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A Linear Programming Approach To Design Robust Input Shaping

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Abstract—Input shaping is an established technique to generate inputs that move flexible mechanical systems with little or no residual vibration. While traditional input shaping design strategies are often analytical, the present paper introduces a design method based on numerical optimization. It is shown that, through a careful selection of the optimization variables, objective function and constraints, it is possible to obtain a linear optimization problem. As a result, it is guaranteed that the globally optimal input shaper be found in a few seconds of computational time. The presented optimization framework is able to handle higher-order, linear time-invariant dynamic systems, as opposed to traditional input shapers, which are mainly based on second-order systems. Moreover, constraints on input, output and state variables are easily accounted for, as well as robustness against parametric uncertainty. Numerical results illustrate the capability of the proposed design approach to reproduce existing input shaping design approaches, while experimental results illustrate its potential for higher-order systems.

I. INTRODUCTION

Input shaping is an established technique to generate reference commands or inputs for point-to-point motions of flexible mechanical systems. The input shaper converts the desired motion to a command input such that very little or no residual vibration occurs upon arrival at the endpoint. The main input shaping concept is to convolve a carefully designed finite sequence of impulses, the *input shaping prefilter*, with the desired motion. The price to be paid for the residual vibration suppression is a short delay, known as move-time penalty, equal to the duration of the prefilter's impulse response.

Figure 1 shows the general input shaping setup. The input shaping prefilter F converts the desired motion r to the input u that drives the system P , of which the output is denoted by y . The prefilter F is a finite-impulse-response (FIR) filter. The objective of input shaping design is to determine the number K , the time locations t_k and the amplitudes f_k of the impulses of F such that the output y does not exhibit residual vibration.

Fig. 1. General input shaping setup. The input shaping prefilter F converts the desired motion r to the input u that drives the system P , of which the output is denoted by y .

Singer and Seering [1] developed the first analytic input shaping design approach, based on analytic expressions for

the response of a continuous-time second-order system to the following sequence of $K + 1$ impulses at time locations t_k

$$f(t) = \sum_{k=0}^K f_k \delta(t - t_k). \quad (1)$$

The amplitude of the resulting vibration at t_K is given by

$$A_{res} = \sqrt{\left(\sum_{k=0}^K B_k \cos \phi_k \right)^2 + \left(\sum_{k=0}^K B_k \sin \phi_k \right)^2}, \quad (2)$$

where

$$\phi_k = \omega_n \sqrt{1 - \zeta^2} t_k \quad (3)$$

$$B_k = \frac{f_k \omega_n}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n (t_K - t_k)}. \quad (4)$$

These expressions only depend on the undamped natural eigenfrequency ω_n and the damping ratio ζ of the system. From (2) and (3)–(4), it follows that zero vibration at time t_K , i.e., $A_{res} = 0$, is obtained provided that:

$$\sum_{k=0}^K f_k e^{-\zeta \omega_n (t_K - t_k)} \sin(t_k \omega_n \sqrt{1 - \zeta^2}) = 0 \quad (5)$$

$$\sum_{k=0}^K f_k e^{-\zeta \omega_n (t_K - t_k)} \cos(t_k \omega_n \sqrt{1 - \zeta^2}) = 0. \quad (6)$$

The minimum number of impulses, required to solve (5)–(6), is two: one at $t_0 = 0$ and one at $t_1 = T_0/2$, where T_0 denotes the period of the damped natural eigenfrequency. However, this basic prefilter design is very sensitive to deviations of ω_n and ζ ; small deviations of these parameters yield large residual vibration amplitudes and hence an ineffective prefilter.

Robust design approaches are required to design a prefilter that is not as sensitive to deviations of the system parameters. The *zero vibration derivative* (ZVD) approach [1] results in first-order sensitivity robustness. This robustness is obtained by imposing, on top of (5)–(6), that the derivatives of these equations be zero with respect to both ω_n and ζ . The *extra insensitive* (EI) approach [2], on the other hand, does not require zero vibration in the nominal system. Conversely, it is required that two systems that are slightly different from the nominal system, be without vibrations. A disadvantage of these and other [3] robust design approaches is that the

additional robustness comes at the cost of extra move-time penalty.

The above mentioned input shaping techniques are used in industry for applications involving cranes, mine-detecting robots [4], ... The classical techniques rely on analytical expressions, such as (5)–(6), for the residual vibration of second-order systems. Extensions to higher-order systems are possible [5], but result in suboptimal (i.e., with very long move-times) prefilters and complex expressions that are tedious to derive.

In contrast, this paper introduces a design method based on numerical optimization. It is shown that, through a careful selection of the optimization variables, objective function and constraints, it is possible to obtain a linear optimization problem (or *linear program*, LP). As a result, it is guaranteed that the globally optimal input shaper be found in a few seconds computational time. This approach overcomes some of the main difficulties that would occur if the input shaper design were formulated as a *nonlinear* optimization problem. In the latter case, the optimization algorithm is likely to get stuck in a local optimum, yielding suboptimal results, while it is impossible to guarantee that the global optimum has been found.

The paper is organized as follows. First, Sec. II develops the linear programming framework, thereby illustrating that constraints on input, output and state variables can be accounted for (as in [6]), as well as robustness against parametric uncertainty. Section III shows that this linear programming framework is able to reproduce the results of the previously discussed robust design methods [1] and [2]. Section IV validates the framework using an experimental 2-degree-of-freedom (DOF) mass-spring-damper system, while Sec. V concludes the paper.

The notation used in this paper is standard. The framework is developed for discrete-time linear time-invariant (LTI) systems where the sampling period is denoted by T_s and the index k refers to the discrete time instants kT_s . System and filters are assumed to have zero initial state.

II. LINEAR PROGRAMMING FRAMEWORK

Designing an input shaping prefilter F with impulse response $f(t)$ amounts to choosing some number K and specifying the time locations t_k and amplitudes f_k of the sequence of $K + 1$ impulses, defined by (1). For a basic input shaper, these design variables need to be selected such that (5)–(6) are complied with, while more complex conditions are imposed to introduce robustness for uncertain system parameters or account for constraints on system inputs and outputs.

The basic motivation for formulating this design problem as a numeric optimization problem is to be able to deal with high-order LTI systems, and to include design specifications that cannot be handled by current analytical approaches.

One optimization approach is to choose K and optimize the time locations t_k and amplitudes f_k of the sequence of $K + 1$ impulses, defined by (1). Although this approach is intuitive, it results in nonlinear optimization problems that feature a lot of local optima. As a result, one is likely to end up with a

suboptimal filter design. The main reason for the nonlinearity of the problem is the fact that $f(t)$ is nonlinear in f_k and t_k .

These difficulties are avoided here by specifying the time instants t_k beforehand, which implies that only the amplitudes f_k are optimized. While $f(t)$ is nonlinear in f_k and t_k , it is, for given time instants t_k , linear in f_k . At first sight, specifying the t_k beforehand, seems to take away much of the design freedom yielding a suboptimal design. However, the main feature of the linear programming framework developed here is that a huge number K of 'available' time instants is specified. For each of these time instants, the optimization algorithm can choose whether it wants to use the time instant or not by selecting a nonzero or zero f_k , respectively.

In order for this approach to work, the optimization framework needs to exhibit two additional features. First, given the large number of variables (typically a few hundreds or thousands), we restrict ourselves to constraints and objective functions that are linear in the optimization variables f_k : hence a linear program is obtained. Solving sparse LPs with thousands of variables is not a problem for current state-of-the-art LP algorithms. The basic linear programming framework is developed in Sec. II-A.

Second, measures need to be taken to keep the move-time penalty introduced by the filter as short as possible. This is achieved by making the duration t_K of the prefilter's impulse response as short as possible. To this end, an additional bisection loop is foreseen, which is discussed in Sec. II-B.

Robustness and other extensions of the basic framework are dealt with in Sec. II-C.

A. Linear Program

The linear programming framework is based on generalizing conditions (5)–(6), which are valid for second-order systems only. Assuming that the plant P is an LTI system with impulse response $p(t)$, and denoting the impulse response of the filter F as $f(t)$, defined by (1), it follows that the output $y(t)$ to a reference input $r(t)$ is given by (see Fig. 1)

$$y = r \otimes f \otimes p, \quad (7)$$

where $\otimes$ denotes the convolution operator. Hence, the output y is constant for $t \geq t_K + t_v$, regardless of the applied reference signal r , if $f \otimes p = 0$ for $t \geq t_K$ and r is constant for $t \geq t_v$. The condition that

$$f \otimes p = 0 \text{ for } t \geq t_K, \quad (8)$$

where P is a given LTI system of arbitrary order, generalizes (5)–(6) and is the basic condition that needs to be fulfilled by filter F .

In order to impose (8) in the optimization framework, a discrete-time representation (sample period T_s) for the plant P dynamics needs to be chosen. Here, a discrete state space model is preferred since it allows us, in a later stage, to impose optional constraints on the state variables:

$$x_{k+1} = Ax_k + Bu_k \quad (9)$$

$$y_k = Cx_k + Du_k. \quad (10)$$

Furthermore, also a description of the filter impulse response f is needed. As already mentioned, the time instants t_k in this definition are specified beforehand. They are chosen at equidistant locations $t_k = kT_s$. Hence

$$f(t) = \sum_{k=0}^K f_k \delta(t - kT_s), \quad (11)$$

where typical values of K are 200...1000.

Based on (9)–(10) and (11), it follows that the zero initial state impulse responses of P and F equal:

$$p(q) = D + \sum_{i=1}^{\infty} CA^{i-1}Bq^i \quad (12)$$

$$f(q) = \sum_{k=0}^K f_k q^{-K}, \quad (13)$$

where q^{-1} represents the delay operator. Note that $f(q)$ is a discrete-time finite-impulse response (FIR) filter of length K .

Convolution of these finite impulse responses of the system and the prefilter yields the following linear expression:

$$\mathbf{E}\mathbf{f} = \mathbf{g}, \quad (14)$$

where $\mathbf{E} \in \mathbb{R}^{(M+1) \times (K+1)}$ represents a matrix containing the impulse response of the system in a lower Toeplitz matrix. $\mathbf{f}$ is a vector of which the $K+1$ elements $f[k]$ equal $f(t_k) = f_k$, $0 \leq k \leq K$. $\mathbf{g}$, on the other hand, is a vector of which the $M+1$ elements $g[k]$ equal $(f \otimes p)(t_k)$, $0 \leq k \leq M$.

Based on the observation that $g[k] = (f \otimes p)(t_k)$, it follows that (14) allows us to express condition (8) for zero residual vibration as

$$\tilde{\mathbf{E}}\mathbf{f} = \mathbf{0}, \quad (15)$$

where $\tilde{\mathbf{E}}$ is the matrix obtained by selecting the last $M-K$ rows of $\mathbf{E}$. An important question to be addressed in this respect is how big M has to be to guarantee zero residual vibration for all $t \geq t_K + t_v$. Based on the Cayley-Hamilton theorem [7], it is straightforward to show that M should be $K+n$, where n denotes the order of the plant P . That is, n linear equality constraints (15) are a necessary and sufficient condition for zero residual vibration for all $t \geq t_K + t_v$. This is in agreement with the fact that the *two* equations (5)–(6) constitute a necessary and sufficient set for zero residual vibration of a *second-order* LTI system.

Based on the above discussion, we can now formulate the basic optimization problem for input shaping:

$$\text{minimize}_{f_k} \quad h(f_k) \quad (16a)$$

$$\text{subject to} \quad \tilde{\mathbf{E}}\mathbf{f} = \mathbf{0} \quad (16b)$$

$$\sum_{k=0}^K f_k = 1, \quad (16c)$$

where $M = K+n$. Constraint (16c) is introduced without loss of generality to avoid the trivial solution $f(q) \equiv 0$. The function $h(f_k)$ denotes the objective function. If h is linear in f_k , (16a)–(16c) constitutes an LP, since (16b)–(16c) are a set of $n+1$ linear equality constraints.

Given that sparse solutions might be preferred, that is, solutions in which many f_k are zero, the objective function $h(f_k)$ could be chosen to promote sparsity. The one-norm is well-known [8] to have such an effect, and has the advantage of resulting in an LP:

$$h(f_k) = \sum_{k=0}^K |f_k|.$$

If no $h(f_k)$ is specified, standard LP solvers consider (16a)–(16c) as a *feasibility problem*. That is, the solver assumes $h(f_k) = 0$ and just tries to find a solution f_k that complies with constraints (16b)–(16c). By solving a sequence of such feasibility problems, it becomes possible to minimize t_K , as explained in the next section.

B. Bisection

In order to minimize the move-time penalty introduced by the input shaper, t_K needs to be minimized. Given that $t_K = KT_s$, it follows that K should be minimized. Hence, the purpose is to find the lowest K , denoted as K^* , such that the corresponding filter still complies with (16b)–(16c).

K^* is found by solving, for various values of K , the LP (16a)–(16c) as a feasibility problem, i.e., with a goal function $h(f_k) = 0$. If the LP solver finds a solution f_k for the selected value of K , we know that $K^* \leq K$. If the LP solver is not able to find a solution f_k for the selected value of K , i.e., if the problem is infeasible, it is obvious that $K^* \geq K$.

In order to minimize the required number of trials for K , the well-known bisection algorithm is used. The bisection starts from a lower bound K_l (e.g., $K_l = 0$) and an upper bound K_u on K^* , and a tolerance ϵ that determines the accuracy of the solution. The algorithm proceeds as follows [8]:

```

repeat 1)  $K = (K_u + K_l)/2$ 
        2) solve the LP (16a)–(16c) with  $h(f_k) = 0$ 
        3) if the LP is feasible:  $K_u := K$ ; else  $K_l := K$ 
until  $K_u - K_l \leq \epsilon$ 

```

Exactly $\lceil \log_2((K_u - K_l)/\epsilon) \rceil$ steps are required before the algorithm terminates. The numerical results ($\epsilon = 0.5$) of Sec. III illustrate the effectiveness of this approach, by showing that it is able to reproduce known analytical results with a minimum move-time penalty.

C. Extensions

This basic framework can be extended with the constraint that all prefilter impulses be positive to avoid actuator saturation [1]. As this constraint introduces quite big an extra move-time, the input saturation constraints are introduced in a similar but more generic way than in [6], [9] by adding the following inequality constraint to (16a)–(16c):

$$-U \leq R \sum_{k=0}^i f_k \leq U, \quad i = 0, \dots, \quad (17)$$

where U denotes the input saturation level, which is assumed to be identical for positive and negative inputs, and R the required total displacement. This constraint is a sufficient condition for general dwell-rise-dwell motions except if the desired motion r is a step when it is also a necessary condition. Furthermore, this constraint is linear in the optimization variables f_k , such that adding it to the basic LP does not destroy the LP structure. In order to avoid the conservatism for motions other than steps, the saturation constraint (17) can be tailored to a given reference motion r :

$$-U \leq \sum_{k=0}^i f_k r_{i-k} \leq U, \quad i = 0, \dots, \quad (18)$$

where

$$r_i, \quad i = 0, \dots \quad (19)$$

denotes the discrete time sequence representing the desired motion. For a given reference motion, this is again a constraint that is linear in f_k .

Overshoot and undershoot constraints on any state variable can be included in a similar way.

Robustness with respect to uncertainty on one or several system parameters $\theta \in \Theta$ is accounted for by considering a limited set of system parameters $\theta_v \in \Theta$, $v = 1, \dots, V$, equally distributed within Θ , and imposing a bound γ on residual vibrations for each of these systems. In that case, the zero vibration constraint (16b) is relaxed to a finite bound γ :

$$1 - \gamma < \sum_{k=0}^i g_k(\theta_v) < 1 + \gamma; \quad i = K+1 \dots M+1; \quad \forall \theta_v \in \Theta. \quad (20)$$

This constraint is necessary and sufficient if the reference input is a step, but only sufficient and hence conservative for other reference inputs. However, similarly as for the saturation constraints, this constraint can be tailored to a given reference motion r , so as to obtain a necessary and sufficient condition. As γ diminishes to zero, the zero vibration constraint is better approximated, but this results in longer filters.

III. BENCHMARKING

This section discusses the validation of the developed input shaping framework by comparing this framework with the input shaping approaches presented in [1] and [2]. The considered test case is a continuous-time second-order system:

$$P(s) = \frac{\omega_n^2}{s^2 + 2 \cdot \omega_n \cdot \zeta \cdot s + \omega_n^2}, \quad (21)$$

where $\omega_n = 2\pi$ rad/s and $\zeta = 0.5$. According to [1], the optimal prefilter, assuming positive impulses only, consists of two impulses ($K = 1$):

$$t_0 = 0 \quad f_0 = \frac{1}{1 + R} = 0.86, \quad (22)$$

and

$$t_1 = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} = 0.5773 \quad f_1 = \frac{R}{1 + R} = 0.14, \quad (23)$$

where

$$R = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}. \quad (24)$$

To reproduce these results with the developed input shaping framework, the system dynamics are first discretized for a certain sampling period T_s using the ‘zero-order-hold’ (ZOH) equivalent. The number of candidate prefilter samples is set equal to $K + 1 = 2000$. Fig. 2 shows the results for $T_s = 0.001s$ and $0.00115s$ (which is indistinguishable of the analytic solution). The developed framework reproduces the prefilter obtained with the method of Singer and Seering [1], that is equations (22)-(24), exactly if t_1 (23) is an integer multiple of T_s , which is the case for $T_s = 0.00115s$. This is not the case if $T_s = 0.001s$, yielding three impulses instead of two: one at $t_0 = 0s$, with an amplitude exactly equal to f_0 (22), and two impulses at the discrete time locations just before and after t_1 . The sum of the amplitudes of these two impulses equals exactly f_1 (23).

The linear program is modeled in `matlab` and solved using `linprog` of `Mosek`. For this example, the total required CPU time is 18s on a Mobile Pentium 2GHz processor with 1GB RAM.

Fig. 2. Impulses of a prefilter where the ideal prefilter is not possible due to the chosen discretization step T_s . The acquired solution (red $\circ$) is very close to the ideal solution (blue $\times$). (b) zooms in on a linear scale on (a) in the neighborhood of the second impulse.

Next, the input shaping design is robustified for uncertainty on the undamped natural eigenfrequency ω_n . It is shown how the developed framework can imitate the robust ZVD filter [1] and EI filter [2] designs. The ZVD filter design uses a local

sensitivity approach to include robustness, i.e. robustness is obtained by setting the derivative of the amplitude A_{res} of the residual vibration (2) with respect to the eigenfrequency and damping, to zero (first-order sensitivity). The developed framework yields similar results by considering two systems in (20) with γ equal to zero and eigenfrequencies that are slightly different from the nominal value:

$$\omega_n = 2\pi \pm 0.01\pi [\text{rad/s}]. \quad (25)$$

Fig. 3 shows the vibration error, that is, the maximum amplitude of the residual vibration as a percentage of the amplitude of the desired point-to-point motion, versus the normalized natural eigenfrequency of the system, which is defined as the ratio of the natural eigenfrequency of the perturbed system ω_n and the nominal system $\omega_{n,0} = 2\pi \text{ rad/s}$ [1]. This is shown for both the ZVD approach (red solid line) as for our robust approach (green dashed line). The zoom shows the effect of the different approaches. The maximum difference between these approaches is $0.25 \cdot 10^{-3}$ at the point where $\omega_n/\omega_{n,0} = 1$.

The EI filter design approach uses a global sensitivity approach by imposing zero residual vibration for two systems with eigenfrequencies that are significantly different from the nominal value:

$$\omega_n = 2\pi \pm 0.1\pi [\text{rad/s}]. \quad (26)$$

The developed framework reproduces this filter design exactly by considering two systems in (20) with γ equal to zero and the same eigenfrequencies as in (26).

Fig. 3. Vibration error versus system natural frequency: the ZVD result (red solid line) and the result of the prefilter designed with the developed framework (green dashed line).

IV. EXPERIMENTAL VALIDATION

This section discusses the experimental validation of the developed framework. The test-setup consists of a two-DOF mass-spring-damper system. Fig. 4 shows a picture and a schematic drawing of the setup. The system is excited by a position controlled hydraulic piston with position $p(t)$. The system input is the reference signal for the piston position controller. The position of the upper mass $x_1(t)$ is chosen as the system output.

A fifth-order continuous-time state space model is identified for this system based on frequency response function measurements that are obtained from a multisine excitation with a frequency content between 0.1Hz and 10Hz [10]. To apply the developed framework, this model is transformed to discrete time with a sample period of $T_s = 0.01s$.

Fig. 4. Picture and schematic drawing of the test-setup.

This model contains two pairs of complex conjugated poles originating from the two flexible modes of this system, and one real pole that is introduced by the band limited piston position controller (Table I).

TABLE I
POLES OF THE FIFTH ORDER SYSTEM

frequency [rad/s]	damping [-]
$\omega_0 = 2.6205 \times 2\pi$	$\zeta_0 = 0.157\%$
$\omega_1 = 7.7926 \times 2\pi$	$\zeta_1 = 0.293\%$
1 real pole at 214	/

A nonrobust input shaping design is performed first with the following specifications: an input constraint of $U = 1V$ (17) and a maximum overshoot of 5% of the upper mass displacement. The lower and upper bounds for the bisection algorithm are $K_l = 0$ and $K_u = 200$, respectively. The upper bound corresponds to 5 times the period of the lowest eigenfrequency of the system. The desired motion $r(t)$ is a ramp with a duration of 0.2s and a displacement of 0.4cm (see fig. 5, green dashed line). A ramp motion instead of a step is used in combination with the input constraint because of limited oil flow in the hydraulic system (a nonlinear effect).

Fig. 5 shows the system response, that is the position x_1 , if the ramp input (green dashed line) is applied directly (blue dotted line) and if the designed prefilter is used (red solid line). The ramp input clearly excites the first system resonance. With prefilter, the resulting residual vibrations are negligible at the cost of an additional move-time of 0.1s.

Fig. 5. Validation of the nonrobust input shaping prefilter design: desired motion $r(t)$ (green dashed line), and system response without prefilter (blue dotted line) and with prefilter (red solid line).

Fig. 6. Comparison of robust and nonrobust design: desired motion $r(t)$ (green dashed line), and system response with nonrobust (blue dotted line) and with robust prefilter (red solid line). The plant has a 20% deviation of the model.

Secondly, a robust input shaping design is performed. In order to verify the benefits of a robust design, and since the setup has fixed resonances, the nominal system model used in the design is changed: the first resonance is shifted to $\omega_n = 2.1 \times 2\pi$ rad/s, which corresponds to a change of 20%. Robustness is introduced in the design by considering constraint (20) with $\gamma = 0$ for three ω_n values:

$$\omega_n = 2.1 \times 2\pi, \quad 2.1 \times 2\pi \pm 10\% [\text{rad/s}].$$

Fig. 6 compares this robust design with a nonrobust design for the ramp input presented above (green dashed line). The robust prefilter (red solid line) yields considerably smaller residual vibrations than the nonrobust prefilter (blue dotted line) but introduces an additional move-time of 0.15s. The nonzero residual vibration is due to the use of too restricted a range of model uncertainty in the robust design. This is confirmed by simulations.

V. CONCLUSION

This paper proposes a framework to design input shaping prefilters for flexible motion systems. While traditional approaches are mainly analytic and restricted to second order systems, this paper presents a numerical optimization approach

that formulates the prefilter design as a linear program that can be solved efficiently for any system order. Constraints on input, output and state variables as well as robustness against parametric uncertainty can be accounted for in the design. The developed approach is capable of reproducing the results of existing design approaches. Experimental results on a 2DOF mass-spring-damper system illustrate the importance of including robustness in the design for a higher order system.

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