A Nonlinear Contour Preserving Transform for Geometrical Image Compression

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Part I

Problem description

A Nonlinear Contour Preserving Transform for Geometrical Image Compression

Introduction

Beyond wavelets

- 1 Smoothly coloured regions, separated by
- 2 Smooth contours
- In a 3D presentation the contours show up as discontinuities lying on a smooth curve in the domain.

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Introduction

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Definition (Horizon Class $\mathcal{H}$)

$$\mathcal{H} = \{f_{\Omega} \mid$$

- $f_{\Omega}(x, y) := \begin{cases} 0 & \mathbf{c}(x) > y \\ 1 & \mathbf{c}(x) \leq y \end{cases}$
- $\mathbf{c}(x) \in \mathcal{C}^2$
- $\Omega = \square = [0, 1]^2$ }

Remark Horizon Class Images
are completely defined by $\mathbf{c}(x)$

Why not use wavelets?

Approximation rate on Horizon Class Images

Tensor product wavelet approximants come from spaces $V_n \subset V_{n+1}$ on fixed partitions.

- The number of rectangle subdomains cut by $c(x)$ size exponential w.r.t. j : $n_j \approx O(2^j)$.
- consequence: the total number of nonzero (**significant**) coefficients $N_J = \sum_{j=0}^J n_j = O(2^J)$

Requirements of better methods (beyond wavelets)

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Introduction

Wavelet methods

Beyond wavelets

- ① can compactly represent line singularities
→ adaptive domain partitioning
- ② possess good compression properties
→ sparse representation of smooth contours and smooth surfaces
- ③ fast decoding speed
low inverse transformation complexity ($O(N)$)

Linear methods

- Curvelets
- Contourlets
- etc.

uses directional filterbanks to catch line singularities.
[-] redundant decomposition.

Non linear methods

- Wedgelets
- Binary Space partitioning algorithms
- etc.
- Normal Offsets

→ **adaptive** domain partitioning !

Part II

Normal Approximation :1D

Normal Offsets

Concept

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Concept: normal curves

Normal Offsets

Properties

Wavelet prediction step
(No Update)

Vertical offset: difference between prediction and function *value*

Normal offset step

Normal offset: signed length between prediction and piercing *point*

Normal Offsets already successfully applied for: Smooth manifolds

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Concept: normal curves

Normal Offsets

Properties

- **Remeshing** of smooth 3D surfaces [Guskov et. al]

irregular mesh

Normal Offsets already successfully applied for: Smooth manifolds

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Concept: normal curves

Normal Offsets

Properties

- **Remeshing** of smooth 3D surfaces [Guskov et. al]

semi-regular mesh + 90% scalar coefficients

Normal Offsets already successfully applied for: Smooth manifolds

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Concept: normal curves

Normal Offsets

Properties

- **Remeshing** of smooth 3D surfaces [Guskov et. al]

- Approximation of smooth curves [Daubechies et. al]

Smooth and non-smooth

The nonlinear and highly adaptive character of a normal approximation scheme allows it to mimic both smooth as well as non smooth behaviour, without prior knowledge about the function f .

smooth

piecewise smooth

Geometrical information

The normal offset coefficients information about:

- 1 the function value (z) (the **what**)
- 2 **AND** their **location** (the **where**) (x, y)

Behaviour w.r.t. singularities

- Piercing points are **attracted towards singularities**.

No major advantage in 1D $\implies$ becomes important
in 2D

Geometrical information

The normal offset coefficients information about:

- 1 the function value (z) (the **what**)
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Behaviour w.r.t. singularities

- Piercing points are **attracted towards singularities**.
No major advantage in 1D $\implies$ becomes important in 2D

Part III

Normal Approximation :2D

How to extend to 2D-setting?

Define the normal direction...

...using a le Loop or Butterfly
algorithm as for the 3D surface case

$$\nu_{j+1}^* = \frac{3}{8} \left(\nu_j^{1,i} + \nu_j^{2,i} \right) + \frac{1}{8} \left(\nu_j^{3,i} + \nu_j^{4,i} \right)$$

- global mesh refinement operators to avoid edge flipping
 - non-hierarchical triangulations
 - compression difficulties
- or exception handling needed to stay in the same patch.

How to extend to 2D-setting?

Define the normal direction...

as lying in the vertical plane while $\perp$ on the edge

- Local refinement operators (Edge refinement)
 - nested triangulations i.e. $\mathcal{T}_j \subset \mathcal{T}_{j+1}$
 - suited for compression

Topology

Regular vs. irregular meshes $\Leftrightarrow$ Smooth vs. Nonsmooth

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- In the **smooth** setting the approximating meshes are **semi-regular** (of subdivision connectivity starting from an irregular basemesh).
- In the **nonsmooth** setting **topological information does matter!** We have to **avoid** that edges **cross** contours $\Rightarrow$ **irregular meshes**.

method	image dependent	extra storage	α
subdivision	-	-	small
connectivity	-	-	-
adaptive	yes	2 bits/triangle	greater

α is the rate of approximation $\sigma_{L_p}(f) = O(n^{-\alpha})$

Normal Offsets: Summary

Taylor made for efficient geometric image compression

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Features

- piercing points are attracted towards the contours
- adaptive interconnection $\mapsto$ triangle edges line up against the contour in a tangential manner.
- edge refinement method (projection of normal vector on the projection of edge on the XY -plane) $\mapsto$ local refinement

Normal decomposition & n-terms approximation
algorithm w.r.t. to the L_1 distance norm.

Approximation rate w.r.t. L_1 : $\|f - f_n\|_{L_1} = O(n^{-1})$

Part IV

Encoding of normal coefficients

Lossless encoding

Cross section of an Horizon Class Image

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Lossless encoding

Conclusion

Results

Cross section of an Horizon Class Image

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Conclusion

Results

Lossless encoding

Cross section of an Horizon Class Image

Definition (Normal index)

A normal index is the signed number of pixels between the *location* of the midpoint and piercing point.

- sufficient to define normal offset

- Discrete setting introduces **vertical offsets**
- Distribution of the vertical offsets is a mixture of a Laplace distribution and dirac distribution.

Probability density function of normal indices

- Mapping of indices $\mapsto$ **codewords** (Huffman encoder)
- Can eventually be followed by an arithmetic encoder.

Where lies the compression?

Normal offset transform vector to scalar (in fact $\mathbb{Z}^2$)

Approximation rate w.r.t. $\mathcal{H}$ thanks to Edge Locating property

Entropy encoding because of a priori PDF of projection of normal indices. [\[Lossless\]](#)

Part V

Some pictures...

Images with smoothly colored areas

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Original Image

Normal compressed im-
age 0.005bits/pixel

Images with smoothly colored areas

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Triangulation

Normal compressed im-
age 0.005bits/pixel

Images with smoothly colored areas

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JPEG2000 compressed
image 0.005bits/pixel

Normal compressed im-
age 0.005bits/pixel

Closer look

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normal offsets

Closer look

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JPEG2000 (wavelets)

More complex images

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Figure: domain partitioning 'Lena'

More complex images

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Figure: Normal Mesh representation 'Lena'

More complex images

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Figure: Normal Mesh approximation 'Lena' ▶

The End

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Thank you for your attention!

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Questions?