Proceedings of

SHIRMS 2008

**1st Southern Hemisphere International
Rock Mechanics Symposium**

16–19 September 2008, Perth, Western Australia

Volume Two *Fundamental and Petroleum*

EDITORS

Yves Potvin

Australian Centre for Geomechanics, Australia

John Carter

The University of Newcastle, Australia

Arcady Dyskin

The University of Western Australia, Australia

Rob Jeffrey

CSIRO Petroleum, Australia

ISRM Regional
Symposium

CSIRO | Curtin University | University of WA
Joint Venture

Determining Properties of High Compliance Porosity by Use of an Effective Medium Model Based on Crack Description in Terms of Their Normal and Shear Compliance Parameters

V.Yu. Zaitsev *Institute of Applied Physics Russian Academy of Sciences, Russia*

P. Sas *Katholieke Universiteit Leuven, Belgium*

Abstract

An effective medium model recently developed by the present authors is applied for interpretation of known experimental data on pressure dependences of elastic wave velocities in dry and saturated rocks in order to elucidate the effect of the highly compliant fraction of the porosity. Our approach allows determining essential features of the soft defects in terms of their shear and normal compliances. For the dry defects, we found that the ratio of the normal to shear compliance may be as high as 5–7, which contrasts to the range 0.8–2.2 expected for such widely used defect models as Hertz–Mindlin contacts or elliptical cracks. The high normal compliance of the real defects results in strong decrease of Poisson's ratio of the rock down to negative values. Here, we consider the case of a dry sandstone exhibiting negative Poisson's ratio. For saturated samples, we found that although the normal compliance of the defects becomes smaller than the shear one, the difference is only 1.2–2.5 times. From the viewpoint of the role of "global" (in the sense of Biot) and local (squirt) fluid flows, the performed examination indicates strong domination of the squirt mechanism including the cases, for which a third, unidentified dispersion mechanism was earlier supposed.

1 Introduction

An ample understanding of the relation between the rock microstructure and characteristics of seismic waves is of key importance for both general seismic wave modelling and exploration seismology. Among pertinent publications, a vast amount relates to modelling the wave dispersion in a wide sense, i.e. the velocity dependence on the frequency, external pressure, fluid saturation, and the rock microstructural features. These features include the effect of pores, cracks and inter-grain contacts (Bourbié et al., 1986 and references herein). The intrinsic compliance of dry defects and the fluid saturation are crucial factors determining the elastic wave dispersion. In modelling the role of saturation one main concept originates from Biot's models accounting for the large-scale motion of the saturating fluid relative to the solid phase (Biot, 1956a; 1956b). Another important concept considers the small-scale fluid flows at grain contacts or inside cracks, i.e. the "squirt flow" concept (Mavko and Nur, 1975; Murphy, 1984; Winkler, 1985; 1986; Mavko and Jizba, 1991). Comparison with experiments indicates that in high-porosity materials (like artificial packages of glass beads), the Biot mechanism is essential (Bourbié et al., 1986). In contrast, the data on elastic wave velocities in real rocks often indicate that the squirt mechanism dominates, although for some cases, the observed dispersion apparently could be satisfactorily described neither by the global, nor by the squirt flow mechanisms (Mavko and Jizba, 1994).

Here, we apply the earlier proposed effective-medium model (Zaitsev and Sas, 2000) to interpretation of experimental data obtained by different authors for pressure dependences of P- and S-wave velocities in rocks. It is known that along with spheroidal and cylinder-shape pores (i.e. equant porosity which gives the main contribution to the visible overall porosity), the high-compliant narrow (crack-like) pores can give a significant (and sometimes dominant) contribution to the variations of the elastic-wave velocities, in contrast to often negligibly small contribution to the visible porosity of the material. We compare and elucidate the role of the high-compliant porosity for both dry and saturated samples in terms of parameters characterising the compliance of the defects with respect to shear loading and normal stresses. We show that the ratio of these parameters can significantly differ from the predictions of conventional models such as elliptical cracks or Hertz–Mindlin contacts. As an interesting byproduct of the performed examination, we found an example of a dry rock (sandstone) exhibiting pronounced negative Poisson's ratio at lower confining pressures.

Physical interpretation of this finding is given. In the context of the effect of the fluid saturation, our interpretation of the experimental data indicates strong domination of the squirt (local-flow) mechanism in agreement with Mavko and Jizba (1994). Moreover, we show that this conclusion is actually valid even for those rock samples for which an additional non-Biot and non-squirt dispersion mechanism was earlier supposed (Mavko and Jizba, 1994).

2 Effective-medium model of rocks containing highly compliant defects

In this section we briefly recall the main features of the effective medium model (Zaitsev and Sas, 2000) describing the elasticity and wave dissipation in microinhomogeneous solids (although in what follows, this model will be used only in the part related to the description of elastic properties). The approach (Zaitsev and Sas, 2000) may be characterised as semi-phenomenological, since at the microstructural level the defect parameters are introduced phenomenologically, and then the resultant macroscopic properties of the medium are derived by consistent micromechanical analysis. An analogous energy-balance approach was used, for example, in the models of crack-containing solids by Walsh (1965) or O'Connell and Budiansky (1974). However, these authors used concrete models containing elliptical cracks, whereas the form of real cracks is rarely close to elliptical (Mavko and Nur, 1978). It is important that particular fracture models normally imply a rather limited variability of the ratio of the normal to shear compliance of the defects. For example, for the dry elliptical cracks this ratio is $(1+\gamma)(2-\gamma) \sim 2$ for most solids, where γ is Poisson's ratio for the host rock (Zaitsev and Sas, 2000). In the model used here, the phenomenological introduction of the compliance parameters of the defects allows us to avoid the initial limitations on the normal-to-shear compliance ratio. A similar approach based on effective compliance (or stiffness) coefficients was used earlier for characterisation of contacts in granular solids (Winkler, 1983), as well as for layered inhomogeneous rocks and solids containing planar compliant discontinuity-like fractures (Schoenberg and Douma, 1988; Schoenberg and Sayers, 1995; Bakulin et al., 2000). A similar representation of the defects as compliant planar inclusions was implemented in numerical simulation of elastic properties of fractured rocks (Saenger and Shapiro, 2002). In this context, our approach combines the description of the defects in terms of the effective compliance parameters with the energy approach and statistical averaging over arbitrary orientations of the defects like in the aforementioned models of crack-containing rocks.

In the model, the material (rock) is considered as a solid matrix containing defect-inclusions (grains, cracks, etc.) which are small compared to the length of elastic waves. The compliance (softness) of the defects is much higher compared to the surrounding defect-free material. It is argued (Zaitsev and Sas, 2000) that these highly compliant defects can be approximated as planar objects. In the results presented below, the defects are assumed to be isotropically oriented. We also make the conventional assumption that the characteristic scale (wavelength) of the average stress field is significantly larger than the inter-defect distance, and the latter is significantly larger than the defect diameter. If necessary, the "self-consistent" procedure (O'Connell and Budiansky, 1974) or the differential approach (Vavakin and Salganik, 1978) accounting for the defect interaction can be applied to improve the model accuracy.

In order to quantitatively characterise defect's compliance in response to normal stress (σ_n) and to shear stress (σ_τ) in the in-plane direction, we introduce small parameters $\zeta \ll 1$, $\zeta \in [0,1]$ and $\xi \ll 1$, $\xi \in [0,1]$, which relate Young's modulus (E) and shear modulus (G) for the matrix material with the effective moduli E_d and G_d of the defects:

$$E_d = \zeta E, G_d = \xi G \quad (1)$$

Thus parameters ζ , ξ are close to the non-dimensional weaknesses used in Bakulin and Molotkov (1998) and Bakulin et al. (2000). These two parameters are enough to completely characterise the defects with rotational symmetry and these may be considered effective parameters in characterising a large ensemble of real defects that may have no rotational symmetry. The orientation of the defects with different compliances is characterised by a distribution function $v(\psi, \varphi, \zeta, \xi)$ that depends on both the orientation angles ψ, φ of the defect normal vector and on the non-dimensional elastic parameters ζ, ξ . For the isotropic orientation, the angular part of the distribution is $v(\psi, \varphi) = 1/(4\pi)$. Summation over all orientations and compliances gives the total volume content of the planar defects in the unit volume of the material.

$$v_i = \int_0^{2\pi} d\varphi \int_0^\pi d\psi \int_0^1 d\zeta \int_0^1 d\xi v(\psi, \varphi, \zeta, \xi) \sin \psi d\psi d\zeta d\xi$$

which is much smaller than unity, according to the above assumptions.

Performing statistical averaging, and considering the amount of the elastic energy stored in the matrix material and at the compliant defects, we obtained the following expressions for the effective bulk modulus ($K_{eff.}$), the effective Young's modulus ($E_{eff.}$), the effective shear modulus ($G_{eff.}$), the longitudinal (compressional) wave effective modulus ($M_{eff.}$), and for the Poisson's ratio ($\gamma_{eff.}$) in the microinhomogeneous material (Zaitsev and Sas, 2000):

$$\tilde{K} = \frac{K_{eff.}}{K} = \frac{1}{1 + \frac{1}{3} N_1 / (1 - 2\gamma)} \quad (2)$$

$$\tilde{E} = \frac{E_{eff.}}{E} = \frac{1}{1 + \frac{1}{3} N_1 + \frac{4}{15} (1 + \gamma) N_2} \quad (3)$$

$$\tilde{G} = \frac{G_{eff.}}{G} = \frac{1}{1 + \frac{2}{15} N_1 / (1 + \gamma) + \frac{2}{5} N_2} \quad (4)$$

$$\tilde{M} = \frac{M_{eff.}}{M} = \frac{1 + \frac{4}{15} N_1 / (1 - \gamma) + \frac{2}{15} N_2 (1 + \gamma) / (1 - \gamma)}{[1 + \frac{2}{15} N_1 / (1 + \gamma) + \frac{2}{5} N_2] \cdot [1 + \frac{1}{3} N_1 / (1 - 2\gamma)]} \quad (5)$$

$$\gamma_{eff.} = \frac{\gamma - \frac{1}{15} N_1 + \frac{2}{15} (1 + \gamma) N_2}{1 + \frac{1}{3} N_1 + \frac{4}{15} (1 + \gamma) N_2} \quad (6)$$

Here, notations E , G , and M are used for the moduli of the matrix material and $\tilde{E}$, $\tilde{G}$, and $\tilde{M}$ are the corresponding non-dimensional moduli for the microinhomogeneous medium. Parameters N_1 and N_2 in Equations (2) to (6) in the quasi-static limit (i.e. neglecting the defect relaxation) have the following meaning:

$$N_1 = \int v(\zeta) \zeta^{-1} d\zeta, N_2 = \int v(\xi) \xi^{-1} d\xi \quad (7)$$

where the partial distributions over the compliance parameters ζ , ξ are introduced:

$$v(\zeta) = \int_0^{2\pi} d\varphi \int_0^\pi d\psi \int_0^1 d\xi v(\psi, \varphi, \zeta, \xi) \sin \psi d\psi d\xi, v(\xi) = \int_0^{2\pi} d\varphi \int_0^\pi d\psi \int_0^1 d\zeta v(\psi, \varphi, \zeta, \xi) \sin \psi d\psi d\zeta \quad (8)$$

In what follows, quantities N_1 and N_2 are called the effective defect densities. It is clear from Equation (7) that these parameters are determined jointly by the compliance of the defects and their concentration. In the case of identical defects with fixed values of ζ and ξ , the distributions (Equation (8)) have the form of delta-functions, so that

$$N_1 = v_i / \zeta, N_2 = v_i / \xi \quad (9)$$

which means that effective densities N_1 and N_2 are proportional to the defect volume content (v_i) divided by the compliance parameters. If the ratio of the shear and normal compliances is the same for all defects, then it directly equals N_1 / N_2 . Otherwise, N_1 / N_2 has the meaning of the averaged compliance ratio which characterises qualitative differences of the effect of different defects on the material elasticity. In the work of Zaitsev and Sas (2000), the difference in the decrease of elastic moduli $E_{eff.}$, $G_{eff.}$ and $M_{eff.}$ is comprehensively classified for different values of the ratio N_1 / N_2 (the characteristic cases demonstrating qualitatively different behaviour correspond to $N_1 / N_2 \gg 1$, $N_1 / N_2 \sim 1$, and $N_1 / N_2 \ll 1$).

It is often assumed that the ratio of normal to shear compliances for the Hertz-Mindlin contacts and dry fractures in a solid matrix is of the order of unity (Schoenberg and Sayers, 1995; Bakulin et al., 2000). For example, for elliptical cracks, it may be shown (Zaitsev and Sas, 2000) via comparison of Equations (2) to

(6) and the results of O'Connell and Budiansky (1974) that the ratio $N_1/N_2 = (1+\gamma)(2-\gamma)$, and thus the ratio is expected to range in a rather narrow interval of 2.1–2.2 for the most part of matrix rocks. It is also conventionally assumed that fluid saturation strongly decreases the normal compliance of the defects without significant effect upon the shear compliance thus shifting the ratio N_1/N_2 towards zero (O'Connell and Budiansky, 1974). Examination of experimental data in the next section will show, however, that properties of real defects in both dry and saturated sandstones may differ from these conventionally accepted assumptions.

3 Comparison between the model and experimental data

In this section, we use an instructive set of experimental data on elastic P- and S-wave velocities as functions of confining pressure. These data were discussed in detail by Mavko and Jizba (1991; 1994) in assessing the applicability of the squirt mechanism. Here, we reexamine the same examples in order to infer the defect properties in terms of the above introduced normal and shear compliance parameters according to Equations (2) to (7). For this purpose it is necessary to know experimentally measured velocities (or elastic moduli) for two different types of waves in the rock with the defects. As the reference values for the matrix rock, following Mavko and Jizba (1994), we shall take the ultimate values of the velocities measured at the highest hydrostatic pressures (50–100 MPa). Note that even at the maximum pressure, the “matrix” elasticity can still be affected by the weakly compliant spheroidal and near-cylinder pores (the equant porosity), whose contribution is dominant in the overall porosity (about 9–12%) of the samples (Mavko and Jizba, 1991). We shall see that the role of the equant porosity, which is weakly dependent on pressure, is essential in order to understand why for the saturated rocks, the high-pressure P-velocity may be noticeably higher than for the dry rocks, although the high-compliant planar fractures should already be practically closed in both dry and saturated samples.

Figure 1 Example of a plot demonstrating sensitivity of the complementary velocities variations to the defect properties. Ratio N_1/N_2 ranges from 0.1 to 1 (with step 0.1) and from 1 to 5 (with step 1) for thin lines. Thick lines are for the ultimate cases $N_1/N_2 \rightarrow 0$ and $N_1/N_2 \rightarrow \infty$

For comparison with the model, it is convenient to present the velocity pressure dependences data in the form of a plot of the relative variations in the S-wave velocity ($\Delta V_s/V_s$) against the variations in the P-wave velocity ($\Delta V_p/V_p$). Figure 1 shows a set of such theoretically modelled dependences plotted for different ratios (N_1/N_2) using Equations (4) and (5). The magnitudes ΔV_s and ΔV_p are counted from the corresponding values for the defectless matrix material. Figure 1 indicates that for the entire range

$0 \leq N_1/N_2 < \infty$ of the ratio between the compliance parameters of the defects, the curves corresponding to the resultant complementary variations in P- and S-wave velocities do not occupy the entire quadrant and are confined within a finite sector enclosed between the thick solid lines in Figure 1. One ultimate curve corresponding to the value $N_1/N_2 = 0$ is for the case where defect's normal compliance is negligibly small and the opposite limit $N_1/N_2 \rightarrow \infty$ corresponds to the case where the shear compliance is negligibly small.

Considering Figure 1, one may pose a question: which pair of the velocities or other elastic parameters should be plotted one against another in order to get the maximum sensitivity of the plot to the variations in the compliance ratio N_1/N_2 for the defects? An evident answer is to choose such combinations of the elastic moduli, which are equal to the introduced “normal” and “shear densities” N_1 and N_2 themselves. Such a plot, besides the relative parameter N_1/N_2 , will directly show the effective defect densities $N_{1,2}$. Equations (2) to (5) readily yield the required combinations of the elastic moduli:

$$N_1 = 3(K/K_{eff} - 1)(1 - 2\gamma) \quad (10)$$

$$N_2 = (5/2)(G/G_{eff} - 1) - (K/K_{eff} - 1)(1 - 2\gamma)/(1 + \gamma) \quad (11)$$

Equation (10) is intuitively quite clear: the normal (compressional) compliance of the defects controls the variation in the bulk elastic modulus. However, the combination of elastic moduli given by Equation (11), which is exclusively sensitive to the shear compliance of the defects, is not obvious at all. In terms of elastic-moduli combinations (10) and (11), the sector of their variations widens up to the entire quadrant, instead of a relatively narrow sector of variations in terms of $\Delta V_s/V_s$ versus $\Delta V_p/V_p$ shown in Figure 1. In what follows, we shall use both the plot of $\Delta V_s/V_s$ versus $\Delta V_p/V_p$, and the plot of N_1 against N_2 .

Following the work of Mavko and Jizba (1994) experimental data for Navajo sandstone are presented in Figure 2. In Figure 2(a), the raw experimental points for the velocities are re-plotted in terms of $\Delta V_s/V_s$ and $\Delta V_p/V_p$, and theoretical curves based on Equations (4) and (5) have been fitted to these data.

Figure 2 Experimental points for Navajo sandstone (data from Coyner, 1984). (a) $\Delta V_s/V_s$ against $\Delta V_p/V_p$; and (b) effective “normal density” N_1 against “shear density” N_2 . Open symbols are for the dry sample, and filled symbols for the saturated one. Solid lines are the theoretical fits for $N_1/N_2 = 2.35$ (dry) and $N_1/N_2 = 0.8$ (saturated sample)

The fit yields the compliance ratio $N_1/N_2 \approx 2.35$ for the defects in the dry rock, which is fairly close to the value $N_1/N_2 = (1+\gamma)(2-\gamma) \sim 2$ expected for the elliptical cracks. For the saturated rock, this ratio is almost three times smaller, $N_1/N_2 \approx 0.8$, but is still far from zero value in contrast to conventional assumptions (Bakulin et al., 2000). This rather high residual normal compliance of the saturated defects indicates that, in the elastic wave field, the weakly compressible liquid is not ‘locked’ inside the defects, but squirts forth and back in the narrow compliant pores in agreement with the local-flow concept.

Another remarkable feature of Figure 2(a) is that the experimental data are well fitted by the theoretical curves corresponding to constant values of the ratio N_1/N_2 , which is not an *a priori* evident defect property in a wide pressure range. Thus, this fact may be used as a criterion of adequacy for models describing the defects. Figure 2(b) additionally indicates that at zero pressure, the effective "shear density" (N_2) of the defects remains approximately the same for the dry and saturated Navajo sample (which is not typical of all samples), whereas the "normal density" (N_1) decreases thrice due to the presence of the liquid.

Figure 3 Pressure dependences for Navajo sandstone (data from Coyner, 1984). (a) wave velocities; and (b) Poisson's ratio. Open symbols are the dry sample, and filled symbols are the saturated sample. In plot (a) triangles are for the S-wave, circles are for the P-wave. Lines are the theoretical fits for $N_1/N_2 = 2.35$ (dry) and $N_1/N_2 = 0.8$ (saturated). For comparison, the inset shows the same data for the P-wave together with the fitting model curves reproduced from the paper by Mavko and Jizba (1994)

The next plot in Figure 3(a) is presented in the form similar to that used by Mavko and Jizba (1994) and shows the experimental pressure dependences of the wave velocities together with the theoretical curves corresponding to the above inferred values of N_1/N_2 . To plot the theoretical curves in Figure 3(a), we used the experimentally determined high-pressure values for the velocities, then the complementary velocity variations for the two waves were re-plotted from the theoretical dependences presented in Figure 2(a). In the form shown in Figure 3(a), the deviations between the measurement data and the resultant theoretical fits are hardly distinguishable. For comparison, the inset shows the same data and the fitting model curves reproduced from Mavko and Jizba (1994), where the quality of fitting is much poorer because of improper ratio between the normal and shear compliance parameters of the defects.

The next Figure 3(b), in which the same data as in Figure 3(a) are re-plotted in terms of Poisson's ratio, displays more tiny features of the pressure dependence. In Figure 3(b) as well as in Figure 2, minor systematic discrepancies between the model fit and the measurement are noticeable for the dry rock. Probably these slight deviations are related to the fact that the normal to shear compliance ratio N_1/N_2 is not exactly constant in the whole pressure range. Another possible cause is that the theoretical expressions (Equations (2) to (6)) were derived without allowance for the defect interaction at higher concentration. However, for the 13-15% decrease of the velocities in the considered case, the accuracy of the perturbation approach seems to be quite reasonable. The decrease in Poisson's ratio at a higher density of the dry defects, both qualitatively agree with the conclusions obtained in earlier studies (O'Connell and Budiansky, 1974). It was shown in the latter paper, that at a high enough density of dry cracks, the Poisson's ratio of the rock may reach zero, although all of the elastic moduli were predicted to simultaneously become zero as well. The latter means that the rock should be completely fractured at such a high defect density. However, the following example will show that real situations may be significantly different from such predictions.

Following Mavko and Jizba (1994) further, Figure 4 presents the plots analogous to Figure 2 for a sample of Weber sandstone. Figure 4(b) indicates that the effective "shear density" (N_2) at zero pressure for the Weber

sandstone remains practically the same in the dry and saturated states, much like the Navajo sandstone. However, for the saturated Weber sandstone the ratio of the normal to shear compliance is reduced significantly more than in the Navajo sandstone, from $N_1/N_2 \sim 7$ down to $N_1/N_2 = 0.38$. For the plots shown in Figure 4 for the Weber sample, the difference in the normal compliance (and correspondingly, "normal defect densities" (N_1) between the dry and saturated cases is almost 20 times. This is in contrast to only three times difference for the Navajo sample (Figure 2). Besides, for the dry Weber sample the estimated ratio $N_1/N_2 \sim 7$ is significantly higher than value about two expected for basic defect models such as Hertz-Mindlin contacts and elliptical crack models.

Figure 4 The complementary variations in wave velocities and effective defect densities for Weber sandstone (data from Coyner, 1984). Notations are the same as in Figure 2. Lines are the theoretical fits at $N_1/N_2 = 7$ (dry) and $N_1/N_2 = 0.38$ (saturated)

Figure 4(b) additionally indicates that the estimate of N_1/N_2 at different pressures exhibits noticeable variability in the range 5-8. The reason for this may be both the actual variability of the defect parameters and the neglect of the defect interaction in the theoretical description. However, even the mean constant value $N_1/N_2 = 7$ provides a very good theoretical fit for the velocity variations in the whole pressure range, which is shown in Figure 5(a).

Figure 5 Pressure dependences for Weber sandstone (data from Coyner, 1984). Notations are the same as in Figure 3. Lines are the theoretical fits at $N_1/N_2 = 2.35$ (dry) and $N_1/N_2 = 0.38$ (saturated)

Figure 5(b) presents the pressure dependence for the Poisson's ratio for Weber sandstone. The most remarkable feature of this plot is the pronounced negative values of Poisson's ratio for the dry sample at pressures below 20 MPa, at which the wave velocities are only 25% reduced compared to the high-pressure

values. This fact contrasts with the previously mentioned model (O'Connell and Budiansky, 1974), which predicts that the material should become completely fractured when its Poisson's ratio reaches zero. Physically the origin of the negative Poisson ratio is quite clear. Indeed, if the compliant defects had no shear compliance at all ($N_2 = 0$) and were oriented at 45° to the axis of the uniaxial stress, then such defects should produce equal expansions in the direction of the applied stress and in the orthogonal direction. If at high enough density of such soft defects, the contribution of the solid matrix to the overall strain could be neglected, then the effective (but anisotropic) Poisson's ratio should tend to minus unity. For more realistic cases of a non-zero shear compliance, $N_2/N_1 \neq 0$, and for statistically isotropic random orientations of the defects, Equation (6) yields the following prediction for the minimum value of the effective Poisson's ratio:

$$\gamma_{\min.} = \frac{-1 + 2(1 + \gamma)N_2/N_1}{3 + 4(1 + \gamma)N_2/N_1} \quad (12)$$

It is clear from Equation (12) that the absolute minimum $\gamma_{\text{eff}} = -1/3$ corresponds to $N_2/N_1 \rightarrow 0$ (i.e. the normal compliance of the defects should strongly dominate). For the discussed example of the Weber sandstone with $N_1/N_2 \approx 7$, we obtain the estimate $\gamma_{\min.} \approx -0.19$. Hence, the experimentally observed minimum value $\gamma_{\text{eff}} \approx -0.13$ is not so far from the theoretical limit. The transition from the negative to positive values of the Poisson ratio is fairly well described by the theoretical curve (see Figure 5(b)) based on Equations (2) to (6), which may be ever improved by one or another method of accounting for the interaction of the defects.

Figures 6 and 7 show the results for Nugget V sandstone, for which an unidentified dispersion mechanism was supposed in Mavko and Jizba (1994). The plots in Figure 6 indicate that for the saturated Nugget sample, the defect compliance ratio is very close to that for the Weber sandstone, $N_1/N_2 \approx 0.38$, whereas for the dry defects this ratio is $N_1/N_2 \approx 4.3$, being intermediate between the values determined for the Navajo and Weber samples. Further, unlike the previous examples, a remarkable distinction in Figure 6(b) is that the effective shear density (N_2) at zero pressure in the saturated Nugget V sample is significantly higher than that in the dry rock, roughly $0.6/0.4 = 1.5$ times. Thus the presence of the liquid results in the enhancement of the defect shear compliance in the case of the Nugget sample, in contrast to $N_2 \approx \text{const.}$ in the previously discussed cases of the dry and saturated Navajo and Weber samples. Figure 7(a) presents the wave velocities in the Nugget V sample and demonstrates that even at the highest pressures the additional shear compliance of the saturated defects persists, so that the S-wave velocity for the saturated sample is noticeably smaller than for the dry rock. This feature is again strongly different from approximately the same values of S-wave velocities in the cases of either dry or saturated Navajo and Weber sandstones under maximum pressures. A probable explanation of this fact is that a few-molecule lubricating liquid layer at the defect interfaces persists in the Nugget sample even at highest pressures, which appears to be sufficient to keep the increased shear compliance of the defects in the Nugget sample in the entire range of confining pressures. When this increased shear compliance and the resultant "background" variations in the high-pressure wave velocities are taken into account, then the complementary pressure dependences of the P- and S-wave velocities are perfectly approximated by the suggested theoretical expressions (Equations (2) to (6)) as is seen from the solid and dashed curves in Figure 7(a). Consequently, the next Figure 7(b) also demonstrates perfect agreement of the pressure-dependence of Poisson's ratio with the theoretical fits for both dry and saturated Nugget samples.

Figure 6 The complementary variations in the wave velocities and the effective defect densities for Nugget V sandstone (data from Han, 1986). Notations are the same as in Figure 2. Solid lines correspond to the theoretical fits for $N_1/N_2 = 4.3$ (dry) and $N_1/N_2 = 0.38$ (saturated)

Figure 7 Pressure dependences for Nugget V sandstone (data from Han, 1986). Notations are the same as in Figure 3. Solid lines correspond to the theoretical fits for $N_1/N_2 = 4.3$ (dry) and $N_1/N_2 = 0.38$ (saturated)

Therefore, for all of the considered examples, the wave velocities are described by the proposed model with a high accuracy. We point out, once again, that the description of the pressure-dependent variations by the suggested model actually takes into account only the effect of the variation in the high-compliance porosity that is strongly pressure-dependent. Therefore, for the saturated samples only the squirt-type flows inside saturated contacts and cracks are taken into account. The aforementioned perfect agreement with the experimental data indicates strong domination of the squirt-flow dispersion mechanism in the pressure-dependent variations of the elastic moduli. The performed analysis thus provides a clear explanation why, for the Nugget V sample and for other samples with the similar behaviour, an unidentified dispersion mechanism was earlier supposed in the study of Mavko and Jizba (1994). In fact, in that paper, the interpretation of the experiments was rather similar to ours. Equations (2) to (6), in which the contributions of the shear and normal compliances are explicitly separated, allow us to clearly understand the similarity and the difference between the interpretations. Namely, in Mavko and Jizba's (1994) paper, the following basic relation (Mavko and Jizba, 1991) between the dispersion of the shear and the bulk moduli of a rock was used:

$$\left(\frac{1}{G_{\text{eff}}} - \frac{1}{G_{\text{dry}}} \right)_{\sigma} \approx \frac{4}{15} \left(\frac{1}{K_{\text{eff}}} - \frac{1}{K_{\text{dry}}} \right)_{\sigma} \quad (13)$$

Where subscript *uf* refers to the unrelaxed (high frequency) magnitudes and subscript *dry* refers to the relaxed (low frequency) moduli. For the saturated materials, the variation in the same moduli was obtained in Mavko and Jizba (1994) by substituting Equation (11) into the low-frequency Biot-Gassman equations, which yielded approximately:

$$\left(\frac{1}{G_{sat}} - \frac{1}{G_{dry}} \right)_{\sigma} \approx \frac{4}{15} \left[\left(\frac{1}{K_{sat}} - \frac{1}{K_{dry}} \right)_{\sigma} - \left(\frac{1}{K_{sat}} - \frac{1}{K_{dry}} \right)_{high\sigma} \right] \quad (14)$$

This relation was used in Mavko and Jizba (1994) as a criterion of the domination of the squirt-flow mechanism when examining the experimental data. Note that factor 4/15 in Equation (14), which originates due to the averaging over random orientations of the defects, much like the similar factors in our Equations (2) to (6). Comparison between Equations (2) and (4), and Equations (13) and (14), indicates that Equation (13) can follow from Equations (2) and (4). This requires an additional assumption that in Equation (4) describing the shear modulus $\tilde{G}$, the term containing the shear parameter N_2 can be neglected. The latter condition actually means that condition $N_1/N_2 \gg 3(1+\gamma)$ should be assumed. As we have seen, however, real rocks exhibit values $N_1/N_2 \approx 2-7$ for the dry defects and $N_1/N_2 < 1$ for the saturated ones. Therefore, even for the dry samples characterised by the ratio $N_1/N_2 > 1$, the assumption $N_1/N_2 \gg 3(1+\gamma)$ is rather poor in most cases. For saturated samples with $N_1/N_2 < 1$, this condition is obviously invalid.

However, the second criterion (14), which was used in Mavko and Jizba (1994) for comparison between the dry and saturated samples, implies significantly weaker restrictions than Equation (13), so that the assumption $N_1/N_2 \gg 3(1+\gamma)$ is not necessary. Namely, Equation (14) with the proportionality coefficient 4/15 may be readily obtained from Equations (2) and (4) in the assumption that the fluid saturation does not affect the shear compliance N_2 of the defects. Thus Equation (14) implies that the difference between values G_{sat} and G_{dry} is determined, in our terms, exclusively by the pressure and saturation-dependent variation in the compressional parameter N_1 corresponding to the normal compliance of the soft porosity. This means that the possible effect of compressibility of the weakly compliant equant porosity, as well as the variation in the shear parameter N_2 are not taken into account on the left hand side of criterion (14). In contrast, on the right hand side of Equation (14), which in our terms depends only on compressional parameter N_1 , the second term in the parentheses subtracts the "background" variation in the bulk modulus K , which is neglected for the shear modulus. Thus, rigorously speaking, the approximations used for deriving the left- and right-hand sides of Equation (14) are inconsistent, so that the validity of Equation (13) should be broken for samples in which the fluid saturation noticeably affects the shear parameter N_2 .

The experimental pressure dependences for the S-wave velocity shown in Figures 3(a) and 5(a) together with the plots in Figures 2(b), 4(b) and Equation (4) indicate that the assumption about the same value of the shear parameter (N_2) in the dry and saturated states holds fairly well for the Navajo and Weber sandstones. This fact explains the good agreement with criterion (14), which was mentioned by Mavko and Jizba (1994). On the other hand, for the Nugget sandstone, Figure 7(a) indicates rather significant "background" (i.e. persistent even at high pressures) variations for both the P- and S-wave velocities. The latter variation is due to the aforementioned strong increase in the defects' shear compliance (N_2) for the saturated Nugget sandstone (Figure 6(b)) and also due to the effect of saturation of the weakly compliant equant porosity. This fluid-induced variation in the shear parameter (N_2) and thus in the background value of the shear modulus (G) resulted in the contradiction with criterion (14) for the Nugget sample and similar ones. On the other hand, such background variations are consistently taken into account for the moduli K and G in Equations (2) to (6) and in the theoretical curves shown in Figure 7, so that even for the Nugget sample, the pressure-dependent part of the wave dispersion is also very well described by the effect of the high-compliant porosity, including the saturated case.

Thus the squirt-flow mechanism perfectly accounts for the pressure-dependent variations of the P- and S- wave velocities in all of the considered cases, including the Nugget sandstone for which criterion (14) does not hold for the aforementioned reasons. Here we have limited our analysis to instructive examples and

data identical to what was used by Mavko and Jizba (1994) for ease of comparison. A number of other examined experimental records (Gordon and Davis, 1968; Mavko and Jizba, 1991; Seipold et al., 1998) also exhibited very similar features and were well fitted by Equations (2) to (6). It can be noted that for all weakly-porous rocks (Seipold et al., 1998) the determined value of the compliance ratio N_1/N_2 appeared to be close to two, which agrees fairly well with the elliptic-crack model.

4 Conclusions

The analysis performed in the paper has led us to a number of inferences, which are important for interpretation of seismic data and may be summarised as follows:

- (i) The examination of experimental data indicates that the suggested effective medium model (Zaitsev and Sas, 2000) allows for a high-accuracy description of the observed complementary variations in the P- and S-wave velocities both in dry and saturated rocks containing compliant defects (microcracks or intergrain contacts). The results of the performed examination, which may be considered as a development of the approach by Mavko and Jizba (1994), indicate the strong domination of the squirt-flow mechanism in the observed complementary pressure variations in the elastic moduli. This conclusion is shown to be valid even for those cases for which an unidentified non-squirt and non-Biot mechanism was earlier supposed.
- (ii) The suggested approach allowed us to estimate the effective defect densities ($N_{1,2}$) and the compliance ratio (N_1/N_2) for real defects, which indicated a number of unexpected results. Namely, for some dry samples, the compliance ratio was found to be much greater than unity (Weber and Nugget V sandstones) in contrast to conventionally expected values about unity (Bakulin et al., 2000). Such a high normal compliance of real defects may result in *negative* values of Poisson's ratio, which we have found for the dry Weber sandstone at confining pressures below 20 MPa. On the other hand, for saturated samples, instead of the conventionally expected almost zero compliance ratio $N_1/N_2 \rightarrow 0$ (Bakulin et al., 2000), we have found values of N_1/N_2 on the order of unity (for example $N_1/N_2 \approx 0.8$ for the saturated Navajo sandstone). These findings are important for discrimination of dry and saturated rocks using seismic data.
- (iii) Another important inference is that in all examined cases the compliance ratio N_1/N_2 for the defects persistently keeps nearly constant value in a wide pressure range, which is not *a priori* evident. The revealed compliance properties of the soft porosity indicate the necessity of essential improvement of conventional models of defects in rocks. Since the revealed defect properties appeared to be significantly different from the conventionally accepted assumptions, the implications obtained may significantly modify interpretations of seismic data.

Note finally that relaxation properties are intrinsic of real defects (for example, due to thermoelastic effects or viscous effects in the saturated samples), which can also be readily taken into account by the suggested effective medium model. Therefore, the frequency-dependent complementary variations in the wave velocities may be analysed much like the pressure-dependent variations, including the imaginary part of the moduli corresponding to the defect-induced dissipation of different types of elastic wave analysed in Zaitsev and Sas (2000) using basically the same approach. Concerning the dissipation effects, the presence of high-compliant defects results in pronounced amplitude-dependent dissipation intrinsic to rocks due to the combined action of purely elastic nonlinearity and purely linear (relaxation) dissipation, which are localised at the soft defects. In more detail, the discussion of these aspects of the defect manifestations can be found in recent publications (Zaitsev and Matveev, 2006; Zaitsev et al., 2008).

Acknowledgements

The study was partially supported by the Russian Foundation for Basic Research (Projects No 06-02-72550 and 08-05-00692). We thank Professor S. Shapiro, Freie Universität, Berlin for stimulating discussions.

References

- Bakulin, A., Grechka, V. and Tsvankin, I. (2000) Estimation of fracture parameters from reflection seismic data - Pts. I-III. *Geophysics*, 65(6), pp. 1788-1830.

- Bakulin, A.V. and Molotkov, L.A. (1998) Effective models of fractured and porous media, St. Petersburg University, Press (in Russian).
- Biot, M.A. (1956a) Theory of acoustic attenuation, propagation of elastic waves in a fluid-saturated porous solid: I. Low-frequency range, *Journal of the Acoustic Society of America*, 28, pp. 168–178.
- Biot, M.A. (1956b) Theory of propagation of elastic waves in a fluid-saturated porous solid: II. Higher frequency range, *Journal of the Acoustic Society of America*, 28, pp. 179–191.
- Bourbié, T., Coussy, O. and Zinszner, B. (1986) *Acoustique des milieux poreux: publications de l'institut française du pétrole*, Éditions Technip, Paris.
- Budiansky, B. and O'Connell, R.J. (1976) Elastic moduli of dry and saturated cracked solids, *International Journal of Solid Structures*, Vol. 12, pp. 81–97.
- Coyner, K.B. (1984) Effects of stress, pore pressure, and pore fluids on bulk strain, velocity, and permeability in rocks: Ph.D. thesis, Massachusetts Institute of Technology.
- Gordon, R.B. and Davis, L.A. (1968) Velocity and attenuation of seismic waves in imperfectly elastic rock: *Journal of Geophysical Research*, 73(12), pp. 3917–3935.
- Han, D. (1986) Effects of porosity and clay content on acoustic properties of sandstones and unconsolidated sediments: Ph.D. dissertation, Stanford University.
- Mavko, G. and Nur, A. (1975) Melt squirt in the asthenosphere: *Journal of Geophysical Research*, 80, pp. 1444–1448.
- Mavko, G. and Jizba, D. (1991) Estimating grain-scale fluid effects on velocity dispersion in rocks, *Geophysics*, Vol. 56, pp. 1940–1949.
- Mavko, G. and Jizba, D. (1994) The relation between seismic P- and S- velocity dispersion in saturated rocks, *Geophysics*, 59(1), pp. 87–92.
- Mavko, G.M. and Nur, A. (1978) The effect of nonelliptical cracks on the compressibility of rocks, *Journal of Geophysical Research*, Vol. 83, pp. 4459–4468.
- Murphy, W.F. (1984) Acoustic measures of partial gas saturation in tight sandstones, *Journal of Geophysical Research*, 89, pp. 11,549–11,559.
- O'Connell, R.J. and Budiansky, B. (1974) Seismic velocities in dry and saturated cracked solids, *Journal of Geophysical Research*, 79(10), pp. 5412–5426.
- Saenger, E.H. and Shapiro, S.A. (2002) Effective velocities in fractured media: A numerical study using the rotated staggered finite-difference grid, *Geophysical Prospecting*, 50(2), pp. 183–194.
- Schoenberg, M. and Douma, J. (1988) Elastic wave propagation in media with parallel fractures and aligned cracks, *Geophysical Prospecting*, Vol. 36, pp. 571–590.
- Schoenberg, M. and Sayers, C. (1995) Seismic anisotropy of fractured rock, *Geophysics*, 60, pp. 204–211.
- Seipold, U., Mueller, H.-J. and Tuisku, P. (1998) Principle differences in the pressure dependence of thermal and elastic properties of crystalline rocks, *Physics and Chemistry of the Earth*, 23, pp. 357–360.
- Vavakin, A.S. and Salganik, R.L. (1978) Effective elastic characteristics of bodies with isolated cracks, cavities and rigid inclusions, *Mechanics of the solid body [in Russian]*, No. 2, pp. 95–107.
- Walsh, J.B. (1965a) The effect of cracks on the uniaxial elastic compression of rocks, *Journal of Geophysical Research*, 70, pp. 399–411.
- Walsh, J.B. (1965b) The effect of cracks on Poisson's ratio, *Journal of Geophysical Research*, 70, pp. 5249–5247.
- Winkler, K.W. (1983) Contact stiffness in granular porous materials: comparison between theory and experiment, *Geophysical Research Letters*, 10(11), pp. 1073–1076.
- Winkler, K.W. (1985) Dispersion analysis of velocity and attenuation in Berea sandstone, *Journal of Geophysical Research*, Vol. 90, pp. 6793–6800.
- Winkler, K.W. (1986) Estimates of velocity dispersion between seismic and ultrasonic frequencies: *Geophysics*, Vol. 51, pp. 183–189.
- Zaitsev, V. and Sas, P. (2000) Elastic moduli and dissipative properties of microinhomogeneous solids with isotropically oriented defects, *Acta Acustica united with Acustica*, 86, pp. 216–228.
- Zaitsev, V.Yu. and Matveev, L.A. (2006) Strain-amplitude dependent dissipation in linearly dissipative and nonlinear elastic microinhomogeneous media, *Russian Geology and Geophysics*, 47(5), pp. 694–709.
- Zaitsev, V.Yu., Saltykov, V.A. and Matveev, L.A. (2008) Relation between the Tidal Modulation of Seismic Noise and the Amplitude-Dependent Loss in Rock, *Acoustical Physics*, 54, pp. 538–544.

On the Determination of Rock Anisotropy for Stress Measurements

P.M. Dight *Australian Centre for Geomechanics, The University of Western Australia, Australia*

A.V. Dyskin *School of Civil and Resource Engineering, The University of Western Australia, Australia*

Abstract

The traditional methods of in situ stress determination can be seriously affected by rock anisotropy, which necessitates both development of the corresponding interpretation methods and accurate determination of the elastic characteristics of the anisotropic rocks. In this contribution we consider the method of determination of anisotropic moduli by testing small subcores drilled in different directions. Rock anisotropy is often induced by the presence of joints, foliation, schistosity, bedding or similar features. In the case when the spacing between the theses features exceeds the length of the subcores the results of moduli determination in separate subcores shows considerable variability. A mechanism of this variability lies in the fact that a particular subcore may or may not be intersected by a joint. We show that despite this, the averaging procedure used in the method of moduli reconstruction, developed previously by the authors, can still be successfully applied, since the averaging over the subcores recovers the large-scale moduli. However, the standard deviation associated with the randomness of joint-subcore intersection can in some case be very large (potentially unlimited) which explains the observed variability.

1 Introduction

Adequate determination of the stress field is an important aspect of safe mine design. Stresses in and around mineralised systems are influenced by local and regional tectonics and the rock fracture processes associated with the tectonics. The presence of these factors makes the prediction of in situ stresses quite difficult and requires several measurements/or in multiple locations.

The most used class of stress measurement techniques is overcoring which is based on the determination of local rock strain induced by stress relief caused by overcoring (if there is underground access); the stress is then recovered assuming local elastic behaviour of rock. This principle underlines such popular methods as the hollow inclusion cell (HI), door stopper and ANZI cell techniques. A key element in these methods until recently has been the assumption that the rock is isotropic. In many cases the rock anisotropy is too strong to be ignored and hence the necessary correction has to be introduced. Other stress measurement methods such as the rock memory methods, the deformation rate analysis (DRA) and acoustic emission (AE), sensor the stress by comparing with artificially induced stresses without the need to infer them from strain and are insensitive to the deformation behaviour of rock (Goodman, 1963; Kurita and Fujii, 1979; Lavrov, 2002). These methods are more sophisticated and have yet to receive the recognition they deserve. Even if the rock memory techniques are used, the general tendency is to verify their results against the results of measurements performed by traditional stress relief methods. As confidence builds, this will become less necessary. An intrinsic problem of such a comparison is the assumption of rock isotropy in the interpretation of stress relief measurement results. Such an assumption can lead to considerable deviation of the interpretation of the results from the actual stresses (Borsetto et al., 1984; Hakala et al., 2007), which invariably lead to unjustified rejection of rock memory methods. This circumstance necessitates the development of methods of correction of stress relief techniques for rock anisotropy.

Currently, the research is concentrated on the development of the methods of stress reconstruction for given anisotropic rock moduli (Amadei and Goodman, 1982; Amadei, 1983; Borsetto et al., 1984; Hakala et al., 2007), which presents a considerable challenge because analytical solutions for the general case of rock anisotropy are not available. A less recognised challenge is the determination of the moduli of anisotropic rocks. The obvious difficulty is the sheer number of samples to be tested as in the simplest case of transversal isotropic rock five moduli and two angles of orientation of the plane of isotropy have to be reconstructed. A typical way of dealing with this is sub-sampling the core in different directions. This method is capable of providing the number of measurements sufficient for reconstruction of the moduli;