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Generalized Repetitive Control: Better Performance with Less Memory

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Abstract—Most current repetitive controllers apply the internal model principle by embedding a one-period delay into the control scheme. While being simple and often effective, this structure involves a number of disadvantages: (i) in many applications, large memory buffers are needed; (ii) information about the spectrum of the periodic input cannot be included and (iii) accounting for period-time uncertainty results in controllers that are overly robust for lower harmonic frequencies. In order to overcome these disadvantages, the present paper proposes a novel repetitive controller structure that sacrifices the simplicity of the one-period delay structure for the benefit of more design freedom. The resulting generalized repetitive controller encompasses the one-period delay structure as a special case and is designed based on semidefinite programming. Numerical results reveal that a generalized repetitive controller yields significantly better performance with the same amount of memory or, conversely, requires substantially less memory to achieve equivalent performance.

I. INTRODUCTION

Including a repetitive controller in the feedback loop constitutes a common way to improve the attenuation/tracking of periodic disturbance/reference inputs [3], [7], [10], [18]. Repetitive controllers rely upon the internal model principle [5], which states that, if a reference/disturbance signal can be regarded as the output of an autonomous system, including this system in a stable feedback loop guarantees asymptotically perfect tracking/rejection. Most current repetitive controllers, in this paper referred to as “typical” repetitive controllers, embed the periodic signal generator shown in Fig. 1. This system is able to generate any discrete-time signal with period $T_0 = NT_s$ [s], where N is integer and T_s [s] corresponds to the sample period. Typical repetitive controllers generally add a low-pass filter, denoted $Q(z)$, to the delay z^{-N} for guaranteeing robust closed-loop stability [2], [3], [7].

Fig. 1. Signal generator with period $T_0 = NT_s$, embedded in a typical repetitive controller.

In the pass-band of $Q(z)$, a typical repetitive controller yields perfect asymptotic tracking/rejection of *any* input periodic with T_0 , since its gain is infinite at all multiples of $\omega_0 = 2\pi/T_0$ [rad/s]. Its structure is simple and can be adapted easily for varying periods by adjusting N [4], [16]. However, embedding the signal generator of Fig. 1 also involves some

drawbacks. Since in many applications N is large, the implementation of a typical repetitive controller generally requires a large memory buffer. Furthermore, information about the spectrum of the periodic input, such as the relative importance of the various harmonics, cannot be exploited for a typical repetitive controller treats all harmonics the same way due to the periodicity of its frequency response function (FRF). In addition, the FRF periodicity renders it hard to robustify a typical repetitive controller for uncertainty on T_0 [12], [14], since such uncertainty implies a linearly growing uncertainty interval on higher harmonic frequencies. Typical repetitive controllers cannot do else but assigning the largest uncertainty interval to all harmonics and are hence overly robust for lower harmonic frequencies.

As a result, typical repetitive controllers are most suited for applications where (i) the only knowledge about the input is its period T_0 , (ii) T_0 is constant or accurately measurable and (iii) T_0 contains an integer number N of samples. However, the sample frequency cannot always be adjusted for (iii) to hold, while requirement (ii) may be jeopardized in practice by clock error drift, jitter, measurement noise, Moreover, more than merely the period may be known about the periodic input: the control engineer often has a (rough) idea about the relative importance of the different input harmonics.

This paper proposes a novel type of single-input single-output (SISO) repetitive controllers that sacrifices the simple structure of typical repetitive controllers for the benefit of more design freedom to overcome the aforementioned disadvantages. The additional design freedom is created by replacing the ensemble of the delay z^{-N} and the filter $Q(z)$ in a typical repetitive controller by a finite impulse response (FIR) filter $X(z)$ of order K with design variables X_k (τ will be specified in Sec. III):

$$X(z) = \sum_{k=\tau}^K X_k z^{-k}. \quad (1)$$

Since $Q(z)$ is generally designed as a linear-phase FIR filter [2], [7], [15], typical repetitive controllers are still encompassed as a special case. Therefore, *generalized* repetitive control is the name coined for the novel repetitive controller structure. The design parameters X_k are computed such that the generalized repetitive controller yields an optimal trade-off between two performance indices that quantify its effect on the closed-loop performance for the non-periodic and the

period-uncertain inputs, similar to [12]. In addition, $X(z)$ must guarantee robust closed-loop stability. The numerical results of Sec. IV illustrate that, compared to a typical repetitive controller, a generalized repetitive controller yields significantly better performance with the same amount of memory or, conversely, requires substantially less memory to achieve equivalent performance.

To the best of our knowledge, [8] constitutes currently the sole repetitive controller design that accounts for the periodic input spectrum. In contrast to the optimization based approach proposed here, [8] is based on pole-placement techniques. Moreover, neither the effect of the repetitive controller on the non-periodic inputs, nor uncertainty on the period T_0 are accounted for.

This paper is organized as follows: Section II details the control setup and analyzes the closed-loop system without and with repetitive controller. The generalized repetitive controller design is elaborated in Sec. III, while Sec. IV numerically illustrates its potential. Section V concludes the paper. The notation used in this paper is standard. To alleviate notation, the FRF of a discrete-time system $H(z)$ is denoted by $H(\omega)$ instead of $H(e^{j\omega T_s})$.

II. ADD-ON REPETITIVE CONTROL SYSTEM

The repetitive controller is considered as an add-on device, implying that it is added to the loop of an existing feedback system. This is the more common control scheme in literature [3]. Section II-A details the corresponding control setup, while Sec. II-B analyzes the closed-loop system without and with repetitive controller. Section II-C defines the two relevant performance indices of the repetitive controller.

A. Control Setup

Figure 2 shows the control setup of an add-on repetitive control system. The controller is designed in discrete time, where the sampling period is denoted by T_s [s] and the index k refers to the discrete time instants kT_s . The plant is a SISO linear time-invariant system with transfer function $P(z)$. The signals $r(k)$, $d(k)$ and $e(k)$ correspond to the reference input, the disturbance input and the tracking error, respectively. In closed loop, $e(k)$ is given by

$$e(k) = S(z) \underbrace{(r(k) - d(k))}_{v(k)}, \quad (2)$$

where $S(z)$ denotes the closed-loop sensitivity function. The signal $v(k)$ is referred to as the *input* of the closed-loop system and features both a periodic and a non-periodic part. The periodic part has a nominal period T_0 [s], where the corresponding fundamental frequency is denoted by ω_0 [rad/s]. Uncertainty on T_0 is modeled as a relative uncertainty on ω_0 , bounded by Δ :

$$\omega_{0,\delta} = \omega_0 (1 + \delta), \quad |\delta| \leq \Delta. \quad (3)$$

The index l indicates the harmonics of the periodic part of the input $v(k)$, where $\mathcal{L} \subset \mathbb{Z}$ corresponds to the set of harmonics to be suppressed. To each harmonic $l \in \mathcal{L}$ a weight V_l is

Fig. 2. Control setup of an add-on repetitive control system.

attributed, quantifying its relative importance in $v(k)$. Hence, V_l generally corresponds to the amplitude of the corresponding Fourier coefficient, or a (rough) estimate thereof.

B. Closed-Loop System Without and With $K_{RC}(z)$

The system without $K_{RC}(z)$ is referred to as the original feedback system and its closed-loop sensitivity and complementary sensitivity function are denoted by $S_o(z)$ and $T_o(z)$, respectively. The feedback controller $K_o(z)$ is assumed to be designed properly, which implies [13] (i) a stable closed-loop system; (ii) high-gain feedback at low frequencies; (iii) sufficient roll-off of $|T_o(\omega)|$ at high frequencies; and (iv) a small modulus margin $\|S_o(z)\|_\infty$.

The combination of properties (i), (iii) and (iv) guarantees robust stability of the original closed-loop system. Property (ii) is referred to as *good non-periodic performance*, since it yields small $|S_o(\omega)|$ at low frequencies, resulting in a small tracking error $e(k)$ for *any* low-frequency input $v(k)$. Defining the bandwidth ω_{BW} of the original closed-loop system as the frequency where $|S_o(\omega)|$ first crosses -3dB from below implies that $K_o(z)$ yields good non-periodic performance up to ω_{BW} .

When $K_{RC}(z)$ is added to the loop, the closed-loop sensitivity changes from $S_o(z)$ to $S(z)$:

$$S(z) = S_o(z) M_S(z), \quad (4)$$

where $M_S(z)$ is called the *modifying sensitivity function* [15], given by

$$M_S(z) = \frac{1}{1 + K_{RC}(z)T_o(z)}. \quad (5)$$

The main purpose of $K_{RC}(z)$ is to improve the tracking/attenuation of the periodic part of the input $v(k)$, that is, the closed-loop *periodic performance*, as much as possible. However, periodic performance improvement implies degraded non-periodic performance since pushing the closed-loop sensitivity down to zero at multiples of ω_0 is paid for by an increased sensitivity at intermediate frequencies, as dictated by the Bode Sensitivity Integral [6]. This inevitable non-periodic performance degradation should be kept within acceptable limits. In addition, $K_{RC}(z)$ must not compromise the robust stability of the original feedback system. These design specifications for $K_{RC}(z)$ are not conflicting provided that all harmonics $l \in \mathcal{L}$ lie well below ω_{BW} : at harmonics near or above ω_{BW} better performance simply cannot be achieved without compromising the modulus margin or the high-frequency roll-off of the original feedback system. This nonrestrictive and classical assumption is made throughout the remainder of this paper.

C. Performance Indices of $K_{RC}(z)$

This section defines two performance indices that quantify the periodic performance improvement and the non-periodic performance degradation caused by $K_{RC}(z)$. These definitions are based on [12], where the periodic performance index is generalized to account for the weights V_l of the input harmonics.

If the input period is known exactly ($\Delta = 0$), the effect of $K_{RC}(z)$ on the closed-loop periodic performance is quantified by the *nominal periodic performance index* γ_p , defined as the smallest relative reduction of $|S(l\omega_0)|$ over all harmonics $l \in \mathcal{L}$, weighted by their relative importance V_l :

$$\gamma_p \equiv \max_{l \in \mathcal{L}} \left\{ V_l \frac{|S(l\omega_0)|}{|S_o(l\omega_0)|} \right\}, \quad (6a)$$

$$= \max_{l \in \mathcal{L}} \{ V_l |M_S(l\omega_0)| \}. \quad (6b)$$

In the presence of uncertainty on the period T_0 , the worst-case value of (6b) is taken over all values $\omega_{0,\delta}$, resulting in the following definition of the *robust periodic performance index* $\gamma_{p,\Delta}$:

$$\gamma_{p,\Delta} \equiv \max_{|\delta| \leq \Delta} \left\{ \max_{l \in \mathcal{L}} \{ V_l |M_S(l\omega_{0,\delta})| \} \right\}, \quad (7a)$$

$$= \max_{l \in \mathcal{L}} \left\{ V_l \max_{\omega \in \Omega_l} \{ |M_S(\omega)| \} \right\}, \quad (7b)$$

where

$$\Omega_l = [l\omega_0(1 - \Delta), l\omega_0(1 + \Delta)]. \quad (8)$$

The effect of $K_{RC}(z)$ on the closed-loop non-periodic performance is quantified by the *non-periodic performance index* γ_{np} , defined as the highest relative amplification of $|S(\omega)|$ below ω_{BW} :

$$\gamma_{np} \equiv \max_{\omega \leq \omega_{BW}} \left\{ \frac{|S(\omega)|}{|S_o(\omega)|} \right\}, \quad (9a)$$

$$= \max_{\omega \leq \omega_{BW}} \{ |M_S(\omega)| \}, \quad (9b)$$

$$= \|M_S(z)\|_\infty. \quad (9c)$$

The equivalence between (9b) and (9c) holds since both a typical and a generalized repetitive controller design enforce $|M_S(\omega)| \approx 1$ for all $\omega > \omega_{BW}$ (see Sec. III).

Since a repetitive controller improves the robust periodic performance at the expense of worse non-periodic performance: $\gamma_{p,\Delta} < 1$, while $\gamma_{np} > 1$.

III. GENERALIZED REPETITIVE CONTROLLER DESIGN

Figure 3 shows the structure of a typical (a) and a generalized (b) repetitive controller. A typical repetitive controller relies on the internal model shown in Fig. 1, where

$$N = \text{int}(T_0/T_s), \quad (10)$$

and the effect of a potential rounding error is modeled as uncertainty on $T_0 \equiv NT_s$. In a *high-order* typical repetitive controller, the delay z^{-N} is extended to a polynomial of delays

$$W(z) = \sum_{m=1}^M W_m z^{-mN}, \quad (11)$$

Fig. 3. Structure of a typical (a) and a generalized (b) repetitive controller.

which allows the designer to improve the performance of the repetitive controller through a proper choice of the parameters W_m [1], [9], [12], [14], [15]. The filters $Q(z)$ and $L(z)$ are added to obtain a robustly stable closed-loop system: $L(z)$ renders the closed-loop system nominally stable, whereas $Q(z)$ guarantees robust stability by preserving the modulus margin and the high-frequency roll-off of the original feedback system. To this end, $Q(z)$ is generally designed as a low-pass linear-phase FIR¹ filter that turns off the repetitive controller at a frequency below ω_{BW} [2], [7], [15]. In addition, all harmonics $l \in \mathcal{L}$ should lie in the pass-band of $Q(z)$ for all current design strategies for $W(z)$ rely on this assumption. $Q(z)$ can comply with both requirements provided that all harmonics $l \in \mathcal{L}$ lie well below ω_{BW} .

A generalized repetitive controller (Fig. 3(b)) replaces the ensemble of $W(z)$ and $Q(z)$ in a typical repetitive controller by the FIR filter $X(z)$ (1) of order K with design variables X_k , where τ denotes the relative degree of $T_o(z)$. Typical repetitive controllers form a subclass of generalized repetitive controllers since a FIR filter design of $Q(z)$ turns $W(z)Q(z)$ into a FIR filter with a particular structure. As detailed in Sec. III-B, the design of $X(z)$ combines the optimal performance and robust stability requirements, handled separately by $W(z)$ and $Q(z)$ in a typical design, while nominal closed-loop stability is still guaranteed by the filter $L(z)$ (Sec. III-A).

A. $L(z)$: nominal stability.

The generalized repetitive controller structure of Fig. 3(b) gives rise to the following expression for $M_S(z)$:

$$M_S(z) = \frac{1 - X(z)}{1 - X(z)[1 - L(z)T_o(z)]}. \quad (12)$$

To obtain a stable closed-loop system, $L(z)$ is set equal to $T_o^{-1}(z)$, where the “zero phase error tracking” inverse [17] is

¹It is beneficial to design $Q(z)$ as a linear-phase FIR filter, since its linear phase behavior, which corresponds to a delay of $n_Q/2$ time samples (n_Q denotes the filter order), can be compensated for by decreasing the delays in $W(z)$ [15].

used if $T_o(z)$ is non-minimum phase. This design is common in typical repetitive control [7], [18], and results in a plant-independent expression for the modifying sensitivity function:

$$M_S(z) = 1 - X(z). \quad (13)$$

Non-causality of $T_o^{-1}(z)$ can be accounted for, since a delay equal to the relative degree τ of $T_o(z)$ is enforced in $X(z)$ [15].

B. $X(z)$: robust stability and optimal performance.

Inspired by the design of $Q(z)$ in a typical repetitive controller, the modulus margin and high-frequency roll-off of the original feedback system are preserved by turning off the generalized repetitive controller at a frequency below ω_{BW} . This is imposed by the following constraint

$$|X(\omega)| \leq \epsilon, \quad \forall \omega \geq \omega_\epsilon, \quad (14)$$

where $\omega_\epsilon < \omega_{BW}$ and ϵ , a small positive number (e.g. 10^{-3}), are specified beforehand. In addition, $X(z)$ should realize an optimal trade-off between the conflicting performance indices $\gamma_{p,\Delta}$ and γ_{np} of the repetitive controller. As a result, computing the optimal design parameters X_k amounts to solving the following optimization problem:

$$\text{minimize } \gamma_{p,\Delta} + \alpha \gamma_{np} \quad (15a)$$

$$\text{subject to } |X(\omega)| \leq \epsilon, \quad \forall \omega \geq \omega_\epsilon, \quad (15b)$$

$$\|M_S(\omega)\|_\infty \leq \gamma_{np}, \quad (15c)$$

$$V_l |M_S(\omega)| \leq \gamma_{p,\Delta}, \quad \forall \omega \in \Omega_l, \quad \forall l \in \mathcal{L} \quad (15d)$$

where α is a user-specified optimization weight, quantifying the relative importance of $\gamma_{p,\Delta}$ and γ_{np} . Optimization problem (15) is numerically not tractable, since the constraints require evaluation for infinitely many values of ω . However, this problem can be rendered numerically tractable by transforming it into a semi-definite program (SDP) based on the (generalized) KYP lemma. For reasons of conciseness, the reader is referred to [12] for more details. The obtained SDPs are very hard to solve with standard SDP-solvers, since their computational cost is $\mathcal{O}(K^6)$ and K is generally large (see Sec. IV). To overcome this problem, the specialized solver developed by Liu and Vandenberghe [11] is used, which exploits the particular structure of LMIs obtained by the (generalized) KYP lemma. Although this results in a significant reduction in computation time, $\mathcal{O}(K^3)$ instead of $\mathcal{O}(K^6)$, the computational effort is still larger than for a typical design.

IV. NUMERICAL RESULTS

A numerical example illustrates the potential of the generalized repetitive controller. The period of the input contains $N = 50$ samples, while the first five harmonics are present in $v(k)$ with equal weight: $V_l = 1, \forall l \in \mathcal{L} = \{1, 2, 3, 4, 5\}$. Robust closed-loop stability is guaranteed by Eq. (14) with $\epsilon = 10^{-3}$ and $\omega_\epsilon = 8\omega_0$. The relative degree of $T_o(z)$ equals $\tau = 2$ and is the only property of the original feedback system that affects the repetitive controller design.

In order to compare generalized with typical repetitive controllers, the latter are designed according to the same specifications. In contrast to the combined treatment in a generalized design, a typical repetitive controller design splits up the optimal performance and robust stability specifications among $W(z)$ and $Q(z)$, respectively. Using the methodology discussed in [12], $W(z)$ is designed such that the typical repetitive controller yields an optimal trade-off between the conflicting performance indices $\gamma_{p,\Delta}$ and γ_{np} . To guarantee robust stability without compromising the obtained performance, $Q(z)$ is designed as the lowest order linear-phase FIR filter satisfying

$$|Q(\omega)| \approx 1, \quad \forall \omega \leq 5\omega_0 \quad (16a)$$

$$|Q(\omega)| \leq \epsilon, \quad \forall \omega \geq \omega_\epsilon \quad (16b)$$

where the latter constraint is sufficient for $W(z)Q(z)$ to satisfy the robust stability constraint (14). In this way, $Q(z)$ is of order $n_Q = 78$, and for a first-order typical repetitive controller ($M = 1$), $W(z)Q(z)$ corresponds to a FIR filter of order

$$MN + \frac{n_Q}{2} = 89. \quad (17)$$

This typical repetitive controller (tRC) is compared with different generalized repetitive controllers (gRCs), where the notation $\text{gRC}_{\mu\%}$ indicates that the generalized repetitive controller uses $\mu\%$ of the memory required for the tRC. For instance, $\text{gRC}_{100\%}$ has $K = 89$.

Section IV-A compares, for this particular example, the FRF of $M_S(z)$ for a typical and a generalized repetitive controller, while Sec. IV-B analyzes the corresponding trade-off curves between γ_{np} and $\gamma_{p,\Delta}$.

A. Comparison of $M_S(\omega)$

Figure 4 compares for $\Delta = 2\%$ the FRF of $M_S(\omega)$ for the tRC and the $\text{gRC}_{100\%}$ that both yield $\gamma_{np} = 1.4$. While yielding the same level of non-periodic performance degradation, $\text{gRC}_{100\%}$ yields better periodic performance ($\gamma_{p,2\%} = 0.53$) than tRC ($\gamma_{p,2\%} = 0.72$). Besides showing the typical periodic behavior of $M_S(\omega)$ for the tRC, Fig. 4 illustrates how the additional design freedom of a gRC is applied to obtain better performance. First, in contrast to the tRC, $\text{gRC}_{100\%}$ does not waste control effort at harmonics not present in the input, like $l = 0$ and $l = 6$. In addition, $\text{gRC}_{100\%}$ properly accounts for the linearly growing uncertainty intervals Ω_l around the harmonics, as opposed to tRC. This is revealed by the fact that for $\text{gRC}_{100\%}$ constraint (15d) is active for all harmonics $l \in \mathcal{L}$, that is: in each of the grey-shaded uncertainty intervals there is at least one ω at which $|M_S(\omega)| = \gamma_{p,2\%} = 0.53$. For tRC, on the other hand, constraint (15d) is only active for $l = 5$, the highest harmonic in $\mathcal{L}$, such that tRC is overly robust at the lower harmonics.

B. Trade-off curves between $\gamma_{p,\Delta}$ and γ_{np}

To allow for a more systematic comparison between the performance of a typical and a generalized repetitive controller, Fig. 5 shows the corresponding trade-off curves between $\gamma_{p,\Delta}$

Fig. 4. Comparison of the FRF of $M_S(\omega)$ for the tRC and the $\text{gRC}_{100\%}$ that both yield $\gamma_{np} = 1.4$. The $\Delta = 2\%$ relative uncertainty on ω_0 results in the grey-shaded uncertainty intervals on the various harmonics.

Fig. 5. Comparison of the tradeoff-curves between $\gamma_{p,\Delta}$ and γ_{np} for tRC and two gRC's. (a) $\Delta = 0$, (b) $\Delta = 2\%$; where Δ corresponds to the relative uncertainty on ω_0 .

and γ_{np} , for $\Delta = 0\%$ (a) and $\Delta = 2\%$ (b). These trade-off curves are computed by solving (15) and its tRC equivalent [12] for a range of α values. For a given level of robust periodic performance improvement $\gamma_{p,\Delta}$, the curve indicates the minimal level of non-periodic performance degradation γ_{np} that has to be tolerated. Or, vice-versa, for a fixed level of allowable non-periodic performance degradation, the trade-off curve indicates the best robust periodic performance improvement that can be achieved.

If no uncertainty on ω_0 is present ($\Delta = 0\%$) the trade-off curves reveal that, for the same level of non-periodic performance degradation γ_{np} , $\text{gRC}_{100\%}$ improves the periodic performance γ_p with up to 0.23. Since $\text{gRC}_{65\%}$ ($K = 58$) yields for any γ_{np} at least as good a periodic performance as the tRC, a generalized repetitive controller makes it possible to save at least 35% of memory without loss of performance. In addition, the implementation of $\text{gRC}_{65\%}$ requires less floating point operations than tRC, since $X(z)$ ($K = 58$) in $\text{gRC}_{65\%}$ is of lower order than $Q(z)$ ($n_Q = 78$) in tRC.

In the presence of 2% uncertainty on ω_0 , $\text{gRC}_{100\%}$ gains up to 0.39 periodic performance $\gamma_{p,2\%}$ for a given level of non-periodic performance degradation. In this case, $\text{gRC}_{60\%}$ ($K = 54$) is the most memory efficient gRC that outperforms tRC for every γ_{np} value, indicating that a 40% memory reduction is possible. The advantage of a gRC over a tRC gets larger for higher uncertainty on ω_0 due to the former's capability to deal more efficiently with this kind of uncertainty.

V. CONCLUSIONS

This paper proposes the generalized repetitive controller, a novel type of SISO repetitive controllers that generalizes the typical, one-period delay based repetitive controller structure. Whereas the typical structure impedes the repetitive controller to account for the input spectrum and to handle uncertainty on the period of the input efficiently, the generalized structure

provides additional design freedom to overcome these disadvantages. It has been numerically illustrated that in this way a generalized repetitive controller yields significantly better performance with the same amount of memory, or, conversely, requires substantially less memory to achieve equivalent performance.

The generalized repetitive controller design is based on two performance indices, γ_{np} and $\gamma_{p,\Delta}$, that quantify its effect on the closed-loop performance for the non-periodic and the period-uncertain inputs, respectively. The trade-off curves between these performance indices not only provide valuable insight in the fundamental trade-off between γ_{np} and $\gamma_{p,\Delta}$, they also allow for a clear comparison between the performance of typical and generalized repetitive controllers.

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