H-INDEX CONCATENATION

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ABSTRACT. A method for the calculation of a ‘concatenated’ h-index of jointly ranked combined bibliographies is presented in the case when only size and h-index of the original publication sets are known.

1 INTRODUCTION

Despite all attempts to refine, adjust or even improve the h-index, Hirsch’s original measure prevailed, and has already become a standard tool in evaluative bibliometrics. However, handling of this simple but somewhat uncommon statistic proved rather difficult. While classical statistics has comprehensive answers to the questions of how to treat and evaluate shares, means or extreme values, the problem of comparing or combining different h-indexes is still unsolved. Nonetheless, the question of how the h-index of the combination of two bibliographies relates to their original h-indexes is certainly of interest.

Re-ranking the union of two bibliographies is practically the only way to determine their joint h-index. The question, of course, arises of whether it is possible to directly derive the joint h-index from different individual samples without creating and re-ranking the combined publication set. In other words, how can we calculate the *concatenated h-index* of two or more h-indexes if besides the h-indexes no further information is available?

2 THE CONCATENATED H-INDEX

In order to solve this problem, we assume that the samples are based on independent, identically distributed random variables and the samples are disjoint. Analogously to earlier papers by the author (Glänzel, 2006, 2008), it is furthermore assumed for simplicity’s sake that the common citation distribution can be approximated by a non-negative continuous distribution. In particular, a *Lomax* distribution (a *Pareto* distribution of the second kind) is proposed, that is, if X represents the citation rate of a paper then

$$G(x) := P(X \geq x) = \frac{N^\alpha}{(N + x)^\alpha}, \text{ for all } x \geq 0, \quad (1)$$

where $N > 0$ and $\alpha > 0$ are the two free parameters of the *Lomax* distribution. If x is large ($x \gg N$), the parameter in the denominator can be neglected and we have

$$G(x) \approx N^\alpha / x^\alpha, \text{ for } x \gg N.$$

We assume a sample of n observations $\{X_i\}_{i=1,\dots,n}$, where all elements are independent and have a *Lomax* distribution with the same parameters N and α . In an earlier paper (Glänzel, 2008) it was shown that under these conditions the following equality holds.

$$h = N^{\alpha/(\alpha+1)} \cdot n^{1/(\alpha+1)} \Leftrightarrow h^{\alpha+1} = N^\alpha \cdot n. \quad (2)$$

Now we assume that we have two disjoint samples $\{X_i\}$ and $\{X_j\}$ of size n' and n'' , respectively. We also assume that the elements of both samples are independent and identically distributed. The h-indexes associated with the two samples are h' and h'' . In the following the *concatenated h-index* ($h' \circ h$) of the joint ranking list of the combined sample $\{X_i\}_{i=1,\dots,n',n'+1,\dots,n'+n''}$ of size $n := n' + n''$ will be determined. Applying Eq. 2 to the two samples results in $h'^{(\alpha+1)} = N^\alpha n'$ and $h''^{(\alpha+1)} = N^\alpha n''$ with $h'^{(\alpha+1)} + h''^{(\alpha+1)} = N^\alpha \cdot n' + N^\alpha \cdot n'' = N^\alpha (n' + n'') = N^\alpha n = h^{(\alpha+1)}$. Hence we have

$$h = h' \circ h'' = \{h'^{(\alpha+1)} + h''^{(\alpha+1)}\}^{1/(\alpha+1)}. \quad (3)$$

Repeated application of concatenation results in the following theorem.

Theorem 1. Assume that the random variables of k disjoint samples $\{X_{ij}\}_{j=1,\dots,n_i}$ of size n_i ($i = 1, 2, \dots, k$) are independent and are identically distributed. The common distribution is *Lomax* with parameters $N > 0$ and $\alpha > 0$. Assume further that the h-index of each sample is h_i . Then the concatenated h-index ($\circ h_i$) of the joint sample $\{X_j\}_{j=1,\dots,\sum n_i}$ is obtained from the h-indexes of the original samples h_i in the following manner.

$$h = (\circ h_i) = h_1 \circ h_2 \circ \dots \circ h_k = \left\{ \sum_{i=1}^k h_i^{(\alpha+1)} \right\}^{1/(\alpha+1)}. \quad (4)$$

Remark 1. This approximation holds even if $n_i = 1$ for all $i = 1, 2, \dots, k$, provided $N \ll 1$. In this case we have $h_i = 1$, if the paper is cited, and $h_i = 0$, otherwise. The case $h_i = 1$ occurs with probability $P(X \geq 1) = N/(N+1)$. This results in the following approximation if $N \ll 1$.

$$\left\{ \sum_{i=1}^k h_i^{(\alpha+1)} \right\} = \frac{N^\alpha \cdot n}{(N+1)^\alpha} \approx N^\alpha \cdot n = h^{(\alpha+1)}. \quad (5)$$

Remark 2. h takes integer values; therefore $\{\sum_i h_i^{(\alpha+1)}\}^{1/(\alpha+1)}$ has to be rounded.

Remark 3. Let $h = \max(h_1, h_2)$ be the maximum of two h-indexes. Then the following inequality always holds. $h \leq h_1 \circ h_2 \leq \sqrt[\alpha+1]{2} \cdot h$. The lower limit is taken if one of the h-indexes is 0; h takes the upper limit if $h_1 = h_2$. For $\alpha = 2$ this amounts to about $1.26 \cdot h$.

Example 2.1. In earlier papers (Schubert and Glänzel, 2008, Glänzel, 2008), it was shown that the assumption of $\alpha \approx 2$ provides good results for short citation windows comprising an initial period of about three years after publication. In 2002, the two multidisciplinary journals *Science* and *Nature* had an h-index of $h_1 = 106$ and $h_2 = 108$, respectively, if a three-year citation window

TABLE 1. h-index of the journals Science (A) and Nature (B) and the h-index of the jointly ranked publication list of both journals (publication year: 2002, citation window: 2002-2004)

A		B		A ∪ B	
<i>r</i>	<i>R(r)</i>	<i>r</i>	<i>R(r)</i>	<i>r</i>	<i>R(r)</i>
97	112	99	113	125	140
98	111	100	112	126	140
99	111	101	111	127	140
100	111	102	110	128	139
101	110	103	110	129	138
102	110	104	109	130	137
103	110	105	109	131	136
104	109	106	109	132	135
105	106	107	108	133	134
106	106	108	108	134	134
107	106	109	108	135	134

is applied. According to Theorem 1 we obtain $h = h_1 \circ h_2 = \sqrt[3]{h_1^3 + h_2^3} = \sqrt[3]{1063 + 1083} = 134.8$ for the joint ranking list of the combined sample. The empirical value amounts to $h = 134$. Table 1 presents the ranks (r) around the h-index as well as the citation rates of the r th most frequently cited papers ($R(r)$) for both journals as well as for the combined publication list.

TABLE 2. h-index of two bibliometricians and the h-index of the jointly ranked publication list of both authors (Data retrieved in March 2008)

A		B		A ∪ B	
<i>r</i>	<i>R(r)</i>	<i>r</i>	<i>R(r)</i>	<i>r</i>	<i>R(r)</i>
3	23	14	27	15	27
4	22	15	26	16	27
5	21	16	25	17	26
6	17	17	24	18	25
7	17	18	23	19	24
8	16	19	23	20	23
9	15	20	23	21	23
10	15	21	23	22	23
11	14	22	22	23	23
12	8	23	22	24	23
13	8	24	21	25	22

Example 2.2. In contrast to the previous example, I have selected two bibliographies with distinctly different h-indexes. This example refers to Hirsch's original idea of ranking citation rates of individuals. I have selected two bibliometricians who have never co-authored papers.

This guarantees that the two bibliographies are disjoint. Only articles, letters, notes and reviews relevant for the field of bibliometrics are taken into account. Since the intention of this example is merely the illustration of the application of Theorem 1, and not the evaluation of individual scientists, the two authors are treated anonymously. The h-index of author A amounts to 11; that of author B is 22. Table 2 presents the ranks and citation rates for the authors in the same way as Table 1. The concatenated h-index of the two authors is 23.

Unlike in the previous case, citations are cumulated from the publication year till 2008. Both authors are active since the 1980s. Although α might therefore possibly differ from 2, the assumption of $\alpha = 2$ provides acceptable results. According to Theorem 1, we have $h = h_1 \circ h_2 = \sqrt[3]{h_1^3 + h_2^3} = \sqrt[3]{11^3 + 22^3} = 22.9$. This again illustrates the robustness of the formula for Hirsch-index concatenation.

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