DEM simulations of the particle flow on a centrifugal fertilizer spreader

P. Van Liedekerke^{a,*}, E. Tijskens^{a,*}, E. Dintwa^a, F. Rioual^b, J. Vangeyte^c, H. Ramon^a

^a Department BIOSYST, K.U. Leuven, Kasteelpark Arenberg 30, B-3001 Leuven, Belgium

^b CEMAGREF Clermont-Ferrand, Montoldre sur Allier, Domaine des Palaquins, 03150 Montoldre, France

^c ILVO, Burg. Van Gansberghelaan 115 bus, 9820 Merelbeke, Belgium

ARTICLE INFO

Article history:

Received 3 July 2007

Received in revised form 6 August 2008

Accepted 7 August 2008

Available online xxxx

Keywords:

Discrete element method

Granular fertilizer

Centrifugal spreader

Multi particle

ABSTRACT

Usually, the performance of centrifugal spreaders must be evaluated in large halls by capturing the fertilizer distribution patterns in standardized tests, often carrying a big cost to the manufacturers. In contrast, this paper proposes a first attempt to model a particle flow subjected to a spinning disc using the Discrete Element Method (DEM) starting from the particle outflow of a bin, using flat as well as inclined discs. The model is validated by experiments in two different ways. The first manner is the measurement of the cylindrical mass distribution along the edge of the disc by a device that collects the fertilizer particles in a tray of baskets around the disc. A second method consists of collecting the particles on the ground after their ballistic flight through the air. Both validation methods are relatively cheap and fit into the present statistical or qualitative interpretation of DEM simulations. Additionally, a number of rotational disc speeds is chosen (300–650 rpm) to incorporate velocity dependent effects of the particle flow. It was found that the DEM simulations show a good qualitative and considerable quantitative agreement with the experiments. The deviations between the simulations and experiments are profound at high disc rotational speeds (500–650 rpm) and can be identified as (1) an underestimation of the simulated particle velocities at the edge of the disc and (2) a too low dispersion on the (vertical) simulated particle velocities at the edge of the disc. A parameter study revealed that (1) can be resolved by introducing a velocity dependent friction coefficient, in agreement with literature. The influence of other model parameters such as particle damping and stiffness appears to be small, while the introduction of a rolling friction coefficient to mimic rolling resistance or particle shape does not provide any answer either, and hence reason (2) at this moment must be addressed to unknown external factors such as disc plane vibrations appearing at higher disc speeds.

© 2008 Elsevier B.V. All rights reserved.

1. Introduction

For the past few decades there has been a growing concern about the environmental implications associated with the use of soil fertilizers. In Europe, over 90% of fertilizer is distributed using spinning disc spreaders. The popularity of spinning disc spreaders lies with their relative low cost and high accuracy. However, the performance of these machines is highly dependent on the particle properties (e.g. friction coefficients, particle shape...). This results in large differences in spread patterns depending on the fertilizer type as well as the prevailing weather conditions (e.g. air humidity) [8].

The primary concern of a spreader manufactures is the construction of spreading tables which link fertilizer properties and doses (kg/ha) to the machine settings. This can be either done by measuring the spread pattern in spreader halls (expensive, even though nowadays there are techniques which are less time consuming [16], or to interpolate the spreading characteristics from data bases or characteristics which are

easy to measure (mostly inaccurate). Despite their need, these tests are expensive, and require huge amounts of fertilizer. Furthermore, they are not uniquely standardized [14].

Reumers et al. [18] proposed a method based on measurements with a cylindrical collector tray around the disc edge, and photographic tracking for determining particle velocities. By using a ballistic model, the total spread pattern can be determined. To apply the same principle online in the field, determination of particle velocities methods have been proposed by Vangeyte et al. [23], Grift and Hofstee [6]. The primary objective of these models is to replace the expensive spread hall measurements.

The granular flow on a spinning disc with vanes is similar to paddle systems (mixers), which have been studied by Steward et al. [21], Zhou et al. [26], and Kuo et al. [12] using DEM simulations. In these experiments, particles are located within the boundary of a cylinder. Using the Positron emission particle tracking (PEPT) technique, an individual tracer particle in the flow can be monitored instantly and non-invasively. DEM simulations can be validated, although based on a single particle. Experiments with a granular flow on a centrifugal spreader are usually performed by collecting the particles after they leave the disc, resulting in a spatial spread pattern. Although these 70

* Corresponding authors.

E-mail addresses: Paul.VanLiedekerke@biw.kuleuven.be (P. Van Liedekerke), Engelbert.Tijskens@biw.kuleuven.be (E. Tijskens).

Fig. 1. Disc types used; left: lawn spreader disc (type A); right field spreader disc (type B).

experiments provide no information on what happens during the process, they can accurately summarize an experiment and hence be a suitable partner for DEM validations. In this paper, an experimental set up of a particle flow on a spinning disc is given in detail, as well as the comparison with DEM simulations.

This paper has the intention to demonstrate that the discrete element method has the potential to be an even more cost efficient alternative to obtain the spreading pattern for a fertilizer. Therefore, a series of simulations and experiments with matching conditions was conducted. Although the results of the comparison are quantified, they still must be interpreted in a statistical and qualitative way, as with most present-day DEM- simulations. Nevertheless, they can provide the manufacturer valuable information about the particle flow on a spinning disc.

1.1. The discrete element model

In DEM, the contact deformation between two bodies is represented as a single vector describing the deformation of the contact point in the normal and tangential directions. This vector is related to contact forces by a contact force model. For the problem at hand, Van Liedekerke et al. [24] have proposed a Hertz–Kuwabara–Kono model [13,2,20,17] for the normal contact force, $\mathbf{N}$:

$$\mathbf{N} = \min\left(0, -\sqrt{|\delta_N|}\left(k_N\delta_N + c_N\dot{\delta}_N\right)\right)\mathbf{n}. \quad (1)$$

Here, k_N and c_N are the non-linear contact stiffness and damping parameter, respectively. The quantity δ_N is termed the virtual overlap (deformation) of the contacting bodies. The vector $\mathbf{n}$ is a unit vector normal to the contact surface. For the tangential contact force, $\mathbf{T}$, a

linear elastic spring is augmented with dry Coulomb friction, which, formulated as an incremental expression, reads:

$$\mathbf{T}_{t+\Delta t} = -\min\left(\|\mathbf{Q}_t^{t+\Delta t}\mathbf{T}_t + k_T\Delta\delta_T\|, \mu\|\mathbf{N}_{t+\Delta t}\|\right)\mathbf{t} \quad (2)$$

Here, the quantity $\Delta\delta_T$ is the incremental equivalent of δ_N in the normal contact force model over the time step Δt . For a detailed account of how $\Delta\delta_T$ is computed, see [24]. In Eq. (2) $\mathbf{Q}_t^{t+\Delta t}$ is a rotation matrix which maps the orientation of the particle at time step t to that at time step $t+\Delta t$ and accounts for the rotation of the contact plane and the tangential contact force during the time step. In Eq. (2) the min-operator implements the Coulomb friction limit with constant friction coefficient μ between two bodies in contact:

$$\|\mathbf{T}\| \leq \mu\|\mathbf{N}\|, \quad (3)$$

Although more realistic and complex tangential force models exist [19], the choice for this simple is justified if the collisions are mainly driven by friction, and the use of a complex tangential model is less necessary. By simple linear reasoning, such a condition is equivalent to the exceeding of the friction limit at the beginning of contact:

$$\Delta\delta_T k_T \geq \mu k_N \Delta\delta_N \quad (4)$$

Or, by terms of contact velocities, and making abstraction of both normal and tangential stiffness:

$$\frac{\dot{\delta}_T}{\dot{\delta}_N} \geq \mu \quad (5)$$

Eq. (5) reads that such a process should be driven by shear collisions, and is more feasible for lower friction coefficients. It could be argued that the particle flow on a spinning disc fulfils this

Fig. 2. (a) Photo of a testing device for measuring the tangential mass distribution pattern. (Provided by Amazone Werke GmbH, Hasbergen, Germany); (b) Geometry of the testing device for measuring the tangential mass distribution pattern (top view).

condition, because the particles are mainly driven outwards the disc centre, with little difference in normal speeds, while due to the angular velocities and contact with the vanes, the shear velocities are substantial. Hence, a tangential force model as proposed in Eq. (2) was implemented. However, in the case of static a process, the choice of such simple model may not be appropriate.

The use of a rolling friction coefficient to incorporate the resistance to rolling of spherical particles as proposed by Brilliantov et al. [3] or Zhou et al. [25] was primarily not considered in the model. Even for a nearly spherical fertilizer particle, it is hard to determine such a coefficient in a rolling experiment, because of irregularities on its surface. A rough estimation could be obtained by means of its restitution coefficient [3]. However, the use of a rolling friction coefficient could have significant consequences on the results via the complex rolling-sliding regime of the particles and therefore a brief parameter study is discussed in Section 4.2.

The contact forces, together with gravity, drive the motion of the particles, which is described by Newton's and Euler's equations of motion for each particle in the system [1,7]. The equations of motion are solved with a Leapfrog integration method [5].

2. Experimental methods

Testing the spreadability of new kinds of fertilizer spreaders can be done in 2 ways; one possibility is to use a device like the one shown in Fig. 2, which surrounds the disk with a circular collector tray. The method is fast and cheap, but provides only information about the tangential mass distribution, and not about the velocities of the particles leaving the disk. Optical measurement techniques can bring a solution here [6,18,23].

A second method lies with the direct measurement of the total spread pattern produced by the spinning disc in a (large) hall. This however requires a considerable amount of fertilizer particles in order to get a reliable distribution. Furthermore, it is a very time consuming activity, although as previously mentioned, nowadays faster techniques are available. Both kinds of experiments were applied in this paper; the details are discussed below.

The particles used in the experiments are NPK granules with a fairly spherical shape. Their size distribution can be assumed as lognormal. A sample of 100 particles was monitored to determine the radii and density.

The experiments were conducted with two kinds of discs. The first one (Type A) is a disc for a domestic lawn spreader (see Fig. 1, left). A

Fig. 4. Scheme of the TSP experimental set up.

lawn spreader typically features a 4-vane disc, usually made of a light material (e.g. PVC). The size of the disc is small, e.g. 0.15 m radius. To reduce the production cost, the disc is deformed in the regions between the vanes, and the vanes themselves have no horizontal flaps which normally keep the particles lower than the vane height. This kind of a spreader is made predominantly for domestic use.

The second disc (Type B) has a conical inclination ϑ_d of 9° and a radius R_d of 0.29 m. In the experiment, two L-shaped vanes (see Fig. 1, right), of length $L_{\text{vane}} = 0.3$ m and height $H_{\text{vane}} = 0.036$ m were used. As with most current spreaders, the vanes are not placed radially, but have an offset $D_{\text{vane}} = 0.045$ m relative to the centre of the disc. This kind of spreader is made mainly for exhaustive field applications.

2.1. Cylindrical measurements

The cylindrical spread pattern (CSP) is measured by a device (also called Carrousel) where the spinning disc is surrounded by a cylindrical collector tray in which the particles are captured. The disc is driven by a 380 V electrical motor with accurate control of rpm (± 1 rpm). In Fig. 2a, such apparatus (provided by Amazone Werke) is shown. The collectors span sectors of 30° and are positioned at a distance of 0.5 m from the centre of the disk.

In order to prevent the particles from moving inwards, to the centre of the conical disk, a metal body which has the shape of a truncated cone, nearly cylindrical, is mounted in the middle of the disc (see Figs. 2a and 3). To avoid spill over of particles, most current spreaders have vanes with horizontal flaps to keep the particles below the vane height as much as possible.

The fertilizer particles are fed on to the disc by a plastic funnel with an orifice diameter of 0.029 m using a valve which is mounted above the spreader. The funnel is placed at a height $H_{\text{bin}} = 0.11$ m above the disc, a distance $D_{\text{bin}} = 0.1$ m from the centre of the disk and an angular position of 0° .

In each experiment, the funnel was filled with (NPK) fertilizer for domestic use. The particles are assumed to be distributed log-normally in size with a measured average diameter of 0.00126 m and a standard deviation of 0.00014 m. The measured material¹ density of the fertilizer is 1775 kg/m³. The amount of particles in each experiment is 0.5 kg. As the funnel yields a mass flow of 0.11 kg/s, the funnel is emptied in 4.55 s. Thus, at 300 rpm it takes 45 vane passages to empty under the funnel, and 90 times at 600 rpm. The particle flow is initialized by manually opening a valve (see Fig. 3), after which the particles fall onto the disk and are taken by the vanes to be thrown in the collectors. The contents of the collectors are then weighed.

2.2. Rectangular measurements

In a rectangular experiment, the transverse static pattern (TSP) is measured. This is done by aligning baskets at a distance vector (X, Y, Z) from the disc centre (see Fig. 4), where X is the transverse distance, Y is the longitudinal distance, and Z is the height between the baskets

Fig. 3. Side view of the disc and particle bin.

¹ Not bulk density.

Fig. 5. Lined collector tray baskets for measuring the transverse spread pattern.

and the disc. By measuring the TSP over different Y distances (when the tray is shifted incrementally over the total Y range), a total transverse spread pattern (TTSP) can be obtained. The disc is driven by a 1 kW electrical motor, and the speed is monitored by an rpm counter before each experiment.

The relative height Z of the disc to the baskets was 0.28 m and it was kept constant over all experiments. The baskets have dimensions of 0.25 m × 0.25 m. The inside of the baskets is covered with a soft material to prevent particles from jumping out when impacting at high speed. The total length of the tray is 3.5 m, equivalent to 16 baskets (see Fig. 5).

3. Computational setup

Ideally, the simulations should respect the initial conditions and the geometry of the experiment. However, there are some situations where simplifications cannot be avoided. As examples, first, the NPK particles are by production not perfect spheres. Their diameter varies depending on the direction along which it is measured and the also show some surface roughness. In the simulation, the particles are approximated as perfect spheres, and the surface roughness is neglected. Secondly, the particle assembly in the simulation is a random realisation of spherical particles according to the above mentioned particle size distribution and their initial positions are determined by a simulation in which they are dropped into the bin with the valve closed. These initial conditions have been used for all simulations. This is a simplification of the real situation where the particle assembly would be completely random. Lastly, the positions of the vanes at $t=0$, when the valve is opened, is not known. However, this uncertainty is averaged by the number of vane passages in each experiment, and the number of experiments.

3.1. Model parameter settings

Interaction parameters between particles are hard to determine, especially for such small objects as fertilizer grains. Basic methods [24] were used to obtain useful values for contact stiffness, damping and friction properties for impact between a particle and a metal plate. As the experimental determination of particle-particle collision forces for fertilizer particles is not practically feasible, some simple assumptions are introduced to derive particle-particle contact force model parameters from particle-metal plate experiments. Treating the elastic and dissipative part of Eq. (1) independently, then the elastic part is the Hertz' law for 2 spheres which is given by:

$$N = \frac{4\sqrt{R_e E_e}}{3} \delta_N^{3/2} \quad (6)$$

Where N is the normal contact force, δ_N is the virtual overlap between the spheres, R_e is the effective curvature and E_e , the equivalent elasticity modulus, is calculated as

$$\frac{1}{E_e} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}, \quad (7)$$

where ν is the Poisson ratio of the material. Thus, if we move from the contact between a particle and a metal plate, for which ($R_1 \rightarrow \infty$, $E_1 \gg E_2$) to a contact with two identical spheres ($R_1 = R_2$, $E_1 = E_2$) the stiffness of Eq. (6) diminishes by a factor $2\sqrt{2}$. For the damping parameter in Eq. (1), no such derivation can be made. However, Pöschel and Schwager [17] proposed following heuristic form for a visco-elastic contact force model:

$$N = \frac{4}{3} \sqrt{R_e E_e} \left(\delta_N^{3/2} + \frac{1}{2} (C_1 + C_2) \dot{\delta}_N \delta_N^{1/2} \right) \quad (8)$$

where $C_{1,2}$ are damping constants, linked to the material's viscosity [2]. Assuming that the steel plate material reacts purely elastically during collision ($C_1 \approx 0$), this would imply that changing from a particle-hard material interaction to identical particles interaction ($E_1 = E_2$, $C_1 = C_2$), the damping coefficient would be doubled. However, there is no experimental evidence for this. Dintwa [4] showed that a coefficient, approached by:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \quad (9)$$

yields better agreement with finite element analysis, although an offset value was found compared to FEM. Another drawback is that values become unrealistic when one of the materials has zero damping.

The case of a spherical particle colliding with a cylindrical object (see further), is even more complicated. Unlike the previous case, where the contact surface between the objects is circular, it is now generalized to an elliptical shape. For the elastic normal contact, Eq. (6) is then transformed [10] to

$$N = \frac{4}{3} \sqrt{R_e E_e} \delta_N^{3/2} F(R_1/R_2) \quad (10)$$

In this formula, we have

$$\frac{1}{R_1} = \frac{1}{\infty} + \frac{1}{R_s} \text{ and } \frac{1}{R_2} = \frac{1}{R_s} + \frac{1}{R_c}. \quad (11)$$

R_1 and R_2 are defined as the principal equivalent radii of curvature of the two indenting bodies (R_s and R_c are the radii of the sphere and the cylinder respectively). Further, R_e is defined as

$$R_e = \sqrt{R_1 R_2}. \quad (12)$$

The function $F(R_1/R_2)$ is actually a correction which accounts for the eccentricity of the ellipse. Note that if $R_c \gg R_s$, we have $R_1 \approx R_2$ and $F(R_1/R_2) \approx 1$, because the contact area will lose its eccentricity; hence Eq. (10) will again describe the contact between a particle and a plane. Moreover, even if $R_c \approx R_s$, which is the case for the edges of the vanes, $F(R_1/R_2) = 0.98$. Hence it is reasonable to use $F = 1$ in all cases for the simulations.

In the simulations, the contact stiffness between particle and steel plate k_N was set to $2e7 \text{ Nm}^{-3/2} \pm 3e6 \text{ Nm}^{-3/2}$, a value that was obtained by means of an Universal Testing Machine (UTS), while the damping coefficient c_N was determined as $140 \text{ m}^{-3/2} \text{ s}^{-1}$ using trial and error until the model gives comparable results with experimental rebound tests (e.g. comparing restitution coefficients) [24].

The dynamic friction coefficient μ between fertilizer and disc/vane was obtained by letting the particle slide on a steel plate until a constant particle velocity is reached. By measuring the slope of the

Fig. 6. Particle crossing an edge; right: arrows denote the bad contact area in case of a plane with zero thickness (a), beam (b). (c) depicts a polygonal edge which can further reduce the bad contact area.

plate, the friction coefficient can be determined. In the experiment, an average value of 0.35 $\pm$ 0.08 was measured. Since the friction coefficient between fertilizer particles is hard to determine, the same value determined for fertilizer/plate was assumed.²

3.2. Contact detection

Basically, a particle can have 4 different potential contact partners; namely: another particle, a vane, a vane edge or the disc [24]. The search for possible contacts is done pair wise with the machine parts (vanes, disc, and edges). For particle-particle contact, a more efficient algorithm (Verlet List algorithm [17]) is necessary. Note that all machine parts are geometrical surfaces which are *non-closed*. Hence, a particle may make contact with an edge of this surface. In some cases, this can lead to unexpected behaviour, as described in the next section.

3.2.1. Handling particle-edge contacts

An important issue is that a problem may appear when a particle should hit an edge of a plane A. Since all parts are represented as non-closed hyper-surfaces in the 3D space, they actually have no thickness. As a consequence, when a particle approaches the object from a direction as depicted in Fig. 6, a *non-continuous* – and possibly very large – overlap will rise as the particle moves from the half-space I to half-space II, defined by the edge of the plane, in the time step $t' \rightarrow t' + \Delta t$, see Fig. 6. The particle will be immediately forced to move away from the plane. As the contact force increases with the virtual overlap to the power 3/2, contact forces which are many orders of magnitude larger than the realistically plausible can arise from one time step to the other, leading to an ‘explosion’ of the particle assembly. Fig. 6b illustrates that the problem is not related to the zero thickness, but rather to the presence of a singular edge contact zone, i.e. one with an infinitesimally small local radius of curvature. A generic solution to the problem was proposed by Tijssens [22] in the framework of particles with arbitrary shape.

To quantify this problem, one can define the probability P of an edge being badly hit by a particle as

$$P = P_{\pi} P_v \quad (13)$$

with P_{π} being the probability that a particle is in a space-sector defined by the edge of the plane (I in Fig. 6), and P_v the probability the particle has the right approaching velocity. This velocity is dependent on the area (denoted by the double arrows in Fig. 6) which would

result in a non-realistic contact force if a particle would cross it. If we assume the particle to be at a distance d from the edge then:

$$P_v \sim \frac{A}{d^2}. \quad (14)$$

In the case of a single plane (a), we have:

$$P_{\pi} = \frac{2\pi}{4\pi} \text{ and } P_v \sim \frac{2LR_s}{d^2} \quad (15)$$

with L the length of the plane edge. This results in:

$$P \sim \frac{LR_s}{d^2}. \quad (16)$$

In case of a beam (b), we have:

$$P_{\pi} = \frac{\pi}{4\pi} \text{ and } P_v \sim \frac{\sqrt{2}LR_s}{d^2}, \quad (17)$$

hence:

$$P \sim \frac{\sqrt{2}LR_s}{4d^2} \quad (18)$$

yielding a probability of a bad contact with an edge $\sqrt{8}$ times smaller for a beam than for a single plane. Giving this beam a polygonal edge (see Fig. 6c) would further reduce this probability, but one could also use a cylinder which represents the thickness of the vane. Yet, a trade off has to be found in computational efficiency and accuracy. A representation of a vane object for which no false contacts show up is too expensive here.

In this paper, vanes are constructed based on the particle flow direction (see Fig. 7). Single planes with a cylindrical edge are used because the vane thickness is 200 times smaller than the length. As such, the probability for bad contacts is minimal while the computational efficiency is optimal.

3.3. Simulation of the particle flow in the bin

DEM simulations are quite time consuming because at each time step one must find out which particles are in contact and thus subject to contact forces. The time step is determined by the duration of a collision, which decreases with the contact stiffness k_N , and thus with the equivalent Young's modulus. As a rule of thumb, the number of time steps during a collision should not be less than 20, to ensure proper integration of the contact forces. In order to reduce the computational work, the number of particles is reduced by mimicking the funnel as a tube with the same outlet diameter as the funnel (see Fig. 8). The tube is filled with a number of particles generated randomly and depositing them in the tube under the action of gravity, pretty much as how the funnel is filled in the experiment. A random number generator is used to generate the

Fig. 7. Handling the vane edges: double lined edges represent a cylinder attached.

² Particle-particle friction could be higher the wall-particle friction, based on the higher observable smoothness of the vane surface.

Fig. 8. Geometry and filling of the tube that mimics the experimental funnel in the simulations. Left: gravitational deposition of particles in the tube, right: relation between the experimental funnel and the simulated tube.

Fig. 10. (a) Cylindrical plots of experiment (solid lines) and DEM simulation (dashed line) for disc type A, $\Omega_d=300$ rpm; (b) Cylindrical plots of experiment (solid lines), and DEM simulation (dashed line) for disc type A, $\Omega_d=400$ rpm. Error bars are shown.

neighbouring particles. In this way the minimum number of particles that is needed, is determined by the requirement that the tube should never run empty.

particle radius according to the desired statistical distribution. To ensure that the particles come out of the tube at the desired flow rate, the velocity, $\mathbf{v}$, of the particles in the tube is frozen at a value corresponding to the flow rate:

$$\mathbf{v}_{bin} = -Q_{exp}/A\rho_{bulk}\mathbf{e}_z. \quad (19)$$

Here, Q_{exp} is the experimental mass flow, A the cross section of the tube, ρ_{bulk} the fertilizer's bulk density, and $\mathbf{e}_z$ the unit vector in the vertical direction. Since high speed camera observations revealed that $\|\mathbf{v}_{bin}\|$ from Eq. (19) is underestimated and A is actually smaller (the particles moving out of the bin do not use the whole cross-sectional surface, but move instead faster downwards). By means of a high speed camera, A was proportionally recalculated using the observed $\|\mathbf{v}_{bin}\|$. Thus, rather than being accelerated due to the action of gravity, the particles move downwards at constant speed. Also, the wall friction is switched off until just before the particles are going to leave the tube, when gravity and wall friction are switched back on and the particles start to accelerate. The height ε of the zone where the particles are subjected to gravity and wall friction has no significant influence on the outcome, as long as it is smaller than a few centimetres. Subsequently, the particles fall on the disk, are caught by the vane, and are thrown into the collector bins. Once it is known in which bin the particle will end up, it is useless to the simulation. However, rather than taking it out of the simulation, the particle is recycled back into the tube (see Fig. 9) where, once they reach the region indicated by ε , their velocity is frozen to the value of its

Fig. 9. Snapshot of a DEM simulation (for Type B disc): side view and top view.

The rotating disk is implemented as a conical disk while the L-shaped vanes are implemented as two planes of zero thickness. The contact problem discussed in Section 3.2.1 of the contact with the singular edges is solved by ‘gluing’ cylinders along the edges as depicted in Fig. 7. Only three cylinders need to be added, the remaining edges are either hidden by the connection of the vane to the cone, or have zero probability of catching a particle in the given confirmation and dynamics. A snapshot of a simulation is shown in Fig. 9.

In the experiments, a natural variance of the particle positions can be expected since each bin filling is different. The measured bulk density in the funnel was 1040 kg/m^3 versus 1050 kg/m^3 in the simulated tube. By means of Eq. (19) one obtains $A=0.00035 \text{ m}^2$ with $Q_{\text{exp}}=0.11 \text{ kg/s}$ and $||\mathbf{v}_{\text{bin}}||=0.3 \text{ m/s}$. In the simulation, a minimum of 7000 particles is needed, having a theoretical equivalent of approximately 107 g fertilizer. Due to the recycling of particles, the actual mass of fertilizer that is spread in the simulation is $Q_e T_{\text{sim}}$, with T_{sim} the simulated time span.

4. Results and discussion

4.1. Output variables

In order to have a good comparison between simulation and experiment, 4 different rotation speeds of the disc were chosen: 300 rpm, 400 rpm, 500 rpm and 650 rpm. At each speed, the test was repeated 5 times to average out possible variations due to the unknown position of the vanes at $t=0$.

Especially with high disc speeds, particles may suffer high breakage and some are degraded to a coarse dust, which easily sticks to some parts of the collectors. Even when finer remains are removed from the metal parts and added to the respective collector, a loss of a small fraction can be attributed to impacts with the vane edges (bouncing) or the metal divisions between the collectors. In the simulation however, this will not appear since the particles' positions are simply written down when they reach a distance of 0.50 m from the centre. To account for this, it is better to compare particle fractions in each compartment rather than the collected mass.

Table 1

Summarize of the characteristics of the CSP measurements and comparison with DEM

(A)	Experiment		Simulation	
	300	400	300	400
$\Omega_d(\text{rpm})$				
$\bar{\alpha}(\circ)$	165.6	NPK 173.2	167.3	173.6
$RD(\text{exp, sim})(\%)$	–	–	2.7	4.1
β	38.2	40.6	38.5	40.3
$\bar{v}(\alpha > 120^\circ)(\text{m/s})$	N/A	N/A	5.8	7.9
(B)				
	300	500	300	500
$\Omega_d(\text{rpm})$				
Fertilizer type	NPK	NPK	Spheres	Spheres
$\bar{\alpha}(\circ)$	146.5	127.5	150.2	136.4
$RD(\text{exp, sim})(\%)$	–	–	9.1	19.1
β	28.0	23.9	29.9	23.8
$\bar{v}(\text{m/s})$	N/A	N/A	11.9	19.6
(C)	Experiment		Simulation	
	650		650	
Fertilizer	NPK		Spheres	
$\bar{\alpha}(\circ)$	121.8		131.4	
$RD(\text{exp, sim})(\%)$	–		20.6	
β	20.1		22.9	
$\bar{v}(\text{m/s})$	N/A		24.9	

(A) Disc type A with $\Omega_d=300 \text{ rpm}$, 400 rpm. (B), (C) for disc type B with $\Omega_d=300 \text{ rpm}$, 500 rpm and 650 rpm respectively.

Fig. 11. Cylindrical plots of experiment (solid lines) and DEM simulation (dashed line), for (a) $\Omega_d=300 \text{ rpm}$ and (b) $\Omega_d=500 \text{ rpm}$ for disc type B. Error bars are omitted.

To summarize an experiment, the average distribution angle for the CSP is defined as:

$$\bar{\alpha} = \sum_i f_i \alpha_i \quad (20)$$

with α_i the angle of each i -th compartment and $f_i = \frac{M_i}{M_{\text{tot}}}$ the fraction of particles. One may also introduce a measure for the width of the pattern, β :

$$\beta = \sum_i f_i |\alpha_i - \bar{\alpha}|. \quad (21)$$

After manipulation of the data of the model output, a comparison can be made with the data from experiments. Olieslagers [14] proposed a deviation function (RD) for comparing simulations and experiments, using the sum of absolute differences which can be used for both CSP and TSP:

$$RD(\text{exp, sim}) = \frac{\sum_i (|M_{i,\text{exp}} - M_{i,\text{sim}}|)}{M_{\text{tot,exp}} + M_{\text{tot,sim}}} 100. \quad (22)$$

Fig. 12. Cylindrical plots of experiment (solid lines) and DEM simulation (dashed line), for $\Omega_d=650$ rpm for disc type. Error bars are omitted.

4.1.1. Cylindrical measurements

Generally, the results of simulations and experiments are best viewed in a plot of the grain fraction as a function of the compartment degree. The distribution along the compartments is bell-shaped; in the first compartments, there are less grains because these segments are almost at the same angle as the orifice position and most of the particles have not yet left the disc. Conversely, for the furthest compartments, almost all the particles have left the disc, hence the lower amount of grains collecting there. Fig. 10a and b give these distributions for a disc type A with a rotational speed of 300 rpm and 400 rpm respectively. The dashed lines in the figures are for simulations while solid lines are for experimental data. The width of the error bars is $\pm 2\sigma$, where σ is calculated from 5 data points.

Fig. 10 clearly shows a very good agreement between simulation and experiment, with most points lying in the range of the error bars, but with a small deviation in the 200° compartment. For the rotational speed of 400 rpm, the simulation results again are in good agreement, although a small offset to the right side is established.

From Table 1 (A), showing all the characteristics of the simulated and experimental patterns, one concludes that the simulation output is very close to the experimental data of NPK measurements; values for RD(exp, sim) are less than 5%.

Measurements with the field spreader disc (type B) were done for disc rotational speeds of 300 rpm, 500 rpm, and 650 rpm. The results are shown in from Figs. 11 and 12 respectively. One notices that the curves gradually shift to the left (e.g. lower α values), because the particles leave the disc earlier; and also get narrower (e.g. β decreases). The error bars, which are comparable with previous experiments, are omitted to have a better view.

The simulated grain distributions are also shown in Figs. 11 and 12 by dashed lines. A comparison between simulation and experimental curves shows that, for a disc speed of 300 rpm, the DEM simulations approximate the experimental curve quite well (deviations are ~10%), but for the 500 rpm case, the deviations become substantial (19%) while for 650 rpm they are even higher (21%). Moreover, all simulated curves are offset to the right (higher α values), meaning the particles leave the disc later, and the greater the disc speed the more this is pronounced. The results of the experiments are further summarized in Table 1 (B, C).

4.1.2. Rectangular measurements

The measurements of the total spread pattern, were done on an area of 25 m² covered with a plastic foil. Because of the spatial limitations on the measuring hall, the test for this experiment was only performed by a lawn spreader disc (type A). The main advantage of experiments with this type is that their range is small (not more than 3 m working width).

Hence, the TSP can be measured in a relatively small space. Furthermore, the particles have a relatively low velocity, which makes them less susceptible to deviations induced by aerodynamic forces. Measurements with the conical disc were technically not feasible.

Generally, there are 11 experiments and $15 \times 11 = 165$ measuring points required to cover the whole area of the spread pattern. However, one can also skip lines in order to reduce this number but still have enough control points. In total, 3–5 lines were considered at a distance³ from $Y=0$ m, -0.5 m, -1 m, -1.5 m, and -2 m to the disc centre, depending on the disc rotational speed. To minimize the interference from particles in neighboring baskets, a soft layer of material was placed against the walls of the baskets. In this manner, particles cannot jump out of neighboring baskets. Each single experiment was repeated 4 times for each longitudinal distance in order to have a good repeatability. The disc rotation is clockwise.

In Fig. 13, a contour plot of the total area covered by the measured and simulated pattern is given for a disc rotational speed of 400 rpm. The missing lines $Y=-0.25$ m, $Y=-0.75$ m, $Y=-1.25$ m and $Y=-1.75$ m were obtained by interpolating between the measured lines. The total area covered by the measurements is approximately 9 m² which resulted in 94% of the total particle mass used in the experiment. The remaining fraction missing is due to particles that land beyond the basket area and the interpolation deviations between the lines.

The pattern has a typical bended shape around the disc, with a high concentration zone in the front of the disc. Besides this there was also an increase of density on the right side of the disc. This small “peak” corresponds to the particles bouncing from contact with the vane edges and spill-over. The effect is probably somewhat exaggerated with respect to the type B discs, because the latter mostly have horizontal flaps which actually limit this spill, whereas the lawn spreader disc has not.

In Fig. 14, the particle distributions of the lines $Y=0$ m, -0.5 m, -1 m, -1.5 m, and -2 m are plotted against the transverse distance (X direction) for a disc speed of 400 rpm. The error bars representing the $[-2\sigma, 2\sigma]$ range are given only for Fig. 14 and b for clarity. For the validation of the model, the output files from the same simulations as in Section 4.1.1 were used as input for the ballistic model. For the ballistic flight, a drag coefficient of $C_p=0.44$ is used [14]. Subsequently, a post processing program calculates the distributions on the ground, and calculates the respective transverse distributions. The results of the DEM simulations are denoted by the dashed lines.

From Fig. 14a, b and c, giving the patterns for $Y=0$ m, $Y=-0.5$ m and $Y=-1$ m, we conclude that the DEM simulations predict the pattern qualitatively well (e.g. shape of the curve), but some overestimations of the particle collected masses are observed at all distances. In the overall view, Fig. 13 shows that the experimental pattern in radial direction, e.g. distance from the disc centre, is wider than the simulated. In other words, the simulated pattern is generally smaller in surface and more concentrated in the radial direction than the experimental one.

In Table 2, the results are further summarized, giving the total particle mass per line and the deviation of the DEM simulation. The relative average deviation (RD) per line Y , given by:

$$RD_Y(\text{exp, sim}) = \frac{\sum_x (|M_{XY,\text{exp}} - M_{XY,\text{sim}}|)}{M_{Y,\text{exp}} + M_{Y,\text{sim}}} 100 \quad (23)$$

varies around 25%. The fact that these deviations are much larger for the similar cylindrical experiment, is due to a number of reasons:

1. The low angular resolution of the baskets (30°) averages deviations too much.
2. A fraction (2%) of the mass is not captured because of bouncing in the Carrousel.
3. The cylindrical measurements give no information about the velocities.

³ A distance of 1 m means the basket collects the mass for 1 m–1.25 m, and so on.

Fig. 13. Measured (a) and simulated (b) total transverse spread pattern of NPK grains, with disc type A and $\Omega_d = 400$ rpm. Fig. (c) is obtained after adding a normally distributed vertical random component to the particle disc edge velocity (see Section 4.2). The location of the disc is indicated by (*).

4. An additional deviation is made in the ballistic flight calculation (aerodynamic properties).

To exclude the uncertainty about the influence of the measurement errors for D_{bin} , R_{bin} , h_{bin} , Ω_d , and L_{vane} , additional simulations were run with a variation of 2% of these dimensions. This however did not have any significant improvement on the output.

Table 2 also shows that the sum of the predicted mass over the transverse direction (M_Y) is in good agreement with the experiments, which confirms that the qualitative prediction of the model is good.

Additional measurements were carried out for a disc speed of 300 rpm. In Fig. 15 the fertilizer mass distribution (solid lines) is only given at lines $Y = -0.5$ m, $Y = -1$ m and $Y = -1.5$ m, because the range of the pattern is smaller. For the comparison, the DEM simulations are indicated by the dashed lines. Fig. 15 shows that compared to the 300 rpm measurements the agreement is much better here, with a fairly good overlap, except for the collector line most distanced from the disc centre, where a significant underestimation is observed. Table 2B reads that the deviations are generally lower than for the 400 rpm experiment, although the same significant deviation is observed for the most distanced line.

4.2. Discussion of the results

Referring to single particle models and experiments [9,15,24], a higher α value of a single particle trajectory can be explained by (1) an initial position of the particle being nearer to the disc center, (2) a smaller initial outward radial particle velocity on the disc, or (3) a higher opposing force on the particle moving on the disc, resulting in a particle velocity and hence a later exit off the disc. Although no multi particle experimental tests were conducted, explanations (1) and (2) were not considered, based on the fact that the average initial positions and velocities of the particles leaving the bin are quite accurately specified in the experiment. Also, additional test simulations considering the possible variations of the average positions and velocities⁴ of maximum 2% did not give any satisfying improvement. Explanation (3) could be a result of either a measurement error of the friction coefficient in the tangential contact model, or a measurement error of the damping in the normal force model. A model parameter study using a flat disc rotating at 500 rpm, depicted in Fig. 16a, shows that the DEM simulations show a high sensitivity to friction coefficients in the CSP plots (e.g. the curves shift to lower average outlet angles when using lower friction coefficients). On that basis,

one could hypothesize that the deviations between simulations and experiments are due to an overestimation of the friction coefficients.⁵

This brings us to the idea to adapt the model by extending it with a linear velocity dependent friction coefficient:

$$\mu^* = \mu + av \quad (24)$$

where v is the tangential wall-particle velocity. To avoid spurious simulation results, only the wall-particle friction coefficient was taken to be velocity dependent. The reason for this is that particles can approach each other with high relative velocity, leading to possible negative friction coefficients. This is especially the case when recycling the particles into the bin. The friction model as defined by Eq. (3) would therefore be too simple.

To find the optimal value of a in Eq. (24), a Matlab® lsqnonlin algorithm applied to the differences of the average outlet angles between simulation and experiments for disc type B was used. In Fig. 17 the outlet angles are given as a function of the value of a for 3 rotational speeds. The optimal values are also indicated. A single optimal value of $a = 0.02$ s/m was found to lower all deviations between simulations and experiments to a maximum value of 5%.

Since various researchers [11,8,14] have proposed velocity dependent friction coefficients, this strengthens the idea that the friction coefficient is probably overestimated in the measuring methods and should probably be regarded as a speed dependent parameter. From another perspective, the cylindrical measurements might be an interesting (and cheap) tool to investigate (especially) the tangential contact force models and parameters for their speed dependence.

The underlying mechanisms explaining the difference in the transverse spread patterns (TSP) seem to be more complicated than in the cylindrical experiments. One should remark that, regarding the 400 rpm experiments, a similar effect was observed by Reumers et al. (2003) [18] who conducted experiments in a spreader hall and tried to predict the spread patterns by means of a ballistic model and measurements of the particle outlet velocities at the edge of the disc. Reumers suggested that the deviations are most likely due to a slight measurement error on the vertical (Z-direction) start position/velocity at the edge of the disc, giving significantly more spread on the ground positions. To quantify this, assume a projectile with initial velocity $\mathbf{v}$ (v_{0r} , v_{0z}), where v_r and v_z are the horizontal (along the disc surface) and vertical velocity components (perpendicular to the disc) respectively. Assume further the particle leaves at position $\mathbf{r}(r_0, z_0) = (0, 0)$. The

⁴ The initial positions and velocities on the disc of the individual particles are not known in the experiment. Their average values when leaving the bin however are known through the accurate position of the bin and the velocity monitoring of the particles with a high speed camera.

⁵ The deviations are not due to growing numerical errors, as single particle DEM simulations agree well with analytical models [15] for a broad range of disc rotational speeds.

Fig. 14. (a): Mass distribution on the ground as a function of the transverse distance (X) for NPK grains: $Y=0$ m, Plots of experiment (solid lines) and DEM simulation (dashed line) for $\Omega_d=400$ rpm. Error bars are omitted. (b and c): Mass distribution on the ground as a function of the transverse distance (X) for NPK grains. (b): $Y=-0.5$ m, (c): $Y=-1$ m; Plots of experiment (solid lines) and DEM simulation (dashed line) for $\Omega_d=400$ rpm. Error bars are shown. (d and e): Mass distribution on the ground as a function of the transverse distance (X) for NPK grains (d): $Y=-1.5$ m, (e): $Y=-2$ m. Plots of experiment (solid lines) and DEM simulation (dashed line) for $\Omega_d=400$ rpm. Error bars are omitted.

Table 2

Summarize of the TSP experiment for NPK, (A) with disc speed 400 rpm, (B) with disc speed 300 rpm

	Total grain mass M_y , exp. (kg)	Total grain mass M_y , sim. (kg)	RD(exp, sim) (%)
(A)			
$Y=-0$ m	0.40	0.59	35.2
$Y=-0.5$ m	0.163	0.171	25.5
$Y=-1$ m	0.251	0.236	28.4
$Y=-1.5$ m	0.390	0.417	13.3
$Y=-2$ m	0.107	0.64	26.8
(B)			
$Y=-0.5$ m	0.264	0.233	12.1
$Y=-1$ m	0.497	0.455	13.3
$Y=-1.5$ m	0.066	0.035	38.6

vertical and horizontal positions of the particle are given by (assuming no air resistance):

$$\begin{cases} r = v_{0r}t \\ z = v_{0z}t - \frac{1}{2}gt^2 \end{cases} \quad (25)$$

Combining the Eq. (25) for $z_0=0$, the vertical landing position of the particle, yields:

$$r = \frac{2v_{0r}v_{0z}}{g} \quad (26)$$

Differentiation of Eq. (26) yields

$$\Delta r = \frac{2(\Delta v_{0r}v_{0z} + v_{0r}\Delta v_{0z})}{g} \approx \frac{2v_{0r}\Delta v_{0z}}{g} \quad (27)$$

because $v_{0r} \gg v_{0z}$ for most of the fertilizer particles. Eq. (27) reads that a small deviation in the vertical velocity will be multiplied by the horizontal velocity to obtain the horizontal position. For $\Omega_d=400$ rpm, the particles have a simulated disc edge speed of 8 m/s and therefore we conclude that $\Delta r=1.6\Delta v_{0z}$.

The model could be extended by adding a normally distributed artificial parameter $b \gg N(0, \sigma_{vz})$ to the vertical outlet velocities of the particles at the disc edge. From a number of Monte Carlo simulations, an optimal value $\sigma_{vz}=0.35$ m/s was found, resulting in a decrease of RD(exp, sim) to an acceptable level of 15% averaged over the Y-lines for the 400 rpm experiments (the result is depicted in Fig. 13c). Note that σ_{vz} is about 1.5 times the original standard deviation on the disc edge vertical velocity components.

Looking for a parameter which can address the lack of velocity variation, higher sliding friction⁶ as well as higher restitution does increase the variation of the particle positions and velocities at the edge of the disc (see also Fig. 16a, b), but neither simulations with a varying (up to 50%) friction coefficient nor with a varying restitution coefficient and stiffness had any satisfactory improvement on the results. One remaining parameter that could be considered is rolling friction. In literature, both the rolling resistance in perfect spherical spheres and irregular shaped particles is modelled by a single rolling friction coefficient. The rolling friction component $\mathbf{M}_r$ can be added to the torque in the particle-particle contact as well as in the wall-particle contact [26]:

$$\mathbf{M}_r = -\mu_r \|\mathbf{N}\| \frac{\boldsymbol{\omega}_T}{\|\boldsymbol{\omega}_T\|} \quad (28)$$

⁶ Higher friction also slows down the particles, which is difficult rhyme with was concluded previously.

where μ_r is the rolling friction coefficient, and $\boldsymbol{\omega}_T = \boldsymbol{\omega} - \mathbf{n}(\boldsymbol{\omega} \cdot \mathbf{n})$ is the angular velocity of the particle contributing to rolling (i.e. the total angular velocity lowered by the twisting motion between particle and contact surface). The particle flow affected by introducing the rolling friction coefficient is depicted in Fig. 16c. The simulation results are given for $\mu_r=0$ (no rolling friction), $\mu_r=0.0001$ and $\mu_r=0.001$. One clearly observes that a higher rolling friction increases the outlet angles of the particles (thus slowing them down), although the effect is limited because at a certain point the rotational motion of the particles is frustrated and a pure sliding regime is adopted ($\mu_r > 0.001$). Another important observation is that the variation of the particle

Fig. 15. Mass distribution on the ground as a function of the transverse distance (X) for NPK. (a): $Y=-0.5$ m, (b): $Y=-1$ m, (c): $Y=-1.5$ m. Plots of experiment (solid lines) and DEM simulation (dashed line) for $\Omega_d=300$ rpm. Error bars are omitted.

Fig. 16. Simulated cylindrical pattern plots of DEM simulation (0.6 m diameter flat disc rotating at 500 rpm), showing (a) the influence of friction coefficients using values: 0.1 (dotted line) 0.3 (dash-dotted) 0.5 (dashed) 0.7 (solid), (b) the influence of the particle restitution coefficients using values 0.2 (solid line) and 0.8 (dashed line), and (c) the influence of the rolling friction coefficient using values 0.0 (solid line), 0.0001 (dashed line), and 0.001 (dotted line).

velocities and positions at the disc edge is lowered (e.g. CSP width gets smaller). Hence, the rolling friction cannot account for the deviations in the TSP.

In Conclusion, it remains unclear where such an increase in dispersion of the particle speed may originate from. The model parameters, describing the contact force model may have inaccurate values, or the contact model itself may be insufficient. Perhaps, external factors such as small vibrations of the plane of the spinning

disc at higher disc speeds (especially when recalling the disc is made of PVC), or the interaction of the particles with the complex airflow around the disc could be responsible. Additional experiments would be recommended in further research.

5. Conclusions

In this paper, DEM simulations of a fertilizer particle flow on a spinning disc were compared with experiments. The experiments were conducted in two different ways. The first one is a measurement of the particle distribution around the disc edge, yielding the cylindrical spread pattern (CSP). This is achieved by use of a cylindrical collector tray with segmental baskets each having an angle of 30°. The second method is the measurement of the total spread pattern on the ground. Here, a line of 15 baskets was constructed and positioned at different distances from the disc centre to cover the area of the whole fertilizer mass on the ground.

Overall, it was found that the DEM simulations perform well when the comparison with the experiments is made qualitatively. More specifically, the cylindrical measurements revealed that the prediction of the spread pattern is very good (deviations are below 5%) when the particle outlet velocities are low (e.g. <10 m/s). For higher disc rotational speeds, the deviations grow and the simulations lose their quantitative predictive value, mainly due to an offset to higher outlet angles of the simulated pattern. As a consequence, one could posit that the opposing forces on the particle flow, either the frictional contact or the normal damping, are overestimated. A small sensitivity study revealed that an overestimation of friction coefficients can be responsible for this behavior. From another perspective, the cylindrical measurements might be an interesting (and cheap) tool to investigate (especially) the tangential contact force models and parameters for their speed dependence.

For the experiments of the transverse spread patterns, a similar conclusion could be drawn, and even though the disc speeds are relatively low, the deviations are clearly magnified compared to the cylindrical measurements. These deviations are mainly due to an over concentration of fertilizer mass in the radial direction compared to the experiments (especially at higher disc speeds). A similar effect was observed by Reumers et al. (2003) [18] who conducted experiments in a spreader hall and tried to predict the spread patterns by means of a ballistic model and measurements of the particle outlet velocities at the edge of the disc. Likely, one could link these deviations to the conclusions made by Reumers. It is then plausible to assume that they

Fig. 17. Simulated outlet angles as a function of the linear velocity dependent friction model parameter a in Eq. (24) for disc type B with $\Omega_d = 300$ rpm (solid line), $\Omega_d = 500$ rpm (dashed line), $\Omega_d = 650$ rpm (dotted line). The optimal values for reducing the deviations with the relative experiments are indicated with a (*).

are due to miscalculation of the velocities – with emphasize on the vertical components – at the disc edge.

Since a model parameter study did not give any sense for improvement, one could consider additional experiments under well specified conditions to investigate the influence of vibrations and airflow. Other, more sophisticated, contact force models could also give a new direction as well.

Notation

Regular symbols

A	(Cross) section area [m^2]
c	damping coefficient in Kono-Kuwabara model [$\text{Ns}/\text{m}^{3/2}$]
D	distance [m]
e	restitution coefficient [–]
E	elasticity modulus [Pa]
H	height [m]
f	fraction [–]
I	inertia tensor [kgm^2]
k	stiffness [$\text{Nm}^{-3/2}$]
L	Length [m]
m	mass [kg]
M	Moment of contact force [Nm]
N	normal contact force [N]
$\mathbf{n}$	normal direction vector [–]
P	probability [–]
Q	rotation matrix [–]
$\mathbf{r}$	CoM position vector [m]
R	radius [m]
$\mathbf{t}$	tangential direction vector [–]
$\mathbf{T}$	tangential contact force [N]
$\mathbf{v}$	particle CoM velocity [m/s]
$\mathbf{X}$	model parameter [–]

Abbreviations

CoM	Centre of Mass
CSP	Cylindrical Spread Pattern
DEM	Discrete Element Method
IDV	Initial downward velocity (out of bin)
IPA	Initial Particle area (on the disc)
N/A	Not available
RD	Relative error or deviation

Greek symbols

α	segment angle in CSP [°]
β	width of cylindrical spread pattern [°]
δ	contact virtual overlap [m]
Δt	time step [s]
χ	eccentricity [–]
$\angle_d$	conical disc inclination [°]
μ	friction coefficient [–]
ν	Poisson's ratio [–]
ρ	density [kg/m^3]
Ψ	bulk flow [kg/s]
ω	angular velocity of particle [rad/s]
Ω_d	disc rotational speed [rpm]

Acknowledgments

The authors are very grateful to the IWT, FWO and the K.U. Leuven Research Fund for financial support and to Amazone Werke GmbH, Hasbargen, Germany.

References

- [1] V.I. Arnold, Mathematical methods of classical mechanics, Springer Verlag, 1978. 812
- [2] N.V. Brilliantov, F. Spahn, J. Hertzsch, T. Pöschel, Model for collisions in granular gases, Physical Review E 53 (1996) 5382–5392. 813
- [3] N.V. Brilliantov, T. Pöschel, Rolling friction of a viscous sphere on a hard plane, Europhysics Letters 42 (1998) 511–516. 814
- [4] E. Dintwa, Development of accurate contact force models for use with Discrete Element Method (DEM) The Discrete element Method (DEM) modelling of bulk fruit handling processes. PhD Thesis, Faculteit Landbouwkundige en Toegepaste Biologische Wetenschappen, K.U.Leuven, Belgium (2006). 815
- [5] G. Garberoglio, Dynamical properties of H-bonded liquids: A theoretical and computer simulations study. PhD Thesis, Faculty of Physical and Natural Mathematical sciences, University of Trento (2001). 816
- [6] T.E. Grift, J.W. Hofstee, Measurement of velocity and diameter of individual fertilizer particles by an optical method, Journal of Agricultural Engineering Research 66 (1997) 235–238. 817
- [7] E.J. Haug, Intermediate Dynamics, Prentice Hall, Englewood Cliffs, NJ, 1992. 818
- [8] J.W. Hofstee, Handling and spreading of fertilizers Part 5: the spinning disc type fertilizer spreader, Journal of Agricultural Engineering Research 62 (1995) 143–162. 819
- [9] F.M. Inns, A.R. Reece, The theory of the centrifugal distributor II: motion on the disc, off-centre feed, Journal of Agricultural Engineering Research 7 (4) (1962) 345–353. 820
- [10] K.L. Johnson, Contact Mechanics, Cambridge University Press, 1987. 821
- [11] N. Mohsenin, Physical properties of plant and animal materials, Gordon and Breach, 1986. 822
- [12] H.P. Kuo, P.C. Knight, D.J. Parker, M.J. Adams, J.P.K. Seville, Discrete element simulations of a high-shear mixer, Advanced Powder Technology 15 (3) (2004) 297–309. 823
- [13] G. Kuwabara, K. Kono, Restitution coefficient in a collision between 2 spheres, Japanese Journal of Applied Physics 26 (8) (1987) 1230–1233. 824
- [14] B. Olieslagers, Fertilizer distribution modelling for centrifugal spreader design. PhD Thesis, Faculteit Landbouwkundige en Toegepaste Biologische Wetenschappen, K.U.Leuven, Belgium (1997). 825
- [15] D.E. Patterson, A.R. Reece, The theory of the centrifugal distributor I: motion on the disc, near-centre feed, Journal of Agricultural Engineering Research 7 (3) (1962) 232–240. 826
- [16] E. Piron, D. Miclet, Spatial distribution measurement: a new method for the evaluation and testing of centrifugal fertilizer spreaders, 2nd International Symposium on Centrifugal Fertilizer Spreading, 24–25 septembre, Montoldre, France, 2006. 827
- [17] T. Pöschel, T. Schwager, Granular dynamics: Models and Algorithms, Springer, Berlin, 2005. 828
- [18] J. Reumers, E. Tijskens, H. Ramon, J. De Baerdemaeker, Experimental characterisation of dynamical properties of fertilizer grains in a centrifugal disk spreader: towards an alternative for spreading hall measurements, Biosystems Engineering 86 (3) (2003) 327–337. 829
- [19] J. Schäfer, S. Dippel, D.E. Wolf, Force Schemes in simulations of granular materials, Journal de Physique (France) 6 (1996) 5–20. 830
- [20] T. Schwager, T. Pöschel, Contact of viscoelastic spheres, in: D.E. Wolf, P. Grassberger (Eds.), Friction, Arching, Contact Dynamics, World Scientific, Singapore, 1996. 831
- [21] R.L. Stewart, J. Bridgwater, Y.C. Zhou, A.B. Yu, Simulated and measured flow of granules in a bladed mixer – a detailed comparison, Chemical Engineering Science 56 (19) (2001) 5457–5471. 832
- [22] E. Tijskens, Modelling particle shape in DEM simulations, 2nd International Symposium on Centrifugal Fertilizer Spreading 24–25 septembre, Montoldre, France, 2006. 833
- [23] J. Vangeyte, B. Sonck, P. Van Liedekerke, H. Ramon, Comparison of two methods to measure outlet velocity of fertilizer grains from a rotary disc, AgEng 2004 proceeding, 336–337. 834
- [24] P. Van Liedekerke, E. Tijskens, E. Dintwa, J. Anthonis, H. Ramon, A discrete element model for simulation of a spinning disc fertilizer spreader I. Single particle simulations, Powder Technology 170 (2) (2006) 71–85. 835
- [25] Y.C. Zhou, B.D. Wright, R.Y. Yang, B.H. Xu, A.B. Yu, Rolling friction in the dynamic simulation of sandpile formation, Physica. A, Statistical Mechanics and its Applications 269 (2–4) (1999) 536–553. 836
- [26] Y.C. Zhou, A.B. Yu, R.L. Stewart, J. Bridgwater, Microdynamic analysis of the particle flow in a cylindrical bladed mixer, Chemical Engineering Science 59 (6) (2004) 1343–1364. 837