed-loop system. Figure 9 complements the results of Table I by showing the power spectrum of the non-periodic part of the measured error motion $\mathbf{e}_m$ and the control signal $\mathbf{u}$.

Interpretation of the results of Fig. 9(a) and Table I requires special care, for they involve the *measured* error motion $\mathbf{e}_m$. While $\mathbf{n}$ relates to $\mathbf{e}_m$ by the closed-loop sensitivity, its effect on the *actual* error motion $\mathbf{e}$ is determined by the complementary sensitivity. For this reason, Fig. 9(a) and Table I only provide an indication of the measurement noise contribution to the error motion $\mathbf{e}$. However, main conclusions regarding the non-periodic performance remain valid: $\mathbf{K}_{RC1}(z)$ and $\mathbf{K}_{RC2}(z)$ yield a similar, modest, non-periodic contribution to $\mathbf{e}$, whereas $\mathbf{K}_{RC3}(z)$ amplifies the measurement noise with a factor of almost two. This is also clear from the effect of the measurement noise on the control signal (Fig. 9(b)), since this non-periodic contribution is transmitted to the system, thereby directly affecting the output $\mathbf{e}_u$ and the actual error motion $\mathbf{e}$. The simulated response of $\hat{\mathbf{G}}(z)$ to the experimental control signals yields similar results as Fig. 9(a) and Table I.

Fig. 8. Periodic steady-state response (1200 rpm): amplitude spectrum of the periodic part of the horizontal tracking error $e_{m,h}$ (a) and the corresponding reduction (b) achieved by the repetitive controllers.

Inspection of the rms values in Table I reveals that the superior periodic performance of $\mathbf{K}_{RC3}(z)$ comes at too high an amplification of the measurement noise, deteriorating the overall closed-loop performance. From this point of view $\mathbf{K}_{RC2}(z)$ is preferred, as it combines better periodic performance than $\mathbf{K}_{RC1}(z)$ with a similar non-periodic contribution to $\mathbf{e}$.

C. Adaptive response (900 – 1200 rpm)

The adaptive implementation of the repetitive controllers is validated for a run-down from 1200 rpm to 900 rpm in 9.85 s and a run-up from 900 rpm to 1200 rpm in the same time frame. Figure 10(a) shows the relative rotational speed variation corresponding to the constant deceleration profile entered to the frequency converter:

$$\alpha_{0,\text{des}} = \frac{1}{f_{0,\text{des}}} \frac{df_{0,\text{des}}}{dt} . \quad (18)$$

The horizontal error motion $e_{m,h}$ of the axis of rotation measured during this run-down without control and with the three selected repetitive controllers is shown in Figs. 10(b-e). To quantify the adaptive performance of the controllers, Table II summarizes the rms values of the measured error motion $\mathbf{e}_m$ during the 9.85 s run-down and run-up.

To which extent the repetitive controllers preserve their steady-state performance depends on two factors. The first and dominating factor is $\alpha_{0,\text{des}}$: while small relative speed variations are accounted for by the robustness of the repetitive controller's periodic performance, large $|\alpha_{0,\text{des}}|$ -values cause performance degradation. This is revealed by Figs. 10(c-e): while initially all three repetitive controllers preserve their steady-state performance, the error motion deteriorates near the end of the run-down due to the increasing $|\alpha_{0,\text{des}}|$ -value. Second, sudden changes in the amplitude/phase spectrum of the error motion $\mathbf{e}_d$ put the higher order repetitive controllers $\mathbf{K}_{RC2}(z)$ and $\mathbf{K}_{RC3}(z)$ at a disadvantage due to their longer transient response to these changes. These sudden changes are revealed by the non-smooth envelope of $e_{m,h}$ in Fig. 10(b), because for the case without control: $\mathbf{e}_u = \mathbf{0}$ and hence, $\mathbf{e} = \mathbf{e}_d$.

Related to the first factor, $\mathbf{K}_{RC2}(z)$ yields better adaptive performance than $\mathbf{K}_{RC1}(z)$ thanks to its lower $\gamma_{p,2\%}$ -value. On account of the second factor, $\mathbf{K}_{RC3}(z)$ yields worse adaptive performance than $\mathbf{K}_{RC2}(z)$, as its high γ_{np} -value translates into

Fig. 9. Non-periodic steady-state response (1200 rpm): power spectrum of the non-periodic component of the measured horizontal error motion $e_{m,h}$ (a) and the control signal u_h (b) for the three repetitive controllers.

	horizontal		vertical	
	run-down	run-up	run-down	run-up
no RC	80.0 nm	81.0 nm	95.7 nm	96.0 nm
$K_{RC1}(z)$	9.0 nm	8.6 nm	9.4 nm	9.5 nm
$K_{RC2}(z)$	7.9 nm	7.6 nm	8.6 nm	8.9 nm
$K_{RC3}(z)$	10.2 nm	10.7 nm	14.1 nm	13.7 nm

TABLE II

ADAPTIVE RESPONSE (900 – 1200 RPM): ROOT-MEAN-SQUARE VALUES OF THE MEASURED ERROR MOTION OF THE SPINDLE'S AXIS OF ROTATION DURING RUN-DOWN AND RUN-UP BETWEEN 1200 RPM AND 900 RPM, EACH IN 9.85 S.

gruff transients at the sudden changes in e_d . Hence, the steady-state performance indices γ_{np} and $\gamma_{p,\Delta}$, being defined for the steady-state performance, also relate to the adaptive performance of the repetitive controllers.

V. CONCLUSION

Relying on a recently developed repetitive controller design methodology, this paper investigates the implications of the performance trade-off in repetitive control for the reduction of the error motion of a spindle's axis of rotation, supported in an active air bearing setup. As dictated by the Bode Sensitivity Integral, improved suppression of periodic disturbances, quantified by $\gamma_{p,\Delta}$, comes at the price of a degraded performance for non-periodic inputs, quantified by γ_{np} .

The experimental results confirm the theoretical trade-off curve between $\gamma_{p,\Delta}$ and γ_{np} . Although measurement noise constitutes the sole non-periodic input to the considered control problem, the experimental results show that it should not be neglected in the repetitive controller design. If $\gamma_{p,\Delta}$ is improved at the price of too high a degradation of γ_{np} , the fed-back

Fig. 10. Adaptive response (900 – 1200 rpm): relative speed variation $\alpha_{0,des}$ (a) and measured horizontal error motion of the axis of rotation (b-e) for a run-down from 1200 rpm to 900 rpm in 9.85 s.

measurement noise dominates the error motion of the axis of rotation. Moreover, high γ_{np} -values give rise to a gruff transient response.

Although defined for steady-state performance, the indices $\gamma_{p,\Delta}$ and γ_{np} also relate to the repetitive controller's adaptive performance during large variations of the spindle's rotational speed. The adaptive controller implementation benefits from low $\gamma_{p,\Delta}$ -values, since low speed variations are encompassed by the robustness for period-time uncertainty. High γ_{np} -values, on the other hand, translate into adaptive performance degradation due to gruff transients at sudden changes in the error motion's amplitude/phase spectrum.

ACKNOWLEDGMENT

Goele Pipeleers is a Research Assistant of the Research Foundation - Flanders (FWO–Vlaanderen). This work benefits from K.U.Leuven-BOF EF/05/006 Center-of-Excellence Optimization in Engineering and from the Belgian Programme on Interuniversity Attraction Poles, initiated by the Belgian Federal Science Policy Office.

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