Skewed Symmetry: A Nonaccidental Property Used to Perceive Visual Forms

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Mathematically, skewed symmetry is a nonaccidental property because it can be interpreted as bilateral symmetry in depth viewed from a nonorthogonal angle. To find out whether this is a useful property in the perception of visual forms, 4 experiments were designed in which the Ss had to determine whether 2 symmetric or random patterns were the same regardless of possible affine transformations between them. The results provided mixed evidence: Although there was always a large symmetry advantage, skewed symmetry was only perceived as bilateral symmetry in depth for dot patterns with higher order types of symmetry (Experiment 1), when the dots were connected to form closed polygons (Experiments 2 and 4), or when they were surrounded by a frame to enhance their planarity (Experiment 3). In other cases, Ss relied on local groupings on the basis of proximity or curvilinearity, which are qualitatively affine invariant.

How do human perceivers determine that two images result from the same object viewed from different angles? This projective equivalence problem is interesting and far from trivial. It is interesting because it lies at the basis of object recognition in general. It is far from trivial because in projecting objects to images, one dimension (i.e., depth) is completely lost, and most features in one image are changed in the other. Yet some characteristic properties are preserved (invariant) despite the transformations resulting from projection.

As indicated by Stevens (1980), only a few properties in the three-dimensional scene are invariant under projection onto the image, and more important, of those that are, only a few have the necessary feature of having an invariant inverse, so the presence of the relation or property in the image does not necessarily imply the corresponding scene property. For example, if two points are close together in space, they invariably appear so in the image, but two-dimensional proximity in the image does not guarantee close positions in space. More relevant to vision are those

properties that are necessarily present in the three-dimensional scene when present in the image or whose presence in the image can at least be used to constrain the possibly corresponding three-dimensional states.

Much work has been done in recent computational approaches to identify these useful image properties (e.g., Binford, 1981; Kanade, 1981; Lowe, 1987; Stevens, 1980; Witkin & Tenenbaum, 1983). In general, they find that under the assumption of a general viewpoint, some spatial structures (e.g., cotermination, collinearity, curvilinearity, and parallelism) can be used to infer, more or less reliably, similar characteristics in the world. Because of evident reasons, these organized structures in the image are called *nonaccidental properties*. The basic assumption is that these image regularities are not an accidental result of a singular arrangement of objects in the scene with respect to the viewer or camera (i.e., general viewing position or viewpoint-invariance condition). Even though inferences that are based on this kind of evidence do not yield absolutely perfect solutions in all cases (there is one viewpoint for which it does not work), the suggestion is that nonaccidental properties are the kind of information on which the human visual system relies in solving the inverse optics problem as formulated previously.

Of fundamental importance in this approach, then, is to see what kind of image properties are nonaccidental in the mathematical sense of the word. Although not all theories agree in this respect, some properties that definitely do not fulfill the necessary requirements can be indicated (e.g., Biederman & Shiffrar, 1987). For example, metric properties such as length, angle, and degree of curvature are not preserved under projection, be it perspective or parallel. Other characteristics that do occur in most available lists of reliable nonaccidental properties are collinearity, curvilinearity, cotermination, and parallelism. Skewed symmetry, which results from orthographic projection of bilateral or mirror symmetry on a plane oriented arbitrarily in space, is present on some lists but not on others. Stevens (1980) and Kanade (1981; Kanade & Kender, 1983) explicitly studied

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this property, whereas Lowe (1987) did not mention it, and Biederman (1987) added a question mark to it.

The purpose of my research was to find out whether skewed symmetry is a useful perceptual property. Mathematically, skewed symmetry is a nonaccidental property: Skewed symmetry in the image most probably does not result from an accidental view on an asymmetric arrangement of features in space but rather from a general view on a real bilateral symmetry in depth. As a consequence, it might be used to recover bilateral symmetry and to determine projective shape equivalence. In addition, computer vision research showed that skewed symmetry can be detected and used as a nonaccidental property, in principle as well as in some working algorithms. For example, skewed symmetry can be detected because the virtual lines connecting symmetrically positioned elements are still parallel under orthographic projection (parallelism is an affine-invariant grouping). Moreover, the angle between these lines and the axis of symmetry in the image provides some constraints on the possible range for both the slant and tilt components of surface orientation in relation to the viewer (see Friedberg, 1986; Hakalahti, 1983; Kanade, 1981; Kanade & Kender, 1983; Stevens, 1980). From previous experiments, however, it is known that human perceivers can detect skewed symmetry, but serious doubts must be formulated with respect to the speed and efficiency with which this can be achieved (see Wagemans, Van Gool, & d'Ydewalle, 1991, 1992). Whereas the local correspondences are preserved after skewing, the higher order regularities defined on two pairs of elements are destroyed. Although some theories stress the role of local correspondence (e.g., Jenkins, 1983; Julesz, 1971), I found that symmetries created by reflection, translation, and rotation are processed on a much more global basis (Wagemans, Van Gool, Swinnen, & Van Horebeek, in press). Therefore, the question remains whether human perceivers use skewed symmetry as a reliable source of information in the image in solving the problems of recovering the real-world symmetry and determining that two images are projections of the same symmetric object.

Criteria for Evidence About the Perceptual Use of Skewed Symmetry as a Nonaccidental Property

In general, not much experimental research has been devoted to discovering if nonaccidental properties are detected and used by human perceivers. Nevertheless, Biederman's (1987) recognition-by-components theory relies heavily on the requirement that nonaccidental properties are processed preattentively because they are assumed to provide the necessary information to determine the identity of *geons*, or geometric ions (primitives), which are supposed to be the basic building blocks of object recognition. For example, a brick can be distinguished from a cylinder because it has three instead of two parallel edges, only straight edges, no curved edges, three locations where three lines coterminate, and so forth. From that perspective, Biederman listed some "psychological evidence for the rapid use of nonaccidental relations" (pp. 119–120), but a closer look at this so-called

evidence reveals several problems with it (see Wagemans, 1992, for a detailed discussion). For example, with respect to the specific case of skewed symmetry, Biederman referred to Garner's (1974) and Pomerantz's (1977) work as showing that the degree of symmetry is a readily available perceptual distinction. Biederman needed this to enable a rapid discrimination between square- and circle-shaped cross-sections on the one hand and rectangular and elliptic on the other, which is one of the attributes on the basis of which geons are identified. This kind of evidence is not directly relevant, however, because it is focused on perfect symmetry, which is preserved under projection only in particular higher order types of symmetry.

Normally, symmetry is distorted by affine transformations (perfect bilateral symmetry becomes skewed symmetry). Although recent research with different experimental paradigms has shown that human perceivers are able to pick up and use affine information in some situations (e.g., Cutting, 1987; 1988; Todd & Bressan, 1990), it is far from clear what this implies for the status of skewed symmetry as a nonaccidental property used to perceive visual forms. With respect to this more specific issue, only casual and often contradictory observations have been made. For example, Palmer (1983) claimed that in cases such as an ellipse or a parallelogram, skewed symmetry is often readily perceived as arising from a tilted symmetric object or surface (i.e., tilted circle or top surface of a brick), whereas Attneave (1982) demonstrated a case (a dot pattern) in which skewed symmetry is not seen as such.

In the experiments described in this article, the subjects' task was to determine whether two patterns presented side by side on a screen were the same regardless of possible affine transformations of the original patterns. In other words, they had to determine whether the two patterns could be images of the same pattern viewed from a different angle (assuming orthographic projection). The affine transformation used was a combination of rotation in the plane and compression in the plane resulting from slant in depth. In other words, subjects had to assume that the patterns were present on a plane, the three-dimensional orientation of which could be different for the two patterns to be compared. Half of the patterns were symmetric, and half were random. In these conditions, bilaterally symmetric patterns are transformed to skewed symmetric ones. Skewed symmetry, if used to perceive visual forms, might facilitate the comparison in two fundamentally different ways. First, as an affine regularity, skewed symmetry might help to determine the affine equivalence of the two patterns because the parameters defining the skewed symmetry (i.e., orientation of the axis and of the virtual lines) can be used to unravel the transformation that might have occurred between them (e.g., by mentally undoing the transformation, as in the classic experiments of R. Shepard & Cooper, 1982).¹ A second way would constitute more direct evidence for skewed symme-

¹ Such a rectification process has been dismissed by Cutting (1987, 1988) for the case of slanted cinema, but he explicitly suggested that his results could not be generalized to the case of static pictures. Moreover, a linear relation between same-different

try as a nonaccidental property used to perceive visual forms because it implies recovery of bilateral symmetry. The underlying rationale is the following. If skewed symmetry in the image is interpreted as bilateral symmetry in the world, the comparison task should be easier because there is a long research tradition showing that gestalts such as those afforded by bilateral symmetry, are easier to compare than nongestalts such as random patterns (e.g., Bagnara, Boles, Simion, & Umiltà, 1983; Fox, 1975).

Although both ways of using skewed symmetry are interesting, only the latter requires a three-dimensional interpretation. The former makes it possible to solve the affine equivalence problem while remaining in the image plane. Therefore, the data generated by the experimental paradigm as outlined earlier will be analyzed in different ways that imply gradually more stringent criteria to be fulfilled to constitute evidence for skewed symmetry as a nonaccidental property used to perceive visual forms. Only the final two or three imply recovery in the strict sense of the word. First, the least one can expect is that the task would be more difficult with patterns that are completely random than with those that possess skewed symmetry. In other words, there must be a main effect of symmetry on the dependent measures used to investigate performance levels in the affine matching task (e.g., response times and error rate).

Second, the effects of both factors constituting the affine transformation (i.e., rotation and compression) should be larger for the random patterns than for the ones with skewed symmetry. This expectation is based on recent research about mental rotation and size scaling (i.e., Euclidean or similarity transformations), which form a subgroup of the more general class of affine transformations. Although classic work (e.g., Cooper & Podgorny, 1976) showed no effect of complexity (when specified as the number of corners in random polygons), more recent studies indicated that judgments of this kind of shape equivalence are harder for complex shapes than for simple ones (e.g., Jolicoeur & Besner, 1987). Bethell-Fox and R. Shepard (1988) suggested that the degree to which the rate of mental rotation is independent of stimulus complexity depends on the degree to which the stimuli are internally represented as integrated wholes. That symmetry might play a role in this seems to be supported by the finding that with respect to the axis of rotation, natural axes are easier than arbitrary axes (S. Shepard & Metzler, 1988). These results can be extrapolated to expect that judgments of affine equivalence are harder for random patterns than for skewed symmetric ones because rotation as well as compression are both more difficult. In other words, this second test can be specified as interaction effects with symmetry of rotation and compression.

An interaction effect of rotation and compression seems to constitute a third viable test on the basis of the following rationale. If some kind of recovery (of bilateral symmetry

from skewed symmetry) takes place, one expects different effects of rotation at different levels of compression and vice versa.² For example, when there is no compression, only rotation in the image plane that preserves all types of bilateral symmetry but does not suggest any three-dimensional orientation is performed. Similarly, when there is no rotation in the plane, one would predict no recovery of slant and tilt because only compression in the image plane is performed. Especially in the case without perspective (as is the case with the affine transformation here), pure compression along the horizontal axis is very inefficient in suggesting slant in depth (see Todd, 1984, for a similar discussion with respect to structure-from-motion recovery). In short, different effects of one factor at each level of the other are expected for both factors as a consequence of these specific rotation-compression combinations. A similar rationale was followed in research by Kubovy and Podgorny (1981) on two other normalization operations (size and orientation). Note also that the presence of a statistically reliable interaction does not *prove* that there is a single stage underlying the processing. Only the absence of an interaction would indicate the existence of separate processing stages (cf. Townsend & Ashby, 1982). Therefore, if an interaction was observed, further tests are needed to find out whether it is really due to a single-stage recovery operation.

One way to test this further would be to check whether the two factors (rotation and compression) interact positively or negatively, providing evidence for superadditivity or subadditivity, respectively (e.g., Schweickert & Townsend, 1989). In the context of the experiments presented here, the latter is expected for patterns with skewed symmetry. In addition to the perception of skewed symmetry as bilateral symmetry in depth, skewed symmetry can be used to undo the slant and tilt of the underlying plane before deciding whether the two patterns on the screen are affine equivalent. This would be assumed if one adhered to mental-transformation approaches such as mental rotation (R. Shepard & Cooper, 1982). More specifically, instead of mentally undoing (see Footnote 1) compression and rotation in two subsequent stages (which could be the case if one remained in the image plane), one could mentally rotate the patterns about a single axis in depth. Note, however, that this requires the *z*-axis (i.e., the orientation in depth) to be perceived as such and that this would result in shorter response times than resolving both compression and rotation in the plane separately.

² This interaction does not imply different degrees of nonaccidentalness, which would be nonsense from a mathematical point of view. As indicated by a reviewer, "accidents" occur with unlikely alignments of observers to major (axial) coordinates of objects. When accidents do not occur, the angle of regard to the objects (as reflected in the rotation-compression parameters) is irrelevant, and analysis can proceed on the image and three-dimensional reconstruction can follow. Perceptually, however, angle of regard is a continuous variable with likely effects on the efficiency of extracting and processing the information in the image. As this research is aimed at discovering the role of skewed symmetry as a nonaccidental property used to perceive visual forms (cf. the introduction), the Rotation $\times$ Compression interaction and the specifications thereof in the criteria that follow is a valid result to expect.

response times and a parametric difference between the comparison shapes (e.g., rotation angle) need not imply an analog mental process (e.g., mental rotation). The comparison may be only more difficult (see Rock, Wheeler, & Tudor, 1989). The nonlinearities observed in the experiments here seem to conform to that idea.

Note also that a similar additive-factors logic has been used in research on other mental transformations such as size and orientation (e.g., Besner, 1978; Kubovy & Podgorny, 1981), three-dimensional mental rotation (e.g., Parsons, 1987), and so-called mental flipping (e.g., Koriat, Norman, & Kimchi, 1991; see also Tarr & Pinker, 1989). Koriat et al. distinguished between a preattentive stage, which chooses the shortest path, and real mental transformations, which require more time. In sum, a more stringent criterion would be to expect the rotation-compression interaction to be due to subadditivity.

A fifth criterion, the most stringent I could think of, is again a further specification of the previous one. Subadditivity can have several different causes. For example, it is probably easier to locate the appropriate axes in some cases than in others. Moreover, there is evidence that mental rotation is slower around axes that depart from the canonical axes of the environment (Shiffrar & R. Shepard, 1991). If recovery took place on the basis of skewed symmetry, however, one would expect linear regressions for the response times yielded by all rotation-compression combinations because each can be translated into a particularly oriented z -axis with equal levels of in-depth slant angles (see the General Method section). A similar reasoning is followed in studies of apparent motion perception (e.g., Bundesen, Larsen, & Farrell, 1983; Farrell, 1983; for more specific evidence about the role of symmetry in apparent extended body motions, see Proffitt, Gilden, Kaiser, & Whelan, 1988). The intercept could then be interpreted as the time needed to find the axis, whereas the slope would indicate the time needed to undo the slant.

Note that the presence of an axis intrinsic to the pattern has been linked with mental transformations before. For example, Jolicoeur (1990) suggested that intrinsic properties such as elongation and symmetry could explain how the system knows which way to rotate to achieve congruency (which is implied by mental rotation curves that are symmetric around 180°). More generally, symmetry axes have been described as invariant lines around which transformations can be applied (e.g., Carlton & R. Shepard, 1990a, 1990b; Leyton, 1986). Tarr and Pinker (1990) started from a different assumption to explain the conflict between mental rotation phenomena and data indicating orientation-invariant recognition. In their view, mental rotation is needed only when a two-dimensional description is necessary (e.g., asymmetric patterns). Patterns for which a one-dimensional description suffices yield recognition times that are independent of their orientation. In their experiments, Tarr and Pinker (1990) explicitly introduced symmetry, skewed symmetry, and bilateral redundancy to support their claim, although their second type of patterns did not possess skewed symmetry in the mathematical sense used in my experiments. Implicit in our reasoning formulated earlier is the assumption that the presence of an axis (of skewed symmetry) makes the mental transformations easier, not superfluous (as in Tarr and Pinker's view). A similar issue is at stake in Corballis's (1988) theory of shape recognition, when he "wonders if any set of heuristics could ever be powerful enough to locate shape-centered reference frames prior to

the act of recognition itself" (p. 117). He explicitly mentioned axes of symmetry and elongation as sources that could be used for these purposes.

General Method

Subjects

In each of the experiments reported here, 4 highly practiced and 4 unpracticed subjects participated. The 4 practiced subjects included myself, a graduate student, and 2 undergraduate students involved in this research project. They all had extensive experience in experiments with skewed symmetry, but only I knew all the details of the experimental design. The 4 unpracticed subjects were volunteers from a pool of undergraduate students in fulfillment of a course requirement and graduate students involved in other research projects. The practiced subjects participated in all four experiments (in different orders), whereas the latter, who were naive with respect to the purpose of the experiment, took part in only one. All observers had normal or corrected-to-normal vision.

Stimuli

Experiment 1: Dot patterns. In each trial two patterns each consisting of 24 dots in a circular area with a cross-section of 5.5 cm were presented on the screen, one in the left half and the other in the right half and separated by a center-to-center distance of 10 cm. One of the two patterns was always untransformed (i.e., the original pattern), and the other one was a transformed version of either the same original pattern or a different one. The left-right position of the untransformed pattern was chosen at random from trial to trial.

Half of the original dot patterns were completely random, and half were symmetric. How these random and symmetric dot patterns were created is described in detail elsewhere (Wagemans et al., 1991). For the purpose of this experiment, it is sufficient to know that the random dot patterns were created by positioning 24 dots randomly at least 0.5 cm apart within the circular area. The symmetric dot patterns were constructed by reflecting a small number of dots, also pseudorandomly distributed, around one or more straight axes. Seven types of symmetry resulted from using four axis orientations in the following manner. In single symmetry, 12 randomly positioned dots were reflected around a vertical, horizontal, left (which is 45° rotated counterclockwise from vertical, or right (45° clockwise) axis. Two kinds of double symmetry (vertical-horizontal or left-right) were created by first reflecting six randomly positioned dots around a vertical or left axis and then reflecting the resulting 12 dots' pattern around the orthogonal axis (horizontal or right). In quadruple symmetry, a pseudorandom collection of three dots was reflected around all four axes (vertical, horizontal, left, and right). For each kind of symmetry, 192 dot patterns were generated. In addition, 1,344 random dot patterns were made to have an equal number of symmetric and random dot patterns. This large number of original patterns made it possible to use different patterns in every experimental trial. Moreover, 84 additional random dot patterns as well as 12 patterns for each kind of symmetry were generated to be used in practice trials. Examples of all eight kinds of dot patterns (1 random + 7 symmetric) are given in Figure 1A.

Experiment 2: Polygons. Pilot work showed that dot patterns as described previously were not really perceived as lying on a plane with arbitrary three-dimensional orientation when affine transformed. In fact, dots as depicted in these experiments, with

Figure 1. Examples of the eight types of patterns used in the experiments. (Panel A shows dot patterns, and Panel B shows polygons. VHLR denotes vertical–horizontal–left–right, VH denotes vertical–horizontal, LR denotes left–right, V denotes vertical, H denotes horizontal, L denotes left, R denotes right, and RND denotes random.)

unresolvable inner detail or size, can be located almost anywhere in three-dimensional space, even at different depth planes. In recovery tasks such as these, for skewed symmetry to be helpful in providing constraints on the range of possible slant and tilt angles, planarity is a necessary assumption. One way to enhance the planarity is to use closed contours instead of dot patterns. The fact that the spatial order is explicitly given in closed contours should help in perceiving the pattern as lying on a single plane. This difference between dot patterns and closed contours could account for the contradiction in Palmer's (1983) and Attneave's (1982) intuition that skewed symmetry is or is not seen as bilateral symmetry oriented in depth. In Experiment 2, therefore, polygons were created by connecting the coordinates generated for Experiment 1 in such a way as to preserve their kind and type (e.g., random, vertically symmetric, quadruply symmetric, etc.). In addition, no intersections were allowed. Examples of all eight kinds of polygons are given in Figure 1B. As in Experiment 1, 192 polygons were generated for each kind of symmetry. In addition, 1,344 random polygons were made to have equal numbers of symmetric and random patterns.

Experiments 3 and 4: Frames around the patterns. A second way to enhance the perception of planarity of the dots is to surround them by a frame, as if the dots were drawn on a sheet of paper. When affine transformed along with the dot patterns included in it, the frame could be interpreted as the contour of the plane, the three-dimensional orientation of which was changed. In Experiment 3, therefore, the original dot patterns were included in an 8-cm-sided square, whereas the transformed dot patterns were surrounded by a rectangle or a parallelogram, the orientation and size of which was determined by the parameters of the particular affine transformation. As a kind of control of this effect, the same was done for polygons in Experiment 4.

Affine transformation. The transformation used is an instance of the class of affine transformations. More specifically, the affine transformation expressed by the following matrix (T) was applied:

$$T = \begin{pmatrix} \cos \Phi & (-\sin \Phi)\Gamma \\ \sin \Phi & (\cos \Phi)\Gamma \end{pmatrix}. \quad (1)$$

(1) This matrix is the result of multiplying the following matrices expressing a rotation (R) with angle Φ and a compression (C) with factor Γ , respectively:

$$R = \begin{pmatrix} \cos \Phi & -\sin \Phi \\ \sin \Phi & \cos \Phi \end{pmatrix} \quad (2)$$

$$C = \begin{pmatrix} 1 & 0 \\ 0 & \Gamma \end{pmatrix}. \quad (3)$$

Note that this multiplication is not commutative and that compression was applied first. Applying this affine transformation to the original dot patterns (by multiplying the x and y coordinates, as a horizontal vector, with matrix T) simulates what happens when orthographically projecting a pattern lying in a plane P on the image plane I (which is a good approximation of perspective projection for patterns that are relatively small compared with the viewing distance). If the plane P is not parallel to the image plane I (i.e., for general viewing positions), both planes intersect along a straight line L . The influence of the slant of P can be interpreted as turning P away from I around L . The effect in I will then be a compression orthogonal to L with a factor Γ . Assuming that L makes an angle Φ with the y -axis in I , the effect is then given by multiplying $R\Phi$ with $C\Gamma$.

Parameters. The parameters defining the transformation were manipulated at several levels: The parameter expressing the angle of rotation Φ was 0° , $\pm 22.5^\circ$, $\pm 45^\circ$, $\pm 67.5^\circ$, or 90° , whereas the compression factor Γ was 1, .9397, .766, or .5. The former means that the pattern could be presented vertically ($\Phi = 0^\circ$), horizontally ($\Phi = 90^\circ$), or in 22.5° -step intermediate orientations both clockwise and counterclockwise. The latter means that the plane could be slanted in depth by 0° , 20° , 40° , or 60° , respectively (compression is given by the cosine of the angle). Combining these two factors orthogonally yielded 32 kinds of transformations. Figures 2A and 2B show what happens to a vertically symmetric and a random dot pattern, respectively, when they are transformed by T at 8 of the 32 levels. One of

them (i.e., $\Phi = 0^\circ$ and $\Gamma = 1$) expresses no transformation (i.e., resulting in the identical pattern). Examples of the affine-transformed patterns in the other three experiments are shown in Figure 3.

Apparatus

The coordinates of the original dot patterns were generated by a C program on a SUN-3 Workstation with a Motorola MC 68881 floating point board (as described in Wagemans et al., 1991). The coordinates of all patterns were stored in separate files that could be accessed by a Turbo Pascal program automating the experiment on an IBM AT-compatible with an Intel 80286 processor and a

Figure 2. Examples of the parameterizations for a vertically symmetric (Panel A) and a random (Panel B) dot pattern: Experiment 1. (For each type of pattern, the original is shown on top, followed by counterclockwise rotations of 0° , 22.5° , 45° , and 67.5° from top to bottom, and .766 and .5 compressions in the left and the right column, respectively.)

Figure 3. Examples of the parameterizations in the other Experiments 2-4. (The pattern at the left is an untransformed one, whereas the pattern at the right is rotated by 45° and compressed by .5. Panel A shows an unframed random polygon [Experiment 2]. Panel B shows a framed vertically symmetric dot pattern [Experiment 3]. Panel C shows a framed vertically symmetric polygon (Experiment 4).

Hercules graphics board. The dot patterns were presented as black dots on a gray background on a monochrome screen (with a fast white phosphor used in reverse mode) with a 60-Hz temporal resolution and a 720×348 spatial resolution. The subjects were seated at a distance of 114 cm on a chair with adaptable height to get their eyes in front of the center of the screen. At that viewing distance, the individual dots, the patterns they constituted, and the whole stimulus area (both patterns plus the separating space) subtended 6.6 arc min, 2.75°, and 7.75° of visual angle, respectively. The color and size of the polygons were the same. The room was completely dark except for the light reflecting from the screen. A headrest was used to prevent head rotations.

A trial was constituted from the following sequence of events. First, a black fixation cross was presented for 750 ms in the center of a gray background. Second, after a 500-ms blank interval, two dot patterns were presented, one at both sides of the fixation cross. They remained on the screen until the subject pressed one of two response buttons on a panel connected with the PC configuration. Response times were recorded to the nearest 0.1 ms from stimulus onset until button press. Each answer was instantaneously evaluated by the computer so that immediate feedback could be given. A correct answer was followed by a 500-ms high-frequency tone (660 Hz), and a false response was followed by a 500-ms low-frequency tone (440 Hz), to keep the motivation and arousal of the subjects at an optimal level. This feedback could have helped the naive, untrained subjects to learn to perform the task better as they proceeded; however, these learning effects were of secondary importance. Moreover, they were averaged across blocks and trials because their order was randomized for each subject separately.

Task and Procedure

Two simultaneously presented patterns had to be compared in a same-different task. Subjects responded by pressing one of two buttons on a response panel, the identity of which could be chosen by the subjects (i.e., left one indicating same responses, the right one different, or the reverse). They were instructed to consider both patterns the same if they were either identical or affine transformed and different if they could not be brought into correspondence by changing their two-dimensional or three-dimensional orientation. Although the verbal instructions may have sounded complex to most subjects, the task was clear after a block of 168 practice trials, similarly constituted as the experimental ones but with different patterns.

In half of the trials, an original pattern (random or symmetric) was paired with a transformed version of the same pattern. In the other half of the trials, an original pattern was paired with a transformed version of a different pattern (randomly chosen out of the whole collection of available patterns, so on some trials, e.g., a vertically symmetric pattern was paired with a transformed different vertically symmetric pattern). Not a single pattern was used in more than one trial, except for the trials with wrong responses, which were retaken at a random position in the remainder of the block. Each of the seven kinds of symmetric patterns was tested three times for each of the 32 parameterizations (8 rotation angles $\times$ 4 compression factors) of the affine transformation T described before. The same number of positive trials (i.e., 672) was used for the random dot patterns, also equally divided over all levels of the transformation. The number of negative trials (i.e., 1,344) was similarly constituted by using all transformation parameters equally often for all kinds of symmetric and random patterns. The number of trials (i.e., 2,688) was divided in 16 blocks of 168 trials that could be run without a break. Each block, taking about 15 or 20 min, was a random sample from the total number of trials with the constraint that it contained an equal number of positive and negative trials, both subdivided in equal numbers of all kinds of symmetric and random patterns. All subjects took three or four blocks in one session with only minor (e.g., 2- or 3-min) breaks between the blocks. Sessions were distributed across several days within a 1-week period.

Results for Criteria 1-3

Data Analysis Methods

I assessed the first three criteria listed separately for each experiment with an analysis of variance of the response times associated with the correct same responses, in a within-subjects design with four orthogonal variables: kind of pattern (symmetric vs. random), pattern type vertical-horizontal-left-right, vertical-horizontal, left-right, vertical, horizontal, left, right), rotation angle (90°, 67.5°, 45°, 22.5°, 0°, -22.5°, -45°, -67.5°), and compression (1, .9397, .766, .5). Although pattern type does not make much sense for the random patterns, it was incorporated as such to obtain orthogonally manipulated variables. An additional argument might be that the way the patterns were generated (together with the symmetric ones for each type of symmetry) could have caused particular features to have "sneaked" in each set of random patterns. Results obtained with a somewhat simplified design, however, are reported here so as not to overload the general picture with too much superfluous detail (results were very similar anyway). In partic-

ular, kind of pattern and pattern type were combined as number of axes (0, 1, 2, 4), and results were collapsed for clockwise and counterclockwise angles of rotation (5 levels: 0° , $\pm 22.5^\circ$, $\pm 45^\circ$, $\pm 67.5^\circ$, 90°).

I performed similar analyses of variance for the number of errors. The results of these analyses are not reported here because they produced results similar to those for the response times. In addition, they are hard to interpret because each trial was repeated until a correct response was given, so the average number of errors per condition could vary from zero to infinity, in principle. In practice, however, the average number of errors varied only between 0 and 1. For example, the latter would mean that all trials in a particular condition had to be repeated once, or half of the trials had to be repeated twice. An indication of the performance level that is easier to understand is the percentage of errors at the first presentation. Because these percentage errors were too low for an analysis across all separate conditions, they were pooled across all rotation-compression combinations and analyzed in separate analyses of variance for each experiment with a single within-subjects variable: symmetry (vertical-horizontal-left-right, vertical-horizontal, left-right, vertical, horizontal, left, right, random). Note that the data presented here were obtained with analyses across the two types of subjects. Separate analyses for expert and naive subjects produced similar results.

Criterion 1 for Percentage Error

Experiment 1. The effect of symmetry was very reliable: $F(7, 49) = 20.55$, $MS_e = 7.20$, $p < .0001$. The means are given in the first column of Table 1. A priori comparisons showed a large difference between symmetric and random patterns, $F(1, 7) = 49.47$, $MS_e = 12.78$, $p < .0005$, which was replicated for all separate numbers of symmetry axes. Of all pairwise differences among the different numbers of symmetry axes, only 4 versus 2 did not reach statistical significance ($F < 1$).

Experiment 2. In general, the percentage errors were much smaller with polygons compared with dot patterns (see Table 1). Yet the same symmetry effect was obtained: $F(7, 49) = 5.21$, $MS_e = 3.17$, $p < .0005$. A priori compar-

isons showed a reliable difference between symmetric and random patterns, $F(1, 7) = 11.68$, $MS_e = 4.01$, $p < .05$, which was also replicated for all separate numbers of symmetry axes. Of all pairwise differences among the different numbers of symmetry axes, only 2 versus 1 did not reach statistical significance ($F < 1$).

Experiment 3. In general, the percentage errors were somewhat higher again, more like in Experiment 1 (with dot patterns but not surrounded by a frame; see Table 1). Yet the same symmetry effect was obtained: $F(7, 49) = 16.05$, $MS_e = 4.68$, $p < .0001$. A priori comparisons showed a reliable difference between symmetric and random patterns, $F(1, 7) = 57.93$, $MS_e = 7.69$, $p < .0005$, which was once more replicated for separate numbers of symmetry axes. Of all pairwise differences among the different numbers of symmetry axes, only 2 versus 1 failed to reach the standard criterion of statistical significance, $F(1, 7) = 3.94$, $MS_e = 2.25$, $p < .10$.

Experiment 4. In general, the percentage errors were somewhat lower again, more like in Experiment 2 (with polygons but not surrounded by a frame; see Table 1). The symmetry effect, however, was once more very reliable: $F(7, 49) = 6.55$, $MS_e = 3.07$, $p < .0001$. A priori comparisons showed a systematic difference between symmetric and random patterns, $F(1, 7) = 13.04$, $MS_e = 7.80$, $p < .01$, which was replicated for all numbers of symmetry axes (although somewhat less pronounced for 2 and 1). Of all pairwise differences among the different numbers of symmetry axes, only 2 versus 1 failed to reach statistical significance, $F(1, 7) = 3.47$, $MS_e = 0.94$, $p < .15$.

Criteria 1-3 for Response Times

The response times in all conditions, averaged across the four experiments, are shown in Figure 4. Each of the four panels is devoted to one set of stimuli with different symmetry (Panel A shows results for random patterns, Panel B shows onefold symmetry, Panel C shows twofold symmetry, and Panel D shows fourfold symmetry). Within each panel, response time is plotted (in milliseconds) for the four functions (yielded by each compression value) at the five angles of affine rotation (0° , $\pm 22.5^\circ$, $\pm 45^\circ$, $\pm 67.5^\circ$, and 90°).

Table 1
Mean Percentages of Errors in all Four Experiments

Pattern	Experiment			
	1	2	3	4
Symmetric				
Vertical-horizontal-left-right	3.9	1.0	1.8	1.6
Vertical-horizontal	4.3	3.1	4.7	3.5
Left-right	3.0	2.2	3.4	2.6
Vertical	7.0	2.9	5.5	3.2
Horizontal	11.1	3.5	4.7	4.3
Left	8.8	3.2	5.5	3.9
Right	6.4	0.4	4.2	3.0
Random	15.9	4.9	12.2	7.0

Note. The percentage errors for the different kinds of symmetric patterns are based on 96 observations per subject ($n = 8$ in each experiment). For the random patterns, the number of observations per subject is 672.

Figure 4. The effects of affine rotation angle on response times (in milliseconds) at different levels of compression (indicated by different line types), averaged across the four experiments. (The four panels represent the results for the different stimuli: Panel A, random patterns; Panel B, onefold symmetry; Panel C, twofold symmetry; Panel D, fourfold symmetry. Note that the ordinates differ, which reflects the main effect of number of axes.)

These plots help to visualize the pattern of results for the three major stimulus dimensions (symmetry, rotation, and compression). More specific quantitative data are provided for each experiment separately in the following.

To illustrate the main differences between the different stimuli, Figure 5 represents the effects of affine rotation on the response times as a function of two orthogonally manipulated variables: pattern (dot patterns vs. polygons, symbolized by dotted vs. full lines) and frame (present vs. absent, symbolized by open vs. filled squares). As was already apparent from the error rates, polygons are much easier stimuli than dot patterns. What was not clear from the analysis of the error data, however, is that the addition of frames reduced the response times for the polygons but slowed performance considerably for the dot patterns, a result I later discuss further.

Experiment 1. The three main effects were all very reliable: Number of axes, $F(3, 21) = 28.08$, $MS_e = 13,765,398$, $p < .0001$; rotation, $F(4, 28) = 14.42$, $MS_e = 9,017,565$, $p < .0001$; compression, $F(3, 21) = 17.58$, $MS_e = 3,430,870$, $p < .0001$. Rotation interacted significantly with number of axes, $F(12, 84) = 9.45$, $MS_e = 1,115,700$, $p < .0001$, whereas compression did not, $F(9, 63) < 1$. The two-way Rotation $\times$ Compression interaction was not statistically significant, $F(12, 84) = 1.14$, $MS_e = 2,614,313$, $p > .30$, but the three-way Rotation $\times$ Compression $\times$ Number of Axes interaction was reliable, $F(36, 252) = 1.57$, $MS_e = 1,050,547$, $p < .05$. When tested separately for each number of axes, the Rotation $\times$ Compression interaction reached statistical significance for the double symmetries only, $F(12, 84) = 2.17$, $MS_e = 535,204$, $p < .05$; for dot patterns with zero, one, or four axes of symmetry, there was no such effect (all F s between 1.2 and 1.4, all $ps > .20$). In Figure 4A, it is clear that there was not much difference between the rotation effects at different levels of compression for random patterns: Smooth linear functions were obtained in all cases. In contrast, large differences were obtained for twofold symmet-

Figure 5. The effects of affine rotation angle on response times (in milliseconds) for the different experiments. (The two orthogonally manipulated variables are represented by different line types [dotted for dot patterns, full for polygons] and symbols [open for frames, filled for no frames].)

ric patterns (see Figure 4C). For example, at $\Gamma = .766$, there was almost no effect of rotation, whereas the functions were linearly increasing at smaller compression values and invertedly U-shaped at $\Gamma = .5$. Similar invertedly U-shaped functions were obtained for fourfold symmetry (Figure 4D) because of obvious invariances.

Experiment 2. The three main effects were all very reliable: Number of axes, $F(3, 21) = 20.62$, $MS_e = 3,815,650$, $p < .0001$; rotation, $F(4, 28) = 82.19$, $MS_e = 451,175$, $p < .0001$; compression, $F(3, 21) = 28.57$, $MS_e = 257,080$, $p < .0001$. Rotation as well as compression interacted significantly with number of axes, $F(12, 84) = 25.57$, $MS_e = 153,898$, $p < .0001$, and $F(9, 63) = 4.69$, $MS_e = 80,886$, $p < .0001$, respectively. In contrast with Experiment 1 with dot patterns, the two-way Rotation $\times$ Compression interaction was also significant in Experiment 2 with polygons, $F(12, 84) = 4.68$, $MS_e = 263,075$, $p < .0001$. Again, there was a reliable three-way Rotation $\times$ Compression $\times$ Number of Axes interaction, $F(36, 252) = 2.76$, $MS_e = 165,160$, $p < .0001$. When tested separately for each number of axes, the Rotation $\times$ Compression interaction reached statistical significance for the two-fold symmetries, $F(12, 84) = 2.09$, $MS_e = 69,357$, $p < .05$, as well as for the random polygons, $F(12, 84) = 4.70$, $MS_e = 490,085$, $p < .0001$. As in Experiment 1, there was no such effect for patterns with one- or fourfold symmetry (both $F_s < 1$).

Experiment 3. Once again, the three main effects produced statistically significant results: Number of axes, $F(3, 21) = 65.02$, $MS_e = 9,797,098$, $p < .0001$; rotation, $F(4, 28) = 41.59$, $MS_e = 5,469,725$, $p < .0001$; compression, $F(3, 21) = 22.94$, $MS_e = 3,221,759$, $p < .0001$. Rotation as well as compression interacted significantly with number of axes, $F(12, 84) = 31.82$, $MS_e = 886,323$, $p < .0001$, and $F(9, 63) = 4.75$, $MS_e = 522,788$, $p < .0001$, respectively. In contrast with Experiment 1 with unsurrounded dot patterns, but as in Experiment 2 with unsurrounded polygons, the two-way Rotation $\times$ Compression interaction was also significant in Experiment 3 with surrounded dot patterns, $F(12, 84) = 1.92$, $MS_e = 1,447,107$, $p < .05$. Again, there was a reliable three-way Rotation $\times$ Compression $\times$ Number of Axes interaction, $F(36, 252) = 1.54$, $MS_e = 733,083$, $p < .05$. As in the previous experiments, the Rotation $\times$ Compression interaction produced statistically significant results for the double symmetries when it was tested separately for each number of axes, $F(12, 84) = 3.39$, $MS_e = 305,204$, $p < .0005$. In this case, the two-way interaction between the affine parameters yielded marginally significant results for the dot patterns with a single axis of symmetry as well, $F(12, 84) = 1.73$, $MS_e = 728,349$, $p < .08$; there was no such effect for random and fourfold symmetric dot patterns surrounded by a frame, $F(12, 84) = 1.50$, $MS_e = 2,472,738$, $p > .10$, and $F(12, 84) = 1.07$, $MS_e = 140,065$, $p > .35$, respectively.

Experiment 4. As in all other experiments, all main effects were very reliable: Number of axes, $F(3, 21) = 28.18$, $MS_e = 2,017,884$, $p < .0001$; rotation, $F(4, 28) = 27.24$, $MS_e = 1,353,432$, $p < .0001$; compression, $F(3, 21) = 21.04$, $MS_e = 473,516$, $p < .0001$. Rotation interacted

significantly with number of axes, $F(12, 84) = 11.44$, $MS_e = 481,261$, $p < .0001$, but compression did not, $F(9, 63) < 1$. The overall two-way Rotation $\times$ Compression interaction produced statistically significant results, $F(12, 84) = 2.86$, $MS_e = 225,560$, $p < .005$, but there was no reliable three-way Rotation $\times$ Compression $\times$ Number of Axes interaction, $F(36, 252) = 1.12$, $MS_e = 321,046$, $p > .30$. When tested separately for each number of axes, the Rotation $\times$ Compression interaction reached statistical significance only for the random polygons surrounded by a frame, $F(12, 84) = 2.31$, $MS_e = 341,648$, $p < .05$; for polygons with one, two, or four axes of symmetry, there was no such effect (F_s between 1.0 and 1.5, all $p_s > .15$).

Results for Criteria 4 and 5

Criterion 4

I tested the fourth criterion specified, namely whether the Rotation $\times$ Compression interaction is caused by subadditivity, across all four experiments. For each of the 8 subjects in each of the four experiments, I computed the effect of pure compression on the basis of the results for zero rotation (i.e., $\Phi = 0^\circ$), and I computed the effect of pure rotation on the basis of the results for zero compression (i.e., $\Gamma = 1$). For each of the 12 other rotation-compression combinations, I then compared the actually obtained value with the expected value if both affine parameters, rotation and compression, would produce additive effects. Finally, I performed an analysis of variance on the numbers of subadditive values (varying between 0 and 12), which constitute only a very crude measure of the degree of subadditivity, to compare them across the different kinds of patterns. For the practiced subjects, who participated in all four experiments, the numbers were analyzed in a within-subjects design with three variables: frame, pattern, and number of axes. For the naive subjects, who participated in only a single experiment, numbers were analyzed in a mixed design with frame and pattern as between-groups variables and number of axes as a within-subjects variable.

Practiced subjects. The three main effects did not produce significant results: Frame and pattern, $F(1, 3) < 1$; number of axes, $F(3, 9) = 1.72$, $MS_e = 5.57$, $p > .20$. Of all two-way interactions, only that of pattern and number of axes yielded a reliable effect, $F(3, 9) = 5.60$, $MS_e = 6.31$, $p < .05$. This interaction occurs because the highest degree of subadditivity was observed with double symmetry for dot patterns and with random patterns for polygons (see the upper half of Table 2). There was also a marginally significant higher order Pattern $\times$ Number of Axes $\times$ Frame interaction, $F(3, 9) = 2.91$, $MS_e = 4.10$, $p < .10$. This trend is caused by the following differences (see the upper half of Table 2 again): When not enclosed by a four-sided frame, dot patterns produced more subadditive effects of rotation and compression if two axes of symmetry were present, whereas subadditivity was more frequent for random and singly symmetric polygons; when surrounded by a frame, these tendencies shifted to a lower number of axes in that both single and double symmetries yielded

Table 2
Mean Numbers of Subadditive Values for the Response Times Across Experiments

Number of axes	Frame present		Frame absent	
	Dot pattern	Polygon	Dot pattern	Polygon
Practiced subjects				
0	8.00	10.00	7.50	8.50
1	5.75	9.25	9.25	6.25
2	9.75	5.50	10.25	5.75
4	6.25	7.75	4.75	7.75
Naive subjects				
0	8.50	8.75	6.75	10.00
1	7.00	6.75	5.50	6.75
2	9.25	3.75	8.00	6.25
4	6.75	5.75	5.50	7.00

Note. The maximum number is 12. For the practiced subjects data, the same 4 subjects participated in the four experiments. For the naive subjects data, 4 new subjects participated in each experiment.

more subadditivity for the dot patterns, and subadditivity occurred more often for random polygons only.

Naive subjects. Frame and pattern did not produce significant results, $F(1, 12) < 1$, whereas number of axes reached marginal statistical significance, $F(3, 36) = 2.27$, $MS_e = 7.24$, $p < .10$. Of all two-way interactions, only that of pattern and number of axes yielded a reliable effect, $F(3, 36) = 2.98$, $MS_e = 7.24$, $p < .05$. As for the trained subjects, this interaction occurred because the highest degree of subadditivity was observed with double symmetry for dot patterns and with random patterns for polygons (see lower half of Table 2). There was no reliable three-way interaction, $F(3, 36) < 1$, although the differences were very much like those for the trained subjects.

Criterion 5

As suggested earlier, a final way to find out if skewed symmetry was used to perceive visual forms in our affine matching task is to test whether the subadditivity observed in some of the conditions was due to leaving the image plane and performing the matching by removing the affine transformation along the shortest path (i.e., different axes oriented in depth for the rotation-compression combinations). I performed linear regressions for compression at all rotation angles and all numbers of axes. The results were disappointing: In general, these analyses produced few significant results, and for those that were reliable, the explained variance (R^2) was mostly below 10%. Only for the pure compressions ($\Phi = 0^\circ$) were the effects somewhat more linear. As a quantitative example, consider the results for Experiment 1. For the random dot patterns and those with one or two axes of symmetry, the linear regressions for compression explained more than 5% of all the variance produced by compression in only the three cases in which there was no rotation in the image plane (i.e., z-axis equals line of sight): $F(1, 670) = 64.57$, $p < .0001$, $R^2 = .09$; $F(1, 382) = 21.47$, $p < .0001$, $R^2 = .05$; $F(1, 190) = 15.89$, $p < .0001$, $R^2 = .08$, respectively. Because of invariances

for the fourfold symmetric dot patterns, linear regressions were statistically reliable for both 0° and 90° rotations, $F(1, 94) = 12.47$, $p < .001$, $R^2 = .12$, and $F(1, 94) = 13.13$, $p < .0005$, $R^2 = .12$, respectively. In short, other depth axes did not yield linear effects on response times needed to match the affine equivalent dot patterns. These nonlinearities occurred in all other experiments as well.³

A somewhat related analysis consists of comparing the explanatory power of two different trends produced by compression: One is a linear effect with unequal intervals of the compression variable (i.e., 1, .9397, .766, .5), and the other assumes that the compression variable was manipulated at equal intervals (which is true for the slant in depth only, i.e., 0° , 20° , 40° , and 60° ; see the General Method section). Gaito (1965) described a method for deriving orthogonal coefficients for the general case of unequal numbers of observations and unequal intervals between points of a quantitative independent variable. As before, these effects were tested separately for all rotation angles and all number of axes. As with the other test proposed here, the results were disappointing; in general, the linear effects of compression in the plane were larger than the linear effects of slant in depth. For example, in Experiment 1 with unsurrounded dot patterns, only 4 of 20 possible cases yielded the reverse pattern of results. Similar trends were observed in the other three experiments.

General Discussion

Summary of the Results

Criterion 1: Main effect of symmetry. The least stringent criterion that has to be met to provide evidence for the perceptual use of skewed symmetry is that symmetry should help in a task in which affine transformed patterns have to be compared to judge whether they are affine equivalent (i.e., bilateral symmetry is transformed to skewed symmetry in most cases). This was clearly the case. No matter if tested with dot patterns or polygons, surrounded by a frame or not; it was always easier to match a pattern with affine transformed versions of it when the pattern contained symmetry. Less time was needed and fewer errors were made.

Criterion 2: Interaction with symmetry of rotation and compression. In line with some recent data showing that mental transformations such as rotations are easier for simple shapes (e.g., integrated wholes) and when they are performed around natural rather than arbitrary axes, it was expected that the effects of rotation and compression (i.e., the

³ At first, these nonlinearities might have something to do with the procedure of repeating the trials until they are responded to correctly. In some cases of very difficult same trials (i.e., in which the affine equivalence is hard to notice), this might yield short response times when the trial is repeated shortly after a wrong response (which might occur at the end of a block). This cannot be the whole story, however, because analyses with the response times from the trials that were responded to correctly from the first occurrence already yielded similar patterns of results.

two parameters defining our affine transformation) would be smaller for symmetric patterns rather than random ones, at least if skewed symmetry would be detected and used. In general, this proved to be the case. The differences caused by increasing rotation angles decreased with increasing number of symmetry axes, regardless of the type of pattern (dot patterns or polygons, with or without frames). The differences caused by increasing compression factors decreased with increasing number of symmetry axes, at least for the dot patterns with frames and the polygons without frames (which appear to be the most difficult and easiest stimuli, respectively, see Figure 5).⁴

Criterion 3: Interaction between rotation and compression. If skewed symmetry is interpreted as bilateral symmetry in depth, there should be different effects of rotation at different levels of compression (and vice versa) because some of these entail pure rotation in the image plane (i.e., no compression and vice versa), whereas others imply some recovery of depth, which should be easier for symmetric patterns as compared with random ones. The overall Rotation $\times$ Compression interaction was reliable in three of the four experiments (i.e., it was not in Experiment 1 with unframed dot patterns). When tested separately for the different numbers of axes, some important differences arose among the four experiments. When the dot patterns were not surrounded by a frame, rotation interacted with compression for patterns with double symmetry only. On the other hand, the interaction became more reliable for single symmetry as well when the dot patterns were enclosed in a frame and even for random patterns when the dots were connected to form closed polygons. Important perceptual differences between isolated elements or closed contours have been observed many times before (e.g., Todd, 1981, 1982, 1985).

Criterion 4: Subadditivity. Although an interaction between rotation and compression seems a necessary condition for skewed symmetry to be used as a nonaccidental property, it is not sufficient. One alternative cause for such an interaction might be that one of both processes corresponding with both parameters of the affine transformation would take considerably more time (see Kubovy & Podgorny, 1981, for a similar argument with respect to normalization operations for size and orientation). If such an interaction were to result from a real recovery of depth, this would imply that rotation and compression would combine subadditively. The single transformations in depth that are expected for symmetric but not random patterns should require less time than the separate rotations and compressions in the image plane. For this much more stringent criterion, the results were less clear cut. As before, there were some significant differences among the four experiments. For unframed dot patterns, the largest degree of subadditivity was obtained with double symmetry. On the other hand, more subadditive values were observed for single symmetry when the dot patterns were surrounded by a frame, or even for random patterns when they were constituted as closed polygons. It was not the case, however, that there was never subadditiv-

ity for random patterns and always subadditivity for symmetric patterns.

Criterion 5: Linear regressions. Although subadditivity seems a necessary condition when skewed symmetry would be used to recover bilateral symmetry in depth, it is once more not sufficient. Perhaps subadditivity as tested in our analysis is an accidental result of the particular values derived from the separate cases of pure rotation and pure compression (cf. the aforementioned procedure).⁵ Therefore, the most stringent criterion is to require that the response times obtained with the 32 parameterizations of the affine transformation can be plotted as a function of the degree of slant for eight possible z-axes. Here, the results were simply disappointing. One analysis showed that the linear regressions did not produce statistically significant coefficients or explained only a minor fraction of the variance. Another analysis indicated that the effects of the affine transformation corresponded better with a rotation-compression combination in the image plane than with a slant-tilt change in depth.

Mixed Evidence for Skewed Symmetry as a Nonaccidental Property Used to Perceive Visual Forms

The tests proposed to investigate whether skewed symmetry functions as a nonaccidental property in human perception yield mixed evidence. The pattern of results as summarized before suggests that the evidence becomes weaker when the criteria become more stringent. A clear picture emerges when referring to the distinction made between two fundamentally different ways in which a regularity or structure in the image (such as skewed symmetry) might be useful in projective equivalence or recovery problems. First, the property might facilitate performance in a task in which shape equivalence under affine (or more generally, projective) transformations has to be judged simply

⁴ In the other two experiments, this interaction between compression and number of symmetry axes was probably masked by the pronounced nonlinear effects of compression (e.g., almost no difference between 1 and .9397; see also Busey, Brady, and Cutting, 1990, and Cutting, 1987, about indiscriminable distortions). When tested with linear regressions, the intercepts and slopes of the compression-response time functions systematically decreased with increasing number of symmetry axes. For example, in Experiment 1 (dot patterns without frame) the intercepts averaged across the 8 individual subjects (expressed in milliseconds) were 1,184, 964, 724, and 559, for zero, one, two, and four axes, respectively, whereas the corresponding slopes were 162, 115, 98, and 116. Similar results were obtained in the other experiments as well as for the rotation-response time functions.

⁵ Indeed, for a lot of subjects the effects of pure compression and pure rotation yielded quite unexpected differences, most of the time not linear and sometimes even negative (e.g., $\Gamma = .9397$ faster response times than $\Gamma = 1$). As a consequence, subadditive or superadditive results could be obtained on the basis of nonlinear estimates. This cannot be the whole story, however, because analyses with the slopes from linear regressions instead of really observed response times yielded quite similar patterns of results.

because it offers some anchors (e.g., skew axis) to rely on in treating the transformation within the image plane. Second, the property in the two-dimensional image might be interpreted as arising from a similar structure in the three-dimensional world. Only this would indicate real recovery, which is always implied in computational treatments of nonaccidental properties. In addition, for some nonaccidental properties (such as skewed symmetry), the parameters defining it (i.e., the skew angle) might be used to derive (constraints on) the underlying three-dimensional surface orientation. I now discuss the results in relation to these distinctions.

Symmetry advantage. Large and systematic facilitatory effects of skewed symmetry in affine-matching tasks such as the one used in the experiments presented here seem to corroborate with the classically observed advantages of perfect bilateral symmetry in simpler same-different comparisons (e.g., Fox, 1975). Likewise, the parameters determining the affine transformation that had to be disregarded by the subjects—rotation angle and compression—caused more difficulties for the random patterns than for the symmetric ones. This appears to be conform with the previously suggested role of bilateral symmetry in mental rotation tasks (e.g., Carlton & R. Shepard, 1990a; Jolicoeur, 1990).

Skewed symmetry as bilateral symmetry in depth. Although it is tempting to attribute this advantage of skewed symmetry to its being perceived as bilateral symmetry in depth, this is not necessarily so. Indeed, the tests that were more directly focused on this recovery aspect yielded positive results in only a restricted number of cases. In Experiment 1, with unsurrounded dot patterns, the data concerning the Rotation $\times$ Compression interaction and its underlying subadditivity indicated that skewed symmetry was perceived as bilateral symmetry on a planar surface in space for double symmetries only. When the planarity of the patterns was increased by connecting them with straight lines to form closed contours (as in Experiment 2) or by enclosing them in a frame that was itself affine transformed (as in Experiment 3), a three-dimensional interpretation occurred for single symmetries and sometimes even for random polygons. Perhaps the increase of the response times caused by the addition of a frame (see Figure 5) might be interpreted as reflecting the additional time used for recovery, compared with the condition in which the task is solved within the image plane for most patterns (Experiment 1).

This pattern of quantitative data appears to confirm the observers' descriptions of how they performed the experimental tasks. When dealing with isolated dot patterns, subjects had the impression that they often relied on local cues offered by specific groupings such as densely positioned dots (clusters) or salient curvilinearities (examples are given in Figure 6). Because this kind of qualitative perceptual groups are more or less invariant under affine transformations, it is a reliable source of information to rely on in performing this task, without having to deproject the image to its original three-dimensional orientation. Such a role of two-dimensional image regularities on the basis

Figure 6. Examples of random dot pattern pairs with accidental groupings that can be used to perform the affine matching. (Panel A shows a dense cluster in the lower right corner of the original, shifted to middle right in the transformed version. Panel B shows seven collinear dots that form almost a straight vertical line in the right half of the original dot pattern. Panel C shows the curvilinearity of seven dots, an L-shaped group of four dots, and a more or less isolated couple, all of which can be retraced in the transformed version as well.)

of grouping has been noted before (e.g., with respect to structure from motion, Braunstein, Hoffman, Shapiro, Andersen, & Bennett, 1987; Todd, Akerstrom, Reichel, & Hayes, 1988). With particular rotation-compression combinations only, subjects reported perceiving the affine-transformed versions of highly salient types of symmetry (e.g., two- or fourfold symmetry) as being oriented in depth (see Figure 7).

Shape from skewed symmetry. As indicated previously, the parameters of skewed symmetry in the two-dimensional image (i.e., skew angle) could be used to infer (constraints on) the three-dimensional orientation parameters (i.e., slant and tilt). In the computer vision literature, such a recovery problem is formulated as a shape-from problem. In Marr's (1982) view, these and other aspects of depth (e.g., distance to the viewer) could then be represented in a metric surface representation, called a $2\frac{1}{2}$ -dimensional sketch. Doubts about the possibility and usefulness of such an accurate depth map and the so-called modules to compute it have been expressed before, both in computer vision (e.g., Barnard, 1985; Lowe, 1987) and in perceptual psychology (e.g., Todd & Akerstrom, 1987; Wagemans, 1988, 1990). The data gathered here suggest that similar doubts are warranted for this specific case of skewed symmetry as well. In contrast, Stevens (1980, 1983) provided some evidence about the usefulness of

Figure 7. Examples of dot pattern pairs with a salient global structure. (Panel A, vertical-horizonal-left-right; Panels B and C, vertical-horizonal. The affine-transformed versions [on the right] are more easily perceived as being perfectly symmetric in depth.)

skewed symmetry in this more demanding aspect of surface-orientation recovery. The simplicity of his experimental stimuli (i.e., ellipses and crosses), however, might have been a determining factor in this more positive result.

As a follow up to the experiments reported here, an additional set of four experiments explicitly tested the possibilities of human perceivers to derive the three-dimensional orientation of planar surfaces with skewed symmetry. In one task, dot patterns or polygons were surrounded by a frame, the suggested three-dimensional orientation of which was in agreement with the one suggested by the affine-transformed one-, two-, or fourfold symmetry in only half of the cases (i.e., an adaptation of the paradigm used by Busey, Brady, & Cutting, 1990). In another task, the perceivers had to adjust the suggested orientation of the frame by pressing arrow keys (left and right for rotation; up and down for compression) to make it correspond to the one inferred from skewed symmetry. As in the experiments presented here, the evidence was mixed, suggesting perceptual use of local regularities (e.g., proximity or parallelism between pattern and frame borders) in some cases and more global factors (e.g., two- or fourfold symmetry) in others. These results are given in more detail elsewhere (Wagemans, 1992).

Conclusion. The results obtained in this set of experiments show that skewed symmetry is clearly helpful in a task in which subjects have to determine whether two images result from the same object viewed from a different angle. This advantage, however, seems to be based on operations that remain in the image plane in most cases. So, with our stimulus patterns and presentation conditions, hu-

man perceivers do not appear to use skewed symmetry to recover bilateral symmetry in depth. The observers' impressions as well as specific quantitative results from stringent tests support this conclusion. Only when the gestalt character of the patterns is increased by connecting dots with lines or by introducing double symmetry could the global regularity be picked up with sufficient speed and efficiency to allow an almost automatic three-dimensional interpretation. In other cases, human perceivers seem to rely on qualitatively invariant groupings.

Qualitative Invariants

In the previous discussion, I suggested that some properties that are more or less invariant under affine transformations might be used to perform the experimental task. Of course, a quantitative measure is either unchanged (i.e., an invariant) or changed after a transformation. It seems, therefore, ridiculous to denote some property as more or less invariant. A qualitative property, however, can be preserved even when the exact metric values underlying it are transformed. For example, although the degree of curvature is changed by affine transformations, the sign of curvature (concave or convex) is not. Concavities and convexities are, therefore, qualitative affine invariants. Likewise, interelement distances are not identical when viewed from different angles, but relative densities remain quite comparable, so grouping processes will often yield the same salient structures (e.g., clusters and curvilinearities). In affine-matching tasks, these structures can, therefore, be used as reliable landmark features (see also Hochberg & Gellman, 1977; Larsen, 1985). In the case of dot patterns with one-, two-, or fourfold symmetry, these randomly generated structures get multiplied with increased saliency as a result. The same occurs for some polygons that contain obvious peculiarities such as deep concavities, zigzag structures, or large uninterrupted parts.

As a follow-up to the experiments reported here, an additional experiment explicitly manipulated the presence and saliency of these qualitative affine invariant structures. Five categories of dot patterns were distinguished that contained (a) salient groups constituting some regular form (e.g., triangle, ellipse), (b) curvilinearities, (c) isolated groups of one or two dots, (d) a collection of multiple structures, and (e) a control category in which no special structures emerged. The results of a similar affine-matching task as the one used here showed that Categories (a) and (b) provided very helpful regularities, whereas Category (c) seemed to interfere with rather than facilitate the matching process. When multiple regularities were present, only one of them appeared to be used as an anchor point. In general, the effects of these local regularities were larger for random than for symmetric patterns, which seems to suggest that the perceptual system only starts looking for local structures in the absence of a strong global structure (i.e., symmetry). More details about this study will be given in subsequent work.

Although more about the issue of qualitative affine invariants can be found elsewhere (Wagemans, 1992), I want to conclude with three notes. First, the big difference be-

tween the use of qualitative affine invariants on the one hand and skewed symmetry as a nonaccidental property and shape from skewed symmetry on the other is that the latter require some (implicit or explicit) three-dimensional recovery, whereas the former allows the observer performing the matching task to remain in the two-dimensional image plane. A recent exchange about structure from motion, also known as the *kinetic depth effect* (KDE), clarifies this (Braunstein & Todd, 1990; Sperling, Doshier, & Landy, 1990; Sperling, Landy, Doshier, & Perkins, 1989). These researchers distinguish between KDE, KDE-alternative, and artifactual computations. In contrast to real KDE, which requires the computation of three-dimensional depth values, the KDE-alternative computation derives the response directly from the two-dimensional optic flow, without an intermediate stage of perceived three-dimensional depth, whereas an artifactual computation uses incidental stimulus or motion cues from only a small part of the stimulus. In light of the richness of normal visual information and the relativity of the artifact-information distinction (it depends on what one is interested in), it remains unclear whether these qualitative perceptual structures deserve to be called artifactual. I think the qualitative perceptual structures provide useful visual information because they allow subjects to perform the task without having to recover depth, which proves slow and cumbersome in dot patterns without high-order symmetries (see Wagemans, 1992).

A second remark has to do with a similar trend in recent computer vision work (although foreshadowed by an important study by Witkin & Tenenbaum, 1983). With respect to recovery, it has been shown that partial qualitative shape information can be obtained from stereo images without going through a detailed disparity map (Edelman, 1990). Likewise, Weinshall (1990) demonstrated that the sign of the Gaussian curvature (but not the Gaussian curvature itself) and the qualitative direction of motion (but not the exact value) can be computed directly from the local motion disparity field. This corresponds nicely with recent theoretical and psychophysical work about more qualitative, non-metric structural shape descriptors, such as nominal (e.g., Koenderink, 1984; Koenderink & van Doorn, 1982) or ordinal structure (e.g., Todd & Reichel, 1989, 1990).

A final note is needed about the generality of the conclusions drawn from this set of experiments. On the one hand, the somewhat negative conclusion with respect to the usefulness of skewed symmetry for recovery might be qualified by research with less fragmented and arbitrary stimuli in less demanding situations. For example, an extension of the apparent motion paradigm used in similar contexts before (e.g., Bundesen et al., 1983; Corballis & Blackman, 1990; Farrell, 1983; Farrell, Larsen, & Bundesen, 1982; Farrell & R. Shepard, 1981) toward apparent motion between affine-equivalent closed contours (with or without skewed symmetry) seems a hopeful alternative; and it is currently being explored in our laboratory. On the other hand, skewed symmetry is used here as a paradigmatic case for all nonaccidental properties because other experimental work about the perceptual use of nonaccidental properties has simply been lacking. Nevertheless, the observations with respect to the

use of qualitative invariant structures such as salient clusters, curvilinearities, concavities, parallelisms, and proximities suggest that there might be important differences depending on the particular sources of available information. Thus, much more data are needed as to the perceptual status of different mathematical nonaccidental properties.

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