Why do we still put up with WACC?

Piet Sercu

DEPARTMENT OF ACCOUNTANCY, FINANCE AND INSURANCE (AFI)

Why do we Still Put Up with WACC?

Piet Sercu*

May 4, 2007

*KU Leuven, Leuven School of Business and Economics, Naamsestraat 69, B-3000 Leuven; piet.sercu@econ.kuleuven.be, phone +32 16 326 756. I thank Bernard Dumas for (inadvertently) steering me to this topic, and for Nancy Huyghebaert for pertinent and constructive criticisms on earlier drafts, but I remain responsible for any remaining shortcomings.

Abstract

WACC, being designed for situations with perpetual constant leverage, overestimates the NPV when the project's life is finite. The reason is that it sets the weight of the PV'ed tax shields at too high a level initially and ignores the fact that this weight should fall even further as the project matures. As a result, initial risk is underestimated, and subsequent further rises in risk are overlooked. For realistic projects of 5-7 years the overstatement of the PV of the tax shield typically exceeds 50%. One needs 10-year horizons for the error to drop to 40%, and 20 years for a drop to 25%. In contrast, ANPV can correctly handle not only any deterministic cash flow pattern or capital structure, but, after a minor tweak in the programming, also borrowing strategies where the loan amount changes proportionally with gross present value—the situation supposedly best solved with WACC: one just needs to apply NPV() twice.

JEL classification: G31, G32.

Key words: Capital structure, capital budgeting, NPV, WACC, tax effects, TGIF

Why do we Still Put Up with WACC?

Introduction

In a standard Modigliani-Miller (1963) setting, corporate borrowing adds value because the corporate tax bill goes down, so that the total amount of cash that can be distributed to financiers goes up. Familiar issues are that (i) part of the tax savings may be postponed or even lost whenever EBIT are insufficient; (ii) the approach ignores possible differences in tax treatment at the personal level and (iii) part or all of the subsidy may in fact be captured by the bondholders and banks rather than by the shareholders (Miller, 1977); and (iv) borrowing has non-tax effects too, most notably financial-distress costs and repercussions on behavior and decision-making. Subject to these caveats, one can implement the tax-benefit idea in two ways. Either one first values the project assuming full equity financing, and one then adds a second stage, in which all tax savings are PV'ed and added to the first-pass NPV. This is called the Adjusted NPV (ANPV) approach. Alternatively, one builds the effect into the discount rate: rather than using the all-equity cost of capital, one calculates a discount rate that is somewhat lower, so that the tax benefit is captured. Miller and Modigliani (1963) derive the required discount rate for the case of perpetual cash flows and perpetual debt. The resulting formula can be re-interpreted as a weighted average of the expected return on equity and the after-tax cost of debt, and this parametrisation is called Weighted Average Cost of Capital (WACC) (Miller and Modigliani, 1961, 1963).

In this note we comment two standard claims about the choice between WACC and ANPV. Below we list the common views and our additions:

1. “WACC/MM strictly requires perpetuities but works well enough when cash flows and debt are not perpetuities, provided that the D/E ratio is constant and the expected return on equity is measured correctly”, (see *e.g.* Myers, 1974; Brealey and Myers, 1981¹).

Myers’ numerical experiments express the error as a fraction of initial investment, but this number depends not just on the cash flow pattern, as he shows, but also on the size

¹The issue of WACC requiring perpetuities, even with known constant D/E, is not even brought up in the more recent editions of Brealey and Myers in my bookcase. Also Ross, Westerfield and Jaffe (1999) are silent about bias in finite-life cases; they just stress the constant-D/E assumption.

of the stage-1 NPV. Also, many of the cases where WACC's error is small are also cases where the total tax advantage is small; so by that criterion we should discard the entire tax subsidy as a second-order refinement—a step that most would be hesitant to make.

Instead of investment, I use the true tax benefit as our scaler. This percentage error is relevant by implication: judging by the amount of space textbooks allocate to it, the tax effect is deemed to be an important issue for a model project; so if the error is a large fraction of the true tax effect, this must be important too.

We find that WACC systematically and nontrivially overestimates the tax effect, typically by more than half and, in fact, up to by several multiples.

2. “WACC is preferred when the D/E ratio is fixed exogenously, while ANPV is the obvious choice when the amount of debt financing is known upfront and D/E varies substantially.” — see *e.g.* Ross, Westerfield and Jaffe (1999).

This argument is one of convenience: both the WACC or MM formulas indeed presuppose that the leverage ratio is a known constant, in market value terms not book values, while ANPV most naturally works with a pre-determined debt schedule. But convenience may lose its appeal if the approach produces serious errors even when D/E is a known constant. Implementing ANPV when D/E is a known constant, rather than the debt schedule itself, is quite easy and just takes two lines and two NPV(...) statements in a spreadsheet.^{2 3}

The source of bias and its correction are presented in Section 1, with programming hints provided in Appendix A. Section 2 assesses the size and determinants of the bias. Section 3 concludes.

²There also is a simple way to use WACC when D is fixed upfront: iterate, starting from zero NPV and, therefore, a book-value-based D/E ratio. In round 2, use the first NPV to update D/E and WACC, and re-estimated NPV, and so on. This converges rapidly to the WACC solution, but as WACC only works well for perpetuities, this way of finding the market D/E and WACC for a given D is of limited use: it would be even simpler to apply ANPV and just add τD to the first-pass NPV, i.e. divide it by $(1 - \tau D/(D + E))$.

³Ross, Westerfield and Jaffe (1999) advance other arguments. They opine that managers *should* borrow more when market values rise (their italics); they also state that banks think that way, citing the example of real-estate projects. But the example does not make clear whether banks lend on the basis of liquidation value rather than projected going-concern values for modal scenarios. (For liquidation value, book value may be more appropriate; and a known book value would then imply a known D and hence make ANPV the obvious choice.) In the case of real-estate projects, the difference between the going-concern value and the liquidation value is probably lower than for most industrial projects, so this example does not really settle the discussion. The fact that most analysts use WACC not NPV is inconclusive too: most analysts may be unaware of the assumptions and alternatives.

1 Why WACC is Biased, even under Constant Leverage, if Project Lives are Finite

While textbooks do stress WACC's assumption of a pre-set and constant leverage ratio, the assumption of perpetuities seems to get less attention nowadays. Let us see whether it matters. We start with a one-period problem, chosen not for its representativeness but because it reveals the key points.

Let there be a cash flow of 110 next year, and let the unlevered discount rate be 10 percent. We immediately get a first-pass ("unlevered") GPV, GPV_0^u , of 100. Let us borrow 50 at 6%. With a tax rate of 30% this generates a tax saving of $50 \times 0.06 \times 0.3 = 0.9$, whose PV is $0.9/1.06=0.849$. We conclude that the Adjusted GPV equals $100 + 0.849 = 100.849$. The implied average cost of capital is $110/100.849 - 1 = 9.074\%$, and the implied cost of equity is $(110 - 50 \times 0.06 \times 0.7 - 50)/(100.849 - 50) = 13.9\%$. Let's see what the perpetuities model would have suggested:⁴

$$D/E = 50/(100.849 - 50) = 0.9833; \quad (1)$$

$$E(\tilde{R}_{EQ}) = R_f + E(\tilde{R}_u - R_f) \left(1 + (1 - \tau) \frac{D}{E} \right), \quad (2)$$

$$= 0.06 + (0.10 - 0.06) \times (1 + (1 - 0.3) \times 0.98330) = 0.12753; \quad (3)$$

$$D/GPV = 50/100.849 = 0.49579; \quad (4)$$

$$WACC = (1 - 0.49579) \times 0.12753 + 0.49579 \times (1 - 0.3) \times 0.06 = 0.0851263; \quad (5)$$

$$GPV = 100/1.0851263 = 101.37. \quad (6)$$

In the above example—and, in fact, in all realistic cases—WACC underestimates the true total required rate of return. At the root of all this is an overestimation of the sustainable debt. As a result, as we shall see, the assumed tax advantage starts at too high a level and is assumed to remain at that level, while it should start lower and then fall even further as the project's end draws nearer. Thus, the PV'ed tax advantage is overvalued. This implies that the weight of the tax advantage in the total value is overstated too. Thus, total risk is underestimated, and so is the Cost of Capital.

In the one-year example, the true tax-induced drop in the required return is about 1 percent

⁴I compute the expected return on equity from the zero-leverage cost of capital—note that the familiar leverage formula, Equation (2), assumes perpetuities—and then the WACC. It would have been faster to use the MM formula, $E(\tilde{R}_u)(1 - \tau \frac{D}{GPV_L})$.

Table 1: **One Period v Perpetuities: Impact on Weight of tax Benefits**

	one period		perpetual	
	present value	value weight	present value	value weight
X_u	$\frac{110}{1.10} = 100$	$\frac{100}{100.849} = 99.16\%$	$\frac{110}{0.10} = 1100$	$\frac{1100}{1265} = 86.9\%$
$D \cdot R_f \tau$	$\frac{100 \times 0.5 \times 0.06 \times 0.3}{1.06} = 0.849$	$\frac{0.849}{100.849} = 0.84\%$	$\frac{1100 \times 0.5 \times 0.06 \times 0.3}{0.06} = 165$	$\frac{165}{1265} = 13.1\%$
total	100.849	100%	1265	100%

(from 0.10 to 0.09074), but WACC optimistically proposes a drop by about 1.5%. It is easy to see where the bias comes from. The GPV can be regarded as resulting from two streams, each with assumedly a constant risk: the unlevered cash flows, and the tax shields. For finite-life projects, WACC overestimates the weight of the latter. If about half of the unlevered GPV is borrowed, for instance, then tax shields have a weight of just 0.84% in our one-year example; but WACC would credit them with a weight of over 13%, as Table 1 shows.⁵ In Table 1, 99.16% of value is risky, but WACC would estimate this at less than 87%. Thus, the cost of capital is based on underestimated risk.

In a multi-year example, a second mistake shows up: the true weight of the tax gains not only starts from a lower level than what perpetuities would suggest, but it also falls as the project matures. If debt shrinks in line with GPV, the implication indeed is that the PV'ed interest falls even faster. Table 2 shows how the present value of the tax shield, which starts at 3.8%—not WACC's 15%—falls to 0.7% at the end. The shrinking weight of the risk-free tax shields implies that the discount rate should have been rising for more distant years. In short, by assigning too much weight to the low-risk part initially, and doing even more so for more distant years, the required return is systematically underestimated by WACC.

The standard intuitive claim that a constant leverage ratio leads to WACC is flawed for the same reasons. If the required return on equity is calculated from $E(\tilde{R}_u)$ using the standard formula (2), the risk of equity is underestimated from the onset. In addition, the true risk of

⁵In fact, the table underestimates the effect as it takes debt to be half of the unlevered GPV, not half of the Adjusted GPV, which is proportionally much higher in the perpetuity case. The correct calculations yield a fraction of (with δ denoting the D/GPV ratio)

$$\frac{GPV - GPV^u}{GPV} = 1 - \frac{WACC}{E(\tilde{R}_u)} = 1 - (1 - \delta\tau) = 0.49579 \times 0.3 = 0.149.$$

Table 2: **Valuing a Constant Cash Flow: Perpetual v 10 Year**

t	X_t	GPV_t^u	GPV_t	D_t	I_t	τI_t	$\tau PV_t(I)$	$\frac{\tau PV_t(I)}{GPV_t}$	E_t	$\frac{PV_t(X)}{E_t}$
WACC-based calculations										
—	200	2000.0	2352.9	1176.5	58.8	17.6	352.9	15.0%	1176.5	170.0%
Correct calculations										
0	—	1228.9	1277.6	638.8	—	—	48.7	3.8%	638.8	192.4%
1	200	1151.8	1193.3	596.7	31.9	9.6	41.5	3.5%	596.7	193.0%
2	200	1067.0	1101.7	550.8	29.8	9.0	34.7	3.1%	550.8	193.7%
3	200	973.7	1001.8	500.9	27.5	8.3	28.1	2.8%	500.9	194.4%
4	200	871.1	893.1	446.5	25.0	7.5	22.0	2.5%	446.5	195.1%
5	200	758.2	774.6	387.3	22.3	6.7	16.4	2.1%	387.3	195.8%
6	200	634.0	645.4	322.7	19.4	5.8	11.4	1.8%	322.7	196.5%
7	200	497.4	504.5	252.3	16.1	4.8	7.2	1.4%	252.3	197.2%
8	200	347.1	350.9	175.4	12.6	3.8	3.8	1.1%	175.4	197.9%
9	200	181.8	183.1	91.6	8.8	2.6	1.3	0.7%	91.6	198.6%
10	200	—	—	—	4.6	1.4	—	—	—	—

Key The calculations are based on the following: unlevered equity's required return $E(\tilde{R}_u) = 0.10$; target D/GPV ratio 0.50; tax rate $\tau=0.30$, $R_f=0.05$. D , I , E stand for Debt, Interest payments, and Equity. Under WACC I compute GPV as $GPV^u/(1 - \tau\delta)$ with δ denoting the target D/GPV ratio, 0.5; the rest follows immediately. For the 10-year version I follow Appendix A.

equity rises all the time. This can be seen by considering the following expression for equity:

$$E = PV(X_u) + \tau PV(I) - D. \quad (7)$$

The standard assumption is that the beta risk of $PV(X_u)$ is constant. D , a risk-free item, falls proportionally with $PV(X_u) + \tau PV(I)$ so it has a constant weight in E ; but the beta risk of $PV(X_u) + \tau PV(I)$ itself is rising because the weight of $\tau PV(I)$ falls. Thus, equity becomes riskier, and the WACC should have been rising as the project matures.⁶ In Table 2, the true initial risk of equity should reflect the fact that $PV_0(X)$ is 192% of equity, not 170%; in addition, this ratio subsequently rises to 199%. At the root of this again is the fact that interest on the true borrowing capacity starts from a lower initial level, and rapidly falls, implying an even faster drop of the tax gains in PV terms. But, having in mind a world where the value of debt is constant and identical to the PV of interest payments, WACC ignores this. To have constant-risk GPVs and equity one would have to keep not D but $PV(I)$ proportional to GPV , which most would regard as a bizarre financing policy.

⁶The typical reply that there are other cash flows from other projects that sustain ongoing debt is besides the point. One should consider only incremental cash flows and, therefore, only the incremental debt capacity that comes with those cash flows.

Figure 1: True and WACC-based GPVs: three 7- and 5-year examples

	A	B	C	D	E	F	G	H	I
1	tax rate	0.3							
2	target D/GPV	0.5		k	0.00849057				
3	E(R_U)	0.1		E(R_L)	0.128				
4	R_D	0.06		WACC	0.085				
5									
6	year	0	1	2	3	4	5	6	7
7	X		-100.00	200.00	400.00	600.00	500.00	300.00	110.00
8	GPV ^u	\$1,320.96	\$1,553.06	\$1,508.37	\$1,259.20	\$785.12	\$363.64	\$100.00	0
9	GPV	\$1,374.49	\$1,597.42	\$1,541.01	\$1,279.94	\$795.59	\$367.57	\$100.86	0
10	GPV a WACC	\$1,402.38	\$1,621.58	\$1,559.42	\$1,291.97	\$801.79	\$369.94	\$101.38	0
11	error ratio	\$0.52	\$0.54	\$0.56	\$0.58	\$0.59	\$0.60	\$0.61	0
12									
13	year	0	1	2	3	4	5	6	7
14	X		-100.00	200.00	400.00	600.00	800.00	1000.00	1200.00
15	GPV ^u	\$2,461.71	\$2,807.89	\$2,888.68	\$2,777.54	\$2,455.30	\$1,900.83	\$1,090.91	0
16	GPV	\$2,585.41	\$2,915.74	\$2,976.75	\$2,844.12	\$2,500.27	\$1,925.99	\$1,100.25	0
17	GPV a WACC	\$2,646.73	\$2,971.70	\$3,024.29	\$2,881.36	\$2,526.27	\$1,941.01	\$1,105.99	0
18	error ratio	\$0.50	\$0.52	\$0.54	\$0.56	\$0.58	\$0.60	\$0.61	0
19									
20	year	0	1	2	3	4	5	6	7
21	X		700.00	600.00	500.00	400.00	300.00	200.00	100.00
22	GPV ^u	\$2,131.58	\$1,644.74	\$1,209.21	\$830.13	\$513.15	\$264.46	\$90.91	0
23	GPV	\$2,184.68	\$1,681.36	\$1,232.90	\$844.14	\$520.40	\$267.47	\$91.69	0
24	GPV a WACC	\$2,213.51	\$1,701.66	\$1,246.30	\$852.24	\$524.68	\$269.28	\$92.17	0
25	error ratio	\$0.54	\$0.55	\$0.57	\$0.58	\$0.59	\$0.60	\$0.61	0

	A	B	C	D	E	F	G	H	I
1	tax rate	0.3							
2	target D/GPV	0.5		k	0.00849057				
3	E(R_U)	0.1		E(R_L)	0.128				
4	R_D	0.06		WACC	0.085				
5									
6	year	0	1	2	3	4	5	6	7
7	X		-100.00	200.00	400.00	600.00	500.00	0.00	0.00
8	GPV ^u	\$1,095.17	\$1,304.69	\$1,235.16	\$958.68	\$454.55	\$0.00	\$0.00	0
9	GPV	\$1,135.02	\$1,336.71	\$1,257.07	\$970.59	\$458.44	\$0.00	\$0.00	0
10	GPV a WACC	\$1,156.36	\$1,354.65	\$1,269.79	\$977.72	\$460.83	\$0.00	\$0.00	0
11	error ratio	\$0.54	\$0.56	\$0.58	\$0.60	\$0.61	#DIV/0!	#DIV/0!	0
12									
13	year	0	1	2	3	4	5	6	7
14	X		-100.00	200.00	400.00	600.00	800.00	0.00	0.00
15	GPV ^u	\$1,281.45	\$1,509.60	\$1,460.56	\$1,206.61	\$727.27	\$0.00	\$0.00	0
16	GPV	\$1,330.05	\$1,549.14	\$1,488.53	\$1,222.87	\$733.50	\$0.00	\$0.00	0
17	GPV a WACC	\$1,355.87	\$1,571.12	\$1,504.66	\$1,232.56	\$737.33	\$0.00	\$0.00	0
18	error ratio	\$0.53	\$0.56	\$0.58	\$0.60	\$0.61	#DIV/0!	#DIV/0!	0
19									
20	year	0	1	2	3	4	5	6	7
21	X		700.00	600.00	500.00	400.00	300.00	0.00	0.00
22	GPV ^u	\$1,967.37	\$1,464.11	\$1,010.52	\$611.57	\$272.73	\$0.00	\$0.00	0
23	GPV	\$2,010.41	\$1,491.63	\$1,026.27	\$619.03	\$275.06	\$0.00	\$0.00	0
24	GPV a WACC	\$2,034.43	\$1,507.36	\$1,035.48	\$623.50	\$276.50	\$0.00	\$0.00	0
25	error ratio	\$0.56	\$0.57	\$0.59	\$0.60	\$0.61	#DIV/0!	#DIV/0!	0

2 Assessing Size and Key Determinants of the Bias

How important is the valuation error? In our one-year example, it seems like nitpicking. And, in line with Myers' 1974 figures, the error rarely amount to over 20 percent of the gross present value (Appendix Table B).⁷ Yet, upon reflection, such an approach rests on a strange

⁷Myers uses a higher tax rate and expresses the error as a fraction of the initial investment.

schizophrenia: the scenarios where the error is minute are also those where the true tax effect is small. Thus, on the basis of these numbers one should ignore the entire tax issue too, a conclusion that few academics would be willing to accept. Thus, alternatively, one could scale the error in the WACC's PV'ed tax savings by the true tax advantage. The textbook consensus being that taxes matter, an overstatement of the tax advantage by, say, more than half cannot be viewed as trivial. And, lastly, doing the calculations properly is not difficult at all.

The (familiar) math for the longer-lived examples is presented in Appendix A, along with suggestions on how to program this using a simple spreadsheet. We illustrate the approach via the worksheets shown in Figure 1. The shaded areas are exogenous data: 30% taxes, a target debt of 50%, an unlevered return of 10%, and a borrowing rate of 6%. We fix operating cash flows for seven years—first rising and then falling in example 1, rising all the time in example 2, and falling all the time in example 3—all shown in the first worksheet. We find that, in all three examples, the overstatement is nowhere below 50%. In the second sheet we cut out the last two years, making the projects even less perpetual. The results are essentially unaffected.

Tables 3 and 4 show the results of wider-ranging robustness tests. We look at lives of 2, 5, 10 and 20 years, and target D/GPV ratios of 0.1, 0.3 and 0.5; we also vary the risk-free rate (3, 6, 9%) and the risk premium on the unlevered equity (3, 6, 9%), for both linearly falling and rising cash flows. The tax rate is still 0.30 in Table 3, but in Table 4 it is lowered to 0.15. For WACC to have only a trivial bias, we clearly need to go back to a 1970s-80s situation with a high risk-free rate; and even then one needs a low risk premium and a very long horizon. At the other extreme (short lives, low risk-free rate, high underlying risk premium), the bias can be 100 or 200%.

The bias ratios shown in Tables 3 are further summarised, in Table 5, via an OLS regression. This regression is meant to be descriptive at best, but the findings below can also be verified directly from the original numbers. The target debt ratio has but a weak effect, since it affects the true and the WACC-estimated tax shield in essentially the same way. The same holds for the tax rate, incidentally (not included in the regression). The most crucial variables are those that directly bear on the mistake in the weight that the tax refund has in the total PV: a one-point rise in the risk-free rate makes the bias fall by roughly 17 percentage points, a one-point rise in the risk premium makes it rise by 11-12 percentage points, and one extra year reduces the bias by about 3 percentage points. The pattern (rising, falling) has little effect, even though the size of the difference is quite unrealistic. The reason why it has little impact is again that it affects the true and the estimated values in very similar ways.

Table 3: Degree of Overestimation of Tax Advantage when Using WACC instead of ANPV (finite life, constant D/E, $\tau=0.30$)

RISING		riskprem 3%				riskprem 6%				riskprem 9%			
		$\delta=0.10$	$\delta=0.30$	$\delta=0.50$	$\delta=0.10$	$\delta=0.30$	$\delta=0.50$	$\delta=0.10$	$\delta=0.30$	$\delta=0.50$	$\delta=0.10$	$\delta=0.30$	$\delta=0.50$
$R_f=3\%$	life 2	0.92	0.93	0.92	1.78	1.79	1.78	2.57	2.59	2.57			
	life 5	0.86	0.87	0.85	1.61	1.63	1.56	2.25	2.29	2.16			
	life 10	0.77	0.78	0.76	1.35	1.38	1.32	1.78	1.84	1.73			
	life 20	0.60	0.62	0.63	0.92	0.96	0.98	1.06	1.13	1.15			
$R_f=6\%$	life 2	0.44	0.45	0.44	0.86	0.86	0.86	1.24	1.25	1.24			
	life 5	0.40	0.41	0.39	0.75	0.76	0.72	1.04	1.07	0.99			
	life 10	0.33	0.34	0.33	0.58	0.60	0.56	0.77	0.80	0.73			
	life 20	0.21	0.23	0.23	0.32	0.34	0.35	0.35	0.38	0.38			
$R_f=9\%$	life 2	0.29	0.29	0.29	0.55	0.56	0.55	0.80	0.81	0.80			
	life 5	0.25	0.25	0.24	0.46	0.47	0.44	0.65	0.66	0.60			
	life 10	0.19	0.20	0.18	0.33	0.35	0.31	0.43	0.46	0.40			
	life 20	0.09	0.10	0.10	0.13	0.14	0.14	0.12	0.15	0.14			
FALLING		riskprem 3%				riskprem 6%				riskprem 9%			
		$\delta=0.10$	$\delta=0.30$	$\delta=0.50$	$\delta=0.10$	$\delta=0.30$	$\delta=0.50$	$\delta=0.10$	$\delta=0.30$	$\delta=0.50$	$\delta=0.10$	$\delta=0.30$	$\delta=0.50$
$R_f=3\%$	life 2	0.93	0.93	0.92	1.79	1.80	1.78	2.59	2.61	2.57			
	life 5	0.88	0.88	0.85	1.65	1.67	1.56	2.33	2.36	2.16			
	life 10	0.80	0.82	0.76	1.45	1.48	1.32	1.97	2.02	1.73			
	life 20	0.72	0.74	0.63	1.25	1.28	0.98	1.63	1.68	1.15			
$R_f=6\%$	life 2	0.45	0.45	0.44	0.86	0.87	0.86	1.25	1.26	1.24			
	life 5	0.41	0.42	0.39	0.77	0.79	0.72	1.09	1.11	0.99			
	life 10	0.36	0.37	0.33	0.65	0.67	0.56	0.89	0.91	0.73			
	life 20	0.31	0.32	0.23	0.53	0.55	0.35	0.70	0.73	0.38			
$R_f=9\%$	life 2	0.29	0.29	0.29	0.56	0.56	0.55	0.81	0.82	0.80			
	life 5	0.26	0.26	0.24	0.48	0.49	0.44	0.69	0.70	0.60			
	life 10	0.22	0.22	0.18	0.39	0.40	0.31	0.53	0.55	0.40			
	life 20	0.18	0.18	0.10	0.31	0.32	0.14	0.41	0.42	0.14			

Key Projects have cash flows that either start from 100 and linearly rise by step 100 ("RISING"), or start from 200 and linearly fall by step 100 ("FALLING"). Lives are 2, 5, 10 or 20 years. (A)NPV is computed assuming no leverage, or a constant D/GPV ratio (" δ ") of 10, 30 or 50%, and compared to an NPV using conventional WACC (i.e. valid for infinite perpetuities). The corporate tax rate is 0.30. The main part of the table shows the overestimation of the tax advantage under WACC, as a fraction of the true tax advantage.

Table 4: Degree of Overestimation of Tax Advantage when Using WACC instead of ANPV (finite life, constant D/E, $\tau=0.15$)

RISING		riskprem 3%			riskprem 6%			riskprem 9%		
		$\delta=0.10$	$\delta=0.30$	$\delta=0.50$	$\delta=0.10$	$\delta=0.30$	$\delta=0.50$	$\delta=0.10$	$\delta=0.30$	$\delta=0.50$
$R_f=3\%$	life 2	0.92	0.92	0.92	1.77	1.78	1.77	2.56	2.57	2.56
	life 5	0.86	0.87	0.85	1.60	1.61	1.56	2.24	2.26	2.15
	life 10	0.77	0.77	0.76	1.34	1.36	1.31	1.77	1.80	1.71
	life 20	0.59	0.60	0.60	0.91	0.93	0.93	1.05	1.08	1.06
$R_f=6\%$	life 2	0.44	0.45	0.44	0.85	0.86	0.85	1.23	1.24	1.23
	life 5	0.40	0.40	0.39	0.74	0.75	0.71	1.04	1.05	0.98
	life 10	0.33	0.34	0.32	0.58	0.59	0.56	0.76	0.77	0.72
	life 20	0.21	0.22	0.22	0.31	0.32	0.31	0.34	0.35	0.34
$R_f=9\%$	life 2	0.28	0.29	0.28	0.55	0.55	0.55	0.79	0.80	0.79
	life 5	0.25	0.25	0.24	0.46	0.46	0.43	0.64	0.65	0.60
	life 10	0.19	0.19	0.18	0.33	0.34	0.31	0.43	0.44	0.40
	life 20	0.09	0.10	0.09	0.12	0.13	0.12	0.12	0.13	0.11
FALLING		riskprem 3%			riskprem 6%			riskprem 9%		
		$\delta=0.10$	$\delta=0.30$	$\delta=0.50$	$\delta=0.10$	$\delta=0.30$	$\delta=0.50$	$\delta=0.10$	$\delta=0.30$	$\delta=0.50$
$R_f=3\%$	life 2	0.93	0.93	0.92	1.79	1.79	1.77	2.59	2.60	2.56
	life 5	0.88	0.88	0.85	1.64	1.65	1.56	2.32	2.34	2.15
	life 10	0.80	0.81	0.76	1.45	1.46	1.31	1.96	1.99	1.71
	life 20	0.72	0.73	0.60	1.24	1.26	0.93	1.62	1.64	1.06
$R_f=6\%$	life 2	0.45	0.45	0.44	0.86	0.87	0.85	1.25	1.25	1.23
	life 5	0.41	0.41	0.39	0.77	0.78	0.71	1.09	1.10	0.98
	life 10	0.36	0.36	0.32	0.65	0.66	0.56	0.88	0.89	0.72
	life 20	0.31	0.31	0.22	0.53	0.54	0.31	0.70	0.71	0.34
$R_f=9\%$	life 2	0.29	0.29	0.28	0.55	0.56	0.55	0.80	0.81	0.79
	life 5	0.26	0.26	0.24	0.48	0.49	0.43	0.68	0.69	0.60
	life 10	0.21	0.22	0.18	0.39	0.39	0.31	0.53	0.54	0.40
	life 20	0.17	0.18	0.09	0.30	0.31	0.12	0.40	0.41	0.11

Key Projects have cash flows that either start from 100 and linearly rise by step 100 ("RISING"), or start from 200 and linearly fall by step 100 ("FALLING"). Lives are 2, 5, 10 or 20 years. (A)NPV is computed assuming no leverage, or a constant D/GPV ratio (" δ ") of 10, 30 or 50%, and compared to an NPV using conventional WACC (i.e. valid for infinite perpetuities). The corporate tax rate is 0.15. The main part of the table shows the overestimation of the tax advantage under WACC, as a fraction of the true tax advantage.

Table 5: Effects of Project Parameters on Bias in WACC

	rising	falling	pooled
target D/GPV (fraction)	-0.016	-0.248	-0.132
risk premium (%)	0.107	0.120	0.114
R_f (%)	-0.170	-0.177	-0.173
life	-0.033	-0.024	-0.029
rising? (=1, 0 otherwise)	-0.009		-0.061
INT	1.482		1.508
R^2	0.839		0.835

Key Projects have cash flows that either start from 100 and linearly rise by step 100 (“RISING”), or start from 2000 and linearly fall by step 100 (“FALLING”). Lives are 2, 5, 10 or 20 years. ANPV is computed assuming a constant D/GPV ratio (“ δ ”) of 10, 30 or 50%, and is compared to an NPV using conventional WACC (i.e. valid for perpetuities). The corporate tax rate is 0.30. The resulting overestimation of the tax advantage under WACC, as a fraction of the true tax advantage, is regressed on the parameters, once giving the rising/falling subsamples separate regression coefficients, once imposing common coefficients. Like in Tables 3 and 4, the riskfree rate and the risk premium are assumed to be expressed as a percentage, but the target debt ratio is a fraction.

3 Conclusion

WACC is designed for situations with perpetual constant leverage, and overestimates the tax shield when the project’s life is finite. The reason is that a constant D/E ratio no longer means a constant ratio of PVs of operating cashflows and tax savings. As a result, initial risk is underestimated, and subsequent further rises in risk are overlooked. For realistic projects of 5-7 years the overstatement of the PV of the tax shield typically exceeds 50%. One needs 10-year horizons for the error to drop to 40%, and 20 years for a drop below 20%. In contrast, ANPV can correctly handle not only any deterministic cash flow pattern or capital structure, but, after a minor tweak in the programming, also borrowing strategies where the loan amount changes with GPV, the situation supposedly best solved with WACC: one just needs to apply $NPV()$ twice.

This objection to WACC comes on top of other familiar drawbacks: with WACC, it is difficult to correct for episodes where EBIT is predicted to fall short of interest due, and in most countries the perpetual rate is not really observable.

One could shrug the error off as not egregious relative to other errors one can make in capital budgeting. Betas, for instance, are hard to estimate. But while the error in beta has expected value zero, the valuation error is a regular bias. Given that it is easily corrected, it should not be ignored.

References

- Brealey, R. A., and S. C. Myers, 1981, *Principles of Corporate Finance*, McGraw-Hill Education; 2nd edition
- Brealey, R. A., and S. C. Myers, 2000, *Principles of Corporate Finance*, McGraw-Hill Education; 6th edition
- Miller, M. and F. Modigliani, 1958. The Cost of Capital, Corporation Finance and the Theory of Investment, *The American Economic Review*, Vol. 48, No. 3. (Jun., 1958), pp. 261-297
- Miller, M. and F. Modigliani, 1961. Dividend Policy, Growth, and the Valuation of Shares, *The Journal of Business*, Vol. 34, No. 4. (Oct., 1961), pp. 411-433.
- Miller, M. and F. Modigliani, 1963. Corporate Income Taxes and the Cost of Capital: A Correction *The American Economic Review*, Vol. 53, No. 3. (Jun., 1963), pp. 433-443.
- Miller, M. 1977, Debt and Taxes, *The Journal of Finance*, Vol. 32, No. 2 (Papers and Proceedings of the Thirty-Fifth Annual Meeting of the American Finance Association, Atlantic City, New Jersey, September 16-18, 1976), (May, 1977), pp. 261-27
- Myers, S. C., 1974. Interactions of Corporate Financing and Investment Decisions—Implications for Capital Budgeting, *The Journal of Finance*, Vol. 29, No. 1. (Mar., 1974), pp. 1-25
- Ross, S. A., R. W. Westerfield, and J. Jaffe, 1999, *Corporate Finance*, Irwin McGraw-Hill, Boston, International Edition

Appendix A: The ANPV under a preset debt ratio

The convention I use is that the year- s cash flow materialises an integer number of years after the valuation date and that a GPV dated time s includes the cashflows subsequent to s , not the one of the period itself. The tax saving is assumed to be realized together with the cash flow itself, but the formulas are easily adjusted for delays in tax payments.

In general, under the debt-capacity approach the savings would be computed as follows, taking into account the firm's long-run debt/assets ratio (δ), the gross present value (GPV) of the project, corporate tax rate (τ), and the interest rate paid on the debt (R_D , a per annum

rate):

$$[\text{Additional borrowing capacity}]_t = \delta \times GPV_t \quad (8)$$

$$[\text{Tax saving due to project}]_t = \delta \times GPV_t \times R_D \times \tau \quad (9)$$

$$[\text{PV of tax savings}]_t = \sum_{s=t+1}^T \frac{R_D \times \tau \times \delta \times GPV_s}{(1 + R_D)^{s-t}}. \quad (10)$$

There is one extra layer, though: the GPV should include the tax savings. Let us denote the (time-varying) GPV of the unlevered project by GPV_t^u . The PV-ed tax savings are to be added; these change from year to year, so there is no easy WACC-like formula, and the solution must be found by recursive calculations, easily done in e.g. a spreadsheet. We start at the end, time $T - 1$. Equation (10) says that in the last year the last tax saving is proportional to the levered GPV, with proportionality factor $k = \tau \delta \frac{R_D}{(1+R_D)}$. This allows us to back out the levered GPV:

$$\begin{aligned} GPV_{T-1} &= GPV_{T-1}^u + R_f \tau \delta \frac{GPV_T}{1 + R_D}, \\ &= GPV_{T-1}^u + k GPV_{T-1}; \\ \Rightarrow GPV_{T-1} \times (1 - k) &= GPV_{T-1}^u; \\ \Rightarrow GPV_{T-1} &= \frac{GPV_{T-1}^u}{1 - k}. \end{aligned} \quad (11)$$

We now move back one period and proceed analogously:

$$\begin{aligned} GPV_{T-2} &= GPV_{T-2}^u + R_f \tau \delta \left(\frac{GPV_{T-1}}{(1 + R_D)^2} + \frac{GPV_{T-2}}{1 + R_f} \right), \\ &= GPV_{T-2}^u + k \left(\frac{GPV_{T-1}}{1 + R_D} + GPV_{T-2} \right), \\ \Rightarrow GPV_{T-2}(1 - k) &= GPV_{T-2}^u + k \frac{GPV_{T-1}}{1 + R_D} \\ \Rightarrow GPV_{T-2} &= \frac{GPV_{T-2}^u + k \frac{GPV_{T-1}}{1 + R_D}}{1 - k}. \end{aligned} \quad (12)$$

Going back one more step makes clear what the general rule is:

$$\begin{aligned} GPV_{T-3} &= GPV_{T-3}^u + k \left(\frac{GPV_{T-1}}{(1 + R_D)^2} + \frac{GPV_{T-2}}{1 + R_D} + GPV_{T-3} \right), \\ \Rightarrow GPV_{T-3} &= \frac{GPV_{T-3}^u + k \left(\frac{GPV_{T-1}}{(1 + R_D)^2} + \frac{GPV_{T-2}}{1 + R_D} \right)}{1 - k}, \end{aligned} \quad (13)$$

$$\Rightarrow GPV_t = \frac{GPV_t^u + k \left(\frac{GPV_{T-1}}{(1 + R_D)^{T-1-t}} + \frac{GPV_{T-2}}{(1 + R_D)^{T-2-t}} \dots + \frac{GPV_{t+1}}{1 + R_D} \right)}{1 - k}, \quad (14)$$

So you start at the end, at period T , compute GPV_T^u and hence GPV_T ; then you go backward one period and compute the new unlevered GPV, which gives you the levered version, and so

on. The bracketed sum following k can be programmed at one stroke as a NPV() operation. Here is how one can program it:

- GPV^u (line 8 in Figure 1) for the last year is simply NPV(\$B3,I7:\$I7); for earlier years, just copy the formula leftwards.
- GPV (line 9) for the last year is $(G8+NPV(\$B4,I9:$I9)*\$E2)/(1-\$E2)$, where $\$E2=k$; for earlier years, copy the formula leftward.

Appendix B: Valuation Error as a fraction of GPV

(Table overleaf)

Table 6: Degree of Overestimation of Tax Advantage when Using WACC instead of ANPV (finite life, constant D/E, $\tau=0.30$)

RISING		riskprem 3%			riskprem 6%			riskprem 9%		
		$\delta=0.10$	$\delta=0.30$	$\delta=0.50$	$\delta=0.10$	$\delta=0.30$	$\delta=0.50$	$\delta=0.10$	$\delta=0.30$	$\delta=0.50$
$R_f=3\%$	life 2	0.00	0.01	0.00	0.00	0.01	0.00	0.01	0.02	0.01
	life 5	0.01	0.02	0.01	0.01	0.03	0.01	0.01	0.03	0.02
	life 10	0.01	0.03	0.01	0.02	0.05	0.02	0.02	0.06	0.02
	life 20	0.02	0.06	0.12	0.03	0.09	0.17	0.04	0.11	0.22
$R_f=6\%$	life 2	0.00	0.01	0.00	0.01	0.02	0.01	0.01	0.02	0.01
	life 5	0.01	0.03	0.01	0.01	0.03	0.02	0.01	0.04	0.02
	life 10	0.02	0.05	0.02	0.02	0.06	0.02	0.02	0.08	0.03
	life 20	0.03	0.09	0.17	0.04	0.11	0.22	0.04	0.13	0.27
$R_f=9\%$	life 2	0.01	0.02	0.01	0.01	0.02	0.01	0.01	0.02	0.01
	life 5	0.01	0.03	0.02	0.01	0.04	0.02	0.02	0.05	0.03
	life 10	0.02	0.06	0.02	0.02	0.08	0.03	0.03	0.09	0.03
	life 20	0.04	0.11	0.22	0.04	0.13	0.27	0.04	0.14	0.30

Key Projects have cash flows that either start from 100 and linearly rise by step 100. Lives are 2, 5, 10 or 10 years. (A)NPV is computed assuming no leverage, or a constant D/GPV ratio (“ δ ”) of 10, 30 or 50%, and compared to an NPV using conventional WACC (i.e. valid for infinite perpetuities). The corporate tax rate is 0.30. The main part of the table shows the overestimation of the tax advantage under WACC, as a fraction of GPV^u .