Oblique Inlet Pressure Loss for Swirling Flow Entering a Catalyst Substrate

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Abstract

This experimental study investigates the oblique inlet pressure loss for the entry of an annular swirling flow into an automotive catalyst substrate. The results are applicable to a wide range of compact heat exchangers. For zero swirl, the total pressure loss agrees with established expressions for pressure loss in developing laminar flow in parallel channels with finite wall thickness. For positive swirl, the additional pressure loss due to oblique flow entry is correlated to the tangential velocity upstream of the catalyst, measured using laser Doppler anemometry. The obtained oblique inlet pressure loss correlation can improve the accuracy of numerical calculations of the flow distribution in catalysts.

Key words: swirling, oblique, inclined flow, catalyst, heat exchanger, pressure loss, laser-Doppler anemometry

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Nomenclature

D, d	hydraulic diameter of annular duct and substrate channel, m
K	pressure loss, relative to the substrate channel dynamic head
L	catalyst substrate length, m
p	pressure, Pa
R	outer radius of the annular duct, m
Re	Reynolds number, based on d

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S, SW	swirl number and swirl vane setting
U, V, W	axial, radial and tangential velocity, m/s
u, v, w	axial, radial and tangential turbulent fluctuation, m/s
x, r	axial and radial coordinate, m
Greek symbols	
α	incidence angle of the oblique flow ($\tan \alpha = U_t/U$), °
β	LDA alignment angle, °
ε	open frontal area ratio
ϕ	LDA beam intersection half-angle, °
θ	tangential coordinate, °
ρ	density, kg/m^3
Subscripts	
c	substrate channel
i	inner radius of the annular duct
m	cross-sectional average
t	transverse velocity component (e.g. $U_t = \sqrt{V^2 + W^2}$)
w	hot-wire sensing element

1 Introduction

Devices for exchange of mass or heat (e.g. catalytic converter, heat exchanger, particulate filters) operate optimally when the flow distribution over the cross-section is uniform. In this case, the pressure loss is minimal and the efficiency of mass or heat exchange is maximal. Particularly for a catalyst, local degradation or ageing [1] can be avoided by means of a uniform flow distribution. The degree of flow uniformity is mainly determined by the upstream flow configuration. Generally, a diffuser distributes the flow, and allows for a device with a larger cross-section than the connecting ducts. This approach increases the residence time within the exchange device, thereby increasing the overall exchange rate.

For the case of automotive catalysts, a number of experimental studies [2, 3, 4] investigate the influence of stationary and pulsating flow conditions and diffuser geometry on the catalyst flow uniformity. However, industrial design of inlet ducts and headers relies mainly on computational fluid dynamics (CFD). As shown in Fig. 1, a catalyst substrate consists of a large number of small parallel channels. As such, a porous volume approach is used to model such systems in CFD. The lengthwise flow resistance is determined based on the local axial velocity U immediately upstream of the porous volume, and the known pressure loss expressions. The overall pressure loss comprises (i) wall friction in fully developed flow, (ii) the momentum loss due to the boundary layer development, (iii) contraction and expansion losses at the inlet and outlet

23 faces.

24 The upstream flow configuration (particularly the diffuser) causes transverse
25 velocity components U_t . Upon entering the channels, the oblique flow is straight-
26 ened (see Fig. 1). Depending on the flow conditions, this process encompasses
27 an additional pressure loss, here denoted ‘oblique inlet pressure loss’.

28 Haimad [5] and Benjamin et al. [6, 7] have proven the importance of incor-
29 porating the oblique inlet pressure loss in modeling the porous volume. Upon
30 incorporating the oblique inlet pressure loss, CFD yields a flow distribution
31 that agrees much better with the experimental results.

32 Küchemann and Weber [8] discuss the pressure loss for oblique flow in finned
33 heat exchangers. For a heat exchanger with tubes resembling two-dimensional
34 flat plates, the authors propose two models for the oblique inlet pressure loss
35 coefficient K_α . The simplest model assumes that no suction can be sustained
36 on the backside of the tubes, causing complete flow separation after the leading
37 edge of the tubes. This condition is depicted schematically in Fig. 1 and yields
38 a pressure loss $K_\alpha^{(1)} = \tan^2 \alpha$. This model is used by Benjamin et al. [6, 7].
39 Since $\tan^2 \alpha \cdot \rho U^2 / 2 = \rho U_t^2 / 2$, the entire transverse dynamic head is lost, and
40 equals the oblique inlet pressure loss.

41 A second, less conservative model [8] assumes that no loss occurs up to a
42 critical angle α_{crit} . The value of α_{crit} is such that the pressure rise due to
43 the streamtube expansion upon entering the channel equals the overall pres-
44 sure loss in the channel for axial inlet flow, represented by K_0 . Symbolically,
45 $\cos \alpha_{crit} = 1 / \sqrt{1 + K_0}$ and the pressure loss is defined as:

$$K_\alpha^{(2)} = \begin{cases} 0 & ; |\alpha| \leq \alpha_{crit} \\ \tan^2 \alpha - K_0 & ; |\alpha| > \alpha_{crit} \end{cases} \quad (1)$$

46

47 Küchemann and Weber [8] provide some experimental data that confirm the
48 proposed models. For laminar flow (inside the heat exchanger), the first model
49 best agrees with the experiments. For turbulent flow, stronger suction can be
50 sustained, and the inlet pressure loss is better predicted by the second model
51 (Eq. (1)).

52 Haimad [5] performed CFD calculations of oblique flow entering an automotive
53 catalyst, not using a porous volume yet with full discretization of the substrate
54 channels. The CFD results yield a correlation for the oblique inlet pressure
55 loss of $K_\alpha = 0.831 \tan^2 \alpha$ ($R^2 = 1.00$). No experimental validation is provided.

56 Meyer and Kröger [9] perform pressure drop measurements on several types
57 of finned tube heat exchangers, for a broad range of angle of incidence α ,

between 0 and 80° . The results collapse reasonably well to the first model of Küchemann and Weber [8] ($K_\alpha = \tan^2 \alpha$), unlike the expression $(1/\sin \theta - 1)^2$ suggested by Meyer and Kröger [9]. The experimental setup consists of a heat exchanger, placed at an incidence to the flow in a wind tunnel test section. In such a setup, the flow will deviate towards the part of the heat exchanger inclined away from the direction of flow. Therefore, the *local* angle of incidence can deviate significantly from the inclination angle of the heat exchanger.

A number of studies [10, 11] examine the pressure drop and heat transfer of louvered fin heat exchangers. A higher louver angle α causes increased heat transfer, and a penalty in terms of pressure loss.

Lee [10] discusses experimental results for the heat transfer coefficient and pressure drop in an array of inclined short plates, where the angle of attack α varies between 20 and 35° ($350 < Re < 5000$). Based on Fig. 18 [10], the contribution to the pressure loss due to the plate inclination behaves roughly as $K_\alpha = \tan^2 \alpha$, for Re between 700 and 2500 , in spite of the very short plates ($L/d \simeq 0.4$). Lee [10] provides an interesting comparison of the heat transfer for a long flat plate and an array of inclined short plates with the same pressure loss, yielding an optimum angle $\alpha \simeq 30^\circ$.

DeJong and Jacobi [11] discuss experimental heat transfer and pressure drop results for a louvered-fin heat exchanger, with louver angles α between 18 and 28° . An increase in pressure drop may be noted for increasing louver angle, yet the range of α is too limited to assess the significance of the pressure loss results.

Springer and Thole [12, 13] provide detailed velocity data for the entrance region of louvered-fin heat exchangers, yet no pressure loss data are provided.

Wang and Tao [14] perform numerical calculations of laminar flow in an array of short inclined plates, as a function of the incidence angle α . The results show a strong relationship between the pressure drop and α . Upon reworking the data in Fig. 7 [14] for $Re = 50$, the contribution to the overall pressure loss due to the inclination varies as $K_\alpha = \tan^2 \alpha$, according to the simple model of Küchemann and Weber [8].

Yocum and O'Brien [15] performed wind tunnel experiments on inclined two-dimensional compressor blade cascades. The experimental pressure loss coefficient in Fig. 18 [15] (for $Re = 200\,000$) behaves according to the second model [8], with $\alpha_{crit} \simeq 15^\circ$.

Elvery and Bremhorst [16] provide experimental data for the pressure inside the tubes of a heat exchanger, placed at an incidence to the flow. The results show that the size of the recirculation region near the entrance grows for increasing values of α , from $0.5d$ for $\alpha = 0$ to about $2d$ for $\alpha = 60^\circ$, while the

97 maximum subpressure is roughly independent of α .

98 This paper provides accurate pressure loss data for oblique inlet flow condi-
99 tions. The research uses an axisymmetric setup instead of a two-dimensional
100 configuration, to eliminate flow redistribution across the cross-section. Fur-
101 thermore, the local velocity is measured as close as possible to the inlet face
102 of the channels. Although the research uses an automotive catalyst, the find-
103 ings are relevant to all devices exchanging heat or mass, that consist of small
104 parallel channels, operating in low Re conditions.

105 2 Experimental approach

106 This section discusses the experimental setup, the instrumentation, and ex-
107 plains how the oblique inlet pressure loss is determined.

108 2.1 Annular swirling flow rig

109 Figure 2a shows a schematic overview of the experimental setup. A Roots
110 blower with ISO standardized flow rate orifice supplies air to a swirl generator,
111 designed by the International Flame Research Foundation (IFRF) [17] for use
112 on a 300 kW burner. The moveable block swirl generator allows to continuously
113 adjust the swirl from zero to a maximum, corresponding to a ratio of tangential
114 to axial velocity of about unity.

115 The swirling flow is guided into a concentric annular duct by a smooth tran-
116 sition. The annular duct features an outer and inner radius of $R = 38.0$ mm
117 and $R_i = 28.3$ mm, corresponding to a hydraulic diameter $D = 19.4$ mm. The
118 outer wall is PVC, the inner wall is aluminium, both with a smooth surface
119 finish.

120 Figure 2b shows a detail of the annular duct, which is connected to an auto-
121 motive catalyst substrate of length L (400 cells per square inch, wall thickness
122 4.3×10^{-3} inch, unwashcoated). The distance between substrate and swirl
123 generator is $18D$. Downstream of the catalyst, an outlet sleeve of length $2R$
124 is mounted. A cylindrical polar coordinate system $[x, r, \theta]$ is defined according
125 to Fig. 2b.

126 The catalyst substrate is mounted tight against the inner annulus wall, and
127 sealed with silicon to prevent leakage. The test section and substrate are con-
128 nected to the swirl generator by bolted flanges, taking care for proper align-
129 ment. This allows to test various substrate lengths.

130 2.2 Velocity measurements

131 2.2.1 Laser-Doppler anemometry

132 The axial and tangential velocity components U and W are measured inside
133 the annular duct, in front of the catalyst substrate. A two-component laser-
134 Doppler anemometer (LDA) is used with a 5 W Ar^+ laser, with Bragg cell
135 frequency shifting on both components. The system is operated in backscat-
136 tering mode. Optical access is through a 1 mm thick glass microscope slide,
137 flush with the outer annulus wall. As shown in Fig. 2b, the glass insert causes
138 a minor defect, about 0.2 mm in height.

139 To maximize the signal-to-noise ratio when measuring close to the inner annu-
140 lus wall or the catalyst substrate, a second glass insert is mounted flush with
141 in the inner annulus wall. Inside the annulus core, a beam dump reduces the
142 amount of reflected laser light.

143 The velocities are determined using a Dantec Flow Velocity Analyzer^{TM} with
144 accompanying software, which is only used for data collection. Velocity weight-
145 ing and statistics are performed using in-house software. An aerosol of DEHS
146 oil is used as seeding, with particle diameters between 0.2 and 1 μm .

147 The LDA system is carefully aligned and focused. The combination of a
148 310 mm focal lens, a beam separation of 60 mm and a beam diameter of
149 1.2 mm results in a measurement volume of $\varnothing 0.17$ mm and 1.7 mm long.
150 The beam intersection angle $2\phi \simeq 11^\circ$. While the LDA measurement vol-
151 ume remains fixed in space, the swirl generator is traversed using a three-axis
152 motorized translation stage, featuring a positioning accuracy within 50 μm .

153 In each measurement point, sufficient samples are collected to reduce the mean
154 axial velocity uncertainty to within 1%, based on a 95% confidence level. The
155 inverse of the velocity magnitude is used as weighting factor, to reduce LDA
156 velocity bias errors [18]. The mean axial velocity is verified using an ISO
157 standardized flow rate orifice (see Fig. 2a), to within an uncertainty of $\pm 5\%$.

158 To measure the velocity close to the substrate inlet face, the LDA system is
159 aligned at an angle $\beta (> \phi)$ to the annular duct (see Fig. 2b). As such, the LDA
160 measures the velocity $[U', W']$. Assuming the radial velocity V is negligible,
161 the actual velocity $[U, W]$ is obtained as $W = W'$ and $U = U' / \cos\beta$. A radial
162 velocity measurement is feasible by applying different alignment angles β_1
163 and β_2 in two measurements. These result in $U'_1 = U \cos \beta_1 + V \sin \beta_1$ and
164 $U'_2 = U \cos \beta_2 + V \sin \beta_2$, from which the axial and radial velocity $[U, V]$ are
165 readily obtained.

166 For an incompressible fluid, the conservation of mass in a cylindrical polar

coordinate system is expressed as:

$$\frac{\partial U}{\partial x} + \frac{1}{r} \frac{\partial(rV)}{\partial r} + \frac{1}{r} \frac{\partial W}{\partial \theta} = 0 \quad (2)$$

where $\partial W / \partial \theta = 0$ for axisymmetric flow (after careful annulus alignment, the deviation from axisymmetry is below 1 %). Equation (2) shows that the radial velocity $V = 0$ if $\partial U / \partial x = 0$. The experiments reveal that $\partial U / \partial x \neq 0$ close to the substrate, from a critical distance $|x| < x_0$ (see Sect. 3.1). Beyond this critical distance ($|x| > x_0$), $\partial U / \partial x \simeq 0$, and therefore the radial velocity is negligible as well.

The limited width of the annular duct $R - R_i \simeq 0.25R$ limits the radial velocity magnitude. In fact, the annular geometry has been chosen after initial experiments with swirling flow in a circular duct. In swirling flow, the tangential velocity W creates pressure gradients in the radial direction to balance the centrifugal forces:

$$\frac{\partial p}{\partial r} = \frac{\rho W^2}{r} \quad (3)$$

Initial experiments have been carried out using swirling flow in a circular duct, as opposed to the annular duct. In a circular duct, swirl causes a subpressure along the central axis. Since the tangential velocity decays in downstream direction, the central subpressure decreases, resulting in a positive pressure gradient $\partial p / \partial x$ along the duct axis. For strong swirl, $\partial p / \partial x$ becomes large enough to create backflow along the central axis, a phenomenon referred to as ‘vortex breakdown’ [19]. The presence of the catalyst substrate further increases $\partial p / \partial x$, thereby promoting vortex breakdown. In these conditions, LDA measurements proved very difficult near the circular duct axis, due to seeding problems. The annular duct approach eliminates this problem.

2.2.2 Hot-wire anemometry

The axial velocity profile downstream of the catalyst has been measured using hot-wire anemometry. This measurement serves to verify the upstream axial velocity profile, as measured using LDA. To minimize the distortion of the downstream profile (due to the mixing of the laminar jets from the substrate channels) the velocity is measured only 1 mm from the outlet face, at $x = L + 1$ mm.

Since the flow length scale is as small as $d \simeq 1$ mm, the sensor wire of standard hot-wire probes is too long (typically $l_w \simeq 1.5$ mm). Therefore, a probe with shorter wire is custom-made for this purpose. As prescribed by Bruun [20],

the wire length should be at least twice the cold wire length $l_{w,cold}$ to minimize conduction heat loss to the prongs. For a Tungsten sensor wire of $5\ \mu\text{m}$ diameter, the cold length $l_{w,cold} \simeq 0.23\ \text{mm}$ at $U = 15\ \text{m/s}$ [20]. As such, the probe wire is chosen $l_w = 0.5\ \text{mm}$ long. The hot-wire is operated in constant temperature mode, with an overheat ratio of 1.8. The probe is calibrated in an open jet with an accuracy better than 1%.

Due to the close measurement position, the velocity profile shows the individual laminar jets. For each channel, the mean velocity is determined, resulting in a ‘discretized’ reference profile for the axial velocity (see Fig. 6).

2.3 Evaluation of the oblique inlet pressure loss

Static pressure taps are located at $L_p = 9.6D$ upstream of the catalyst substrate (see Fig. 2a). The static pressure measured at that station Δp^* is corrected with the equivalent pressure drop $\Delta p^{(L_p)}$ in an annular duct of length L_p , assuming fully developed turbulent flow (Re based on D varies between 13 000 and 32 600), using the Colebrook-White correlation for smooth walls. This yields the pressure loss Δp ($= \Delta p^* - \Delta p^{(L_p)}$) over the substrate, with respect to the ambient pressure.

Δp can be expressed as the sum of various sources of pressure loss, as described in App. A:

$$\Delta p = (4f_{app}L/d + K_{con} + K_{exp} + K_{\alpha}) \rho U_c^2 / 2 \quad (4)$$

where U_c is the mean velocity [m/s] inside the substrate channels. $U_c = U_m/\varepsilon$, where U_m is the mean axial duct velocity [m/s] and ε is the substrate porosity. For zero swirl, the oblique inlet pressure loss vanishes ($K_{\alpha} = 0$).

From the substrate cell density and wall thickness, d and ε can be determined. For a 400/4.3 substrate, $d \simeq 1.16\ \text{mm}$ and $\varepsilon \simeq 0.835$. Each term in Eq. (4) depends on these values. Using the expressions for f_{app} , K_{con} and K_{exp} in App. A, Eq. (4) is least-square fitted to the measured pressure loss, with the substrate wall thickness as fitting parameter. The hydraulic diameter d and porosity ε are determined as a function of the fixed cell density and the fitting parameter, i.e. the wall thickness.

The fitting procedure results in a slightly different value for d and ε compared to the geometrical specifications. However, this approach is justifiable given the finite tolerances on the substrate geometry. After least-square fitting the wall thickness, Eq. (4) provides an excellent prediction of the measured pressure

235 loss Δp in zero swirl conditions, with coefficients of determination $R^2 > 0.95$.

236 Subsequently, the swirl is stepwise increased. For each swirl setting, Eq. (4)
237 is least-square fitted to the measured pressure loss Δp , yet *without* altering
238 the value of d . Instead, the oblique inlet pressure loss coefficient K_α is used as
239 fitting parameter.

240 This procedure leads to a series of K_α values for each swirl setting. These
241 values are combined with the tangential velocity profile obtained using LDA.
242 The following sections discuss the results for the velocity profiles and the
243 oblique inlet pressure loss.

244 3 Experimental results

245 3.1 Velocity profiles

246 Figure 3 shows the radial profile of velocity components U (axial) and W
247 (tangential), measured using LDA at a distance of $0.5D$ ($\simeq 10$ mm) upstream
248 of the catalyst substrate. The velocity is non-dimensionalized using the mean
249 axial velocity U_m . Figures 3a,b,c,d correspond to different swirl vane settings,
250 indicated by the values of SW between 0 and 100%. The degree of swirl is
251 quantified by the swirl number S , as defined by Gupta et al. [19].

252 The radial velocity profiles are quite uniform, for both axial and tangential
253 velocity U and W . The boundary layer is thicker near the inner wall ($r/R =$
254 $R_i/R \simeq 0.745$), compared to the outer wall. Due to the short distance to the
255 swirl generator ($18D$), the velocity profile is not fully developed. The turbulent
256 fluctuation intensity u/U (not shown in Fig. 3) varies roughly between 5 %
257 in the main flow and 15 % in the boundary layers (w/W varies between 10 %
258 and 20 %).

259 The dashed line indicates the radial overpressure profile $\Delta p(r)$, relative to the
260 wall overpressure Δp . The radial profile is obtained by integration of Eq. (3).
261 The swirl-induced pressure reduction near the inner annulus wall is small, even
262 at maximum swirl setting.

263 Figure 4 shows the axial profile of velocity components U and W , measured
264 in the middle of the annulus gap, at $r = R^* = (R_i + R)/2$. The solid lines
265 in Fig. 4 are fitted exponential curves. The tangential velocity W seems inde-
266 pendent of the axial position, at least up to $x = -1$ mm, which is the closest
267 position to the substrate where valid LDA measurements could be performed.
268 The axial velocity U does vary with x in the vicinity of the substrate, quan-

269 tified by a stagnation length x_0 . Close to the substrate, radial flow spreading
 270 occurs as indicated in Fig. 5.

271 From the continuity equation (2) follows $\partial(rV)/\partial r = -r \partial U/\partial x$, for axisym-
 272 metric flow. Thus, an order of magnitude analysis can be performed:

$$273 \quad \frac{\Delta U}{x_0} \simeq \frac{1}{R} \frac{R V_{max}}{D/4} \Rightarrow \frac{V_{max}}{U_m} \simeq \frac{1/4}{x_0/D} \frac{\Delta U}{U_m} \quad (5)$$

274 where the experimental results show that $\Delta U/U_m \simeq 0.1$. Based on the LDA
 275 results, x_0/D varies between 0.1 and 0.5. As such, V_{max}/U_m is estimated to
 276 vary between 0.25 and 0.05. In terms of dynamic pressure, the radial compo-
 277 nent $\rho V^2/2$ is therefore small (yet not negligible) compared to the tangential
 278 component $\rho W^2/2$.

279 The measurement of the full velocity vector $[U, V, W]$ within $|x| < x_0$ proved
 280 too difficult using LDA, since reflections rapidly reduce the signal quality in
 281 the vicinity of the substrate. This has not been further pursued. Since W
 282 remains unchanged in x -direction, the tangential velocity profile is measured
 283 at $x = -0.5D$, where the LDA signal quality is adequate.

284 In a follow-up paper [21], stereoscopic particle image velocimetry (PIV) is
 285 used in the same experimental setup to study the three-dimensional velocity
 286 field $[U, V, W]$ close to the substrate inlet face ($|x| \simeq 0.2$ mm). The relation
 287 between the stagnation length x_0 , flow uniformization and catalyst pressure
 288 drop K is discussed [21].

289 Figure 6 shows the radial profile of axial velocity, measured using hot-wire
 290 anemometry (HWA), at 1 mm downstream of the catalyst substrate. The
 291 circular markers represent the actual velocity measurements, exhibiting the
 292 features of laminar jets issuing from the substrate channels. The solid staircase
 293 line represents the mean velocity for each individual channel, with vertical
 294 lines representing the 95% error margins. Due to the short substrate length
 295 ($L/d = 22.4$), the individual channel profiles are not fully developed.

296 Upon comparison of Fig. 6a,b to Fig. 3a,d, there is some difference in the
 297 upstream ($x = -0.5D$) U profile and the solid line in Fig. 6a,b representing the
 298 channel-averaged downstream velocity ($x = L+1$ mm). Near the inlet face, the
 299 radial spreading causes a further uniformization of the axial velocity profile.
 300 This effect could not be adequately measured with LDA. Instead, stereoscopic
 301 PIV is used in a follow-up study [21] to investigate the flow stagnation region.

3.2 Oblique inlet pressure loss

The oblique inlet pressure loss coefficient K_α is determined according to the procedure described in Sect. 2.3, for different substrate lengths ($4.3 \leq L/d \leq 44$), and mean annular duct velocity between 10 and 25 m/s. Each substrate features square channels, with the same cell density and wall thickness.

Figure 7 shows the obtained values for K_α , as a function of maximum tangential velocity W_{max} , relative to the mean axial velocity U_m , determined using LDA. The 95% uncertainty margins are indicated with horizontal and vertical lines for each marker. A parabolic correlation of the form $K_\alpha = K_{\alpha,1} (W_{max}/U_m)^2$ is least square fitted to the data, yielding the following expression for the oblique inlet pressure loss:

$$K_\alpha = 0.459 \left(\frac{W_{max}}{U_m} \right)^2 \simeq 0.459 \tan^2 \alpha \quad (\pm 13\%, R^2 = 0.81) \quad (6)$$

Since the tangential velocity profile is quite uniform, $W_{max} \simeq W_m$. Using W_m instead of W_{max} , the proportionality constant $K_{\alpha,1}$ in Eq. (6) becomes $0.550 (\pm 13\%)$. Still, W_{max} is preferred for the correlation fit, since it better represents the equivalent transverse velocity $U_t = \sqrt{V^2 + W^2}$, resulting from the tangential *and* radial velocity.

Figure 8 compares the obtained correlation of K_α to the two models proposed by Küchemann and Weber [8], assuming $W_{max}/U_m = \tan \alpha$. The dashed line represents the simple model ($K_\alpha^{(1)} = \tan^2 \alpha$), which assumes that no suction can be sustained, i.e. all transverse dynamic pressure is lost. The dotted line represents the second model ($K_\alpha^{(2)}$, Eq. (1), which assumes that no losses occur up to a critical flow angle, depending on the overall channel pressure loss. The dotted line corresponds to a substrate length of $L/d = 10$. This model is better suited for turbulent flow, as confirmed by some sources [8, 15].

The literature [8, 7, 6, 5, 9, 14] confirms the validity of the first model ($K_\alpha^{(1)}$) for laminar channel flow, although there remains some uncertainty on the proportionality constant $K_{\alpha,1}$. Benjamin et al. [7, 6] compare CFD predictions of a catalyst velocity distribution to experimental data. Upon incorporating the inlet pressure loss with $K_{\alpha,1} = 1$ into CFD, the agreement is better. However, the model overpredicts the inlet pressure loss near the edges of the catalyst, where the flow angle α is large. This suggest that a value of $K_{\alpha,1} = 1$ is too large, which is confirmed by the findings of this paper (Eq. (6)).

334 4 Conclusions

335 The oblique inlet pressure loss has been determined experimentally for an au-
 336 tomotive catalyst substrate, using an annular swirling flow rig. Laser-Doppler
 337 anemometry (LDA) is used to determine the upstream axial and tangential
 338 velocity profiles, as close as possible to the substrate inlet face.

339 The axial velocity profile exhibits radial spreading close to the inlet face.
 340 However, the tangential velocity profile remains constant up to the closest
 341 measurable distance to the inlet face, $|x| \simeq 1$ mm. In the vicinity of the inlet
 342 face, LDA signal quality rapidly deteriorates due to wall reflections. Therefore,
 343 stereoscopic particle image velocimetry (PIV) is used in a follow-up paper [21]
 344 to examine the axial, tangential and radial velocity closer to the inlet face
 345 ($|x| \simeq 0.2$ mm).

346 Four substrate lengths ($4.3 \leq L/d \leq 44$) are investigated, for six swirl settings
 347 ($0 \leq S \leq 0.6$) and three annular duct Reynolds numbers ($13000 \leq Re \leq$
 348 32600). Based on the substrate channel diameter, $850 \leq Re_d \leq 2100$.

349 For zero swirl, the substrate pressure loss is well predicted by the established
 350 expressions for pressure loss in developing laminar flow in channels with finite
 351 wall thickness, as summarized in App. A.

352 For positive swirl, the additional pressure loss due to oblique flow entry is
 353 correlated to the peak tangential velocity upstream of the substrate. Although
 354 there is evidence of radial velocity near the inlet face, the tangential velocity
 355 is not significantly affected. As such, it is argued that the peak tangential
 356 velocity is most representative for the overall transverse dynamic pressure.

357 Figure 7 confirms that the oblique inlet pressure loss coefficient K_α correlates
 358 satisfactorily to the squared ratio of tangential to axial velocity, or:

$$K_\alpha = 0.459 \left(\frac{W_{max}}{U_m} \right)^2 \simeq 0.459 \tan^2 \alpha$$

($\pm 13\%$, $R^2 = 0.81$)

359 This result agrees with the literature [8, 7, 6, 5, 9, 14]. The proportionality
 360 constant $K_{\alpha,1} = 0.459$ is smaller than unity, where the upper limit value of
 361 $K_{\alpha,1} = 1$ corresponds to a total loss of transverse dynamic pressure [8].

362 For the case of automotive catalysts, the value of $K_{\alpha,1} = 0.459 (\pm 13\%)$ seems
 363 appropriate, based on the experience of Benjamin et al. [7, 6] in implementing
 364 the inlet pressure loss with $K_{\alpha,1} = 1$ in CFD.

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367 initial measurement campaign on the circular duct swirling flow rig.

368 A Pressure loss in channel flow

369 The following sections review the different sources of pressure loss associated
370 with flow through an array of parallel straight channels, particularly for lami-
371 nar flow ($Re < 2300$). Each pressure loss contribution is non-dimensionalized
372 using the dynamic head $\rho U_c^2/2$, based on the mean channel velocity U_c .

373 A.1 Fully developed flow

374 For fully developed flow, the pressure loss is governed by wall friction, and
375 quantified as $4fL/d$. For laminar flow, the product of the Fanning friction
376 factor f and Re is constant, e.g. $f \cdot Re \simeq 14.227$ for square channels [22].

377 A.2 Boundary layer development

378 The velocity distribution at the channel inlet is assumed uniform, since the
379 flow enters multiple parallel channels from a large cross-sectional area. The
380 boundary layer development requires momentum transport from the wall to
381 the channel center. Furthermore, the wall velocity gradient changes for a de-
382 veloping profile, so the wall friction is larger in the developing flow region.
383 Both effects combined cause an increase in pressure loss, quantified by Shah
384 and London [22], Shah [23] as an apparent friction factor f_{app} ($> f$):

$$385 \quad f_{app} Re(x^+) = \frac{3.44}{\sqrt{x^+}} + \frac{f \cdot Re + \frac{K_\infty}{4x^+} - \frac{3.44}{\sqrt{x^+}}}{1 + Cx^{+2}} \quad (\text{A.1})$$

386 where $x^+ = (x/d)/Re$. The effect of the boundary layer development can
387 be singled out as $K_{dev}(x^+) = 4(f_{app} Re(x^+) - f \cdot Re) x^+$. This incremental
388 pressure loss tends to K_∞ for $x^+ \rightarrow \infty$. For a square cross-section, $K_\infty = 1.43$
389 and $C = 0.00029$ [23], although other studies yield different values for K_∞ ,
390 e.g. $K_\infty = 1.55$ [24].

391 A.3 Contraction and expansion loss

392 Upon entering and exiting channels with finite wall thickness, the flow respec-
 393 tively contracts and expands, according to the open area ratio ε . Kays and
 394 London [25], Kays [26] provide empirical data for the associated pressure loss
 395 in heat exchangers. For laminar flow in a square channel:

$$\begin{cases} K_{con} = (1 - C_{c,0}) (1 - \varepsilon^2) + 2 (k_d^{(in)} - 1) \\ K_{exp} = (1 - \varepsilon)^2 - 2 (k_d^{(out)} - 1) \varepsilon \end{cases} \quad (\text{A.2})$$

396 where $C_{c,0} \simeq 0.615$ [26] is the vena contracta area ratio for $\varepsilon = 0$ (i.e. entrance
 397 from an infinite reservoir). $k_d^{(in)}$ and $k_d^{(out)}$ quantify the distribution of momen-
 398 tum, respectively at the channel inlet and outlet ($k_d = \int_A (U/U_m)^2 dA/A \geq$
 399 1). Lundgren et al. [24] explain how k_d is related to K_{dev} from Sect. A.2. As-
 400 suming a fully developed outlet profile, $k_d^{(out)} = 1.3785$ [24], yielding $K_{exp} =$
 401 $(1 - \varepsilon)^2 - 0.757 \varepsilon$.

402 Kays [26] evaluates $k_d^{(in)}$ for a fully developed profile, thereby partially incor-
 403 porating the pressure loss due to the boundary layer development in K_{con} . In
 404 the opinion of the authors, it is preferable to assume a uniform velocity profile
 405 near the inlet, immediately following the vena contracta. As such, $k_d^{(in)} = 1$ and
 406 $K_{con} = 0.385 (1 - \varepsilon^2)$. Thus, the contraction and expansion loss is considered
 407 separate from the boundary layer development loss.

408 A.4 Overall

409 The core of this paper concerns the additional pressure loss due to oblique flow
 410 inlet K_α . All pressure loss contributions (excluding temperature changes¹) are
 411 added to yield the overall pressure loss:

$$412 \quad K = 4f_{app}L/d + K_{con} + K_{exp} + K_\alpha \quad (\text{A.3})$$

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¹ For lengthwise temperature changes, the density and velocity change accordingly, causing an additional pressure loss or rise depending on $\rho(x)/\rho(0)$.

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Fig. 1. Catalyst substrate

Fig. 2. Annular swirling flow rig: (a) overview and (b) detail of the measurement section

Fig. 3. Radial profile of velocity $[U, W]$ and pressure p at $x = -0.5D$, upstream of $L = 26$ mm substrate, for swirl setting (a) 0%, (b) 60%, (c) 90% and (d) 100%

Fig. 4. Axial profile of velocity $[U, W]$ at $r = R^* = (R_i + R)/2$, upstream of (a,b) $L = 26$ mm and (c,d) $L = 51$ mm substrate, for swirl setting (a,c) 0%, (b,d) 100%

Fig. 5. Scaling analysis of radial spreading near the catalyst substrate inlet face

Fig. 6. Radial profile of velocity U at $x = L + 1$ mm, downstream of $L = 26$ mm substrate, for swirl setting (a) 0% and (b) 100%

Fig. 7. Oblique inlet pressure loss K_α versus tangential velocity W , for substrate lengths $4.3 \leq L/d \leq 44$

Fig. 8. Oblique inlet pressure loss K_α in comparison to $K_\alpha^{(1)}$ and $K_\alpha^{(2)}$ [8]