Validating state-dependent queueing models for uninterrupted traffic flows using simulation

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Abstract. In this paper, the effectiveness of state-dependent queueing models for analyzing traffic flows is tested by comparing the speeds generated by the queueing models with the ones obtained by simulation. Simulation is thus used to evaluate speeds generated by the different queueing models. Different state-dependency functions are described and their performance is assessed. An $M/G/1$ queueing model with Gaussian state-dependency outperforms all other state-dependent queueing models. Different test results and insights are provided.

Key words: Queueing theory, traffic flows, state dependency

MSC classification: 60K25, 60K30

1 Introduction

In a previous paper, Van Woensel and Vandaele (2005) demonstrated that queueing models can be used to adequately represent the traffic flow process. In their paper, they assessed the quality of this approach by comparing the speeds obtained by means of the queueing model with the ones observed empirically. The model validation was central in that paper; the parameter validation was of second importance. In other words, the primary objective of Van Woensel and Vandaele (2005) was to demonstrate that queueing models can be used to assess the traffic flow process. It appeared that traffic flows on a highway during non-congested hours were best described using a $M/G/1$ queueing model. During the congested hours however, the state-dependent queueing $GI/G/m$ models were more realistic.

The authors observed as a side remark that more data are needed to adequately pinpoint the specific values of the parameters. Moreover, they found that most of

the existing estimation error was made in the congested periods of the day. This is due to two reasons. The first reason is that the use of empirical data makes good estimations more difficult due to the existence of outliers. These outliers were partially damped by aggregating the observations in 10-minute time buckets but were still existing. Secondly, it appeared that state-dependent queueing models performed better than non state-dependent models in the congested hours, but the state-dependency function used did not completely grasp the traffic process, hence resulting in a reduced performance. These observations are the starting point of the current paper. Two major research questions will be addressed in this paper: (1) Does the use of simulation reduce the data accuracy problem related to outliers incurred when using real-life empirical data? and (2) Are there new state-dependency functions that better grasp the peculiarities during the congested hours of the day?

The contributions are then twofold. First, the relationship between speed, flow and density is simulated resulting in a complete dataset which can be used to test the different queueing models. The advantage of using simulation is that it can be controlled in a rigorous way, that the data are not prone to outliers and that it can be replicated as many times as needed. Compared to empirical data, it is then hypothesized that simulation makes it possible to do a better parameter estimation and validation. As such the experiment can be much more controlled: if deviations exist, this is not due to the underlying data used but due to the model specification. Secondly, new state-dependent functions are incorporated in the queueing based traffic models and are evaluated in a similar way as in Van Woensel and Vandaele (2005). As such, this paper extends the insights generated in that paper and copes with the discussed reduced predictive power of the queueing models in the congested hours.

This paper is organized as follows. First, some traffic flow preliminaries are briefly discussed, then the methodology to simulate the traffic flow process is discussed in detail. Next, new state-dependency functions are analytically described. Finally, these new functions are evaluated using the simulated speed-flow data. The paper ends with conclusions.

2 Traffic flow prerequisites

Without being complete, some traffic flow preliminaries are summarized in this section. The interested reader is referred for more details to Daganzo (1997); May (1990); Hall (1996); and the references mentioned there.

In this paper, uninterrupted traffic flows are considered, i.e. traffic on highways. In the literature, three main types of model representations are considered depending upon the level of detail: microscopic, mesoscopic and macroscopic models¹. More specifically, microscopic models deal with vehicles, where macroscopic models deal with flows. Microscopic models describe each vehicle separately and are

¹ The mesoscopic representation which combines microscopic and macroscopic elements is not discussed here. For more details on traffic flow models and a detailed discussion of macroscopic, mesoscopic and microscopic models, the reader is referred to May (1990).

based on theories of how vehicles maneuver through traffic. Usually, individual vehicles are modelled by means of a micro-simulation. Here one tries to model and visualize transport systems at the basic behavioral level of a vehicle and the driver (e.g. aggression factors). Software tools that can be used here are e.g. AIM-SUN, MICROSIM, PARAMICS, etc., see <http://www.its.leeds.ac.uk/projects/smartest/links.html> for more information and a comparison of the different micro-simulation models. The main disadvantages of these microscopic (and mesoscopic) models are their complexity, their non-analytical character and their demand on computer time. These models are more simulations tools than mathematical models. Due to these drawbacks, the usability of microscopic and mesoscopic models is mostly limited to sections of roads, rather than networks. Therefore, macroscopic models are more suitable for the design of control strategies since they describe the traffic flow process analytically and demand less computational time (Kotsialis and Papageorgiou 2002).

In this paper, macroscopic traffic flow models based on queueing theory are presented and validated. It is important to mention that the queueing models developed assume steady-state conditions, i.e. the same behavior is reproduced and observed every time with the same probability. In terms of traffic, this means that the traffic flows observed are stationary. The assumption of stationary traffic flows (or equivalent, steady-state queueing models) means that all vehicles will always be driving (no matter how slow). The non-stationary traffic however experiences stop's and go's. These non-stationary traffic flows can be modelled using so-called transient queueing models. Heidemann (1999) shows that under non-stationary conditions, the speed-flow-density results deviate from the ones obtained with stationary queueing models. Moreover, phenomena like stop-and-go traffic can be explained using these models. Heidemann (1999) also shows that the non-stationary flow-density diagrams converge to the stationary ones when the time period considered in the non-stationary models grows to infinity. In general, the steady-state results are most appropriate in design and policy recommendations. The transient queueing models are more useful in specific control situations for relatively small networks (Van Woensel 2003).

In general, the study of macroscopic traffic flow models boils down to the analysis of the speed-flow-density relationships (see e.g. the seminal paper of Green-shields 1935). The basic formula of traffic flow theory incorporates the interdependence of traffic flow q , traffic density k , and the speed v (Hall 1996):

$$q = k \times v \quad (1)$$

Using relation (1), the typical speed-flow-density diagrams can be constructed. Figure 1 shows the relationships between the speed-flow, the speed-density, and the flow-density diagram.

The actual form of these diagrams depends upon the prevailing traffic and roadway conditions on the roadway segment under study. Although the diagrams

Fig. 1. The relations between the speed-flow, the speed-density, and the flow-density diagrams

show continuous curves, it is unlikely that the full range of all points will be found at any particular measurement location. Almost all data collected for the calibration of these diagrams are subject to influences of changing environmental conditions, non-homogeneity of vehicles in the traffic stream, and lack of complete isolation from ramps and interchanges (Highway Capacity Manual 1996). If traffic count data are available, traffic flows (q) can be assumed as given, which leaves us to calculate either traffic density or speed to complete the above formula. These diagrams also illustrate a number of significant points, i.e. a zero flow ($q = 0$) occurs under two very different conditions:

1. When there are no cars in the facility, density is zero ($k = 0$), and flow is zero. Speed is purely theoretical for this condition and would be whatever the first driver would select, probably the highest possible value (e.g. at the speed limit or v_f).
2. When density becomes so high that all vehicles stop (speed is zero), the flow q is also zero. This is because there is no movement and vehicles cannot pass a point on the roadway. The density at which all movement stops is called the *jam density* (k_j).

Between these two extreme points, the dynamics of traffic flow produce a maximizing effect. As density k increases from zero, flow q also increases, since more vehicles are on the roadway. While this is happening, speed begins to decline (because of the interaction between vehicles). This decline is virtually negligible at low and medium densities and flows. As density continues to increase, these generalized diagrams suggest that speed decreases significantly before the capacity is achieved. Capacity is reached when the product of density and speed results in the maximum flow. Any flow other than capacity can occur under two conditions: one with a high speed and low density and one with low speed and high density. The high density and low speed side of the diagrams represents forced or breakdown flow (Daganzo 1996). For each time period a different flow is observed and consequently different speeds and densities (following the speed-flow-density diagrams).

Traditionally, traffic flows are modelled by empirically determining these speed-flow-density relationships (e.g. Li 2002). Two main methodologies are described in the literature (Daganzo 1996). First, the speed-flow relationship can be estimated econometrically using data on observed traffic flows and speeds, and fitted into a speed-flow specification. The second method consists in postulating a functional form, followed by calibrating its parameters based on a few observations. Both methods suffer from limited feasibility since traffic data concerning volumes or counts is easily obtainable, the corresponding average speed of the traffic flow is not always readily available (De Borger and Proost 2001). Moreover, these approaches are limited in terms of predictive power and sensitivity analysis.

Vandaele et al. (2000) and Heidemann (1996), showed that queueing models can also be used to explain uninterrupted traffic flows and thus offering a more practical approach, useful for sensitivity analysis, forecasts, etc. Jain and MacGregor Smith (1997) describe in their paper a state-dependent $M/G/C/C$ queueing model for traffic flows. Part of their logic is used to extend our queueing models to state-dependent ones. Moreover they show that state-dependent queueing models where the service rate is a function of the flow already on the road, result in a better representation of the traffic flow process compared to non state-dependent queueing models. Also a lot of research is done on a travel time-flow model originating from Davidson (1978). The model is based on some concepts of queueing theory but a direct derivation of the model has not been clearly demonstrated (Akçelik 1991 and 1996).

In a queueing approach to traffic flow analysis, roads are subdivided into segments, with length equal to the minimal space needed by one vehicle on that road. Define k_j as the maximum traffic density (i.e. maximum number of cars on a specific road length). The segment length is then equal to $1/k_j$ and matches the minimal space needed by one vehicle on that road. Each road segment is then considered as a service station, in which vehicles arrive at a certain rate λ and get served at another rate μ (Vandaele et al. 2000; Van Woensel et al. 2001; Heidemann 1996). It then can be shown that the speed v is calculated as the division of the length of

the road segment $\frac{1}{k_j}$ by the total time in the system W or:

$$v = \frac{1/k_j}{W} \quad (2)$$

Consequently, data are needed for the total time in the system W and the length of the road segment $\frac{1}{k_j}$. The latter value can be estimated using the empirically observed maximum flow. The total time in the system will be the result of the simulation to be performed.

3 Simulation methodology

Data are available for flows only; speed data are in general not available or only for a short period in time (Van Woensel 2003). The flow data are collected by the Department of Environment and Infrastructure of the Ministry of the Flemish Community. This department is responsible for collecting and reporting the counting data for all Flemish roads. The dataset consists of observations of the number of vehicles per hour (i.e. flow q) for 273 days of the A1/E19 highway, section Mechelen North–Rumst, headed for Brussels.

Taking into account the typical bimodal character of flow data (i.e. morning congestion and evening congestion are different from the other hours of the day), a straightforward application of a stationary distribution for the arrival process is not appropriate. Therefore, several authors have put forward procedures for which the underlying distribution of the arrival process is non-stationary (Leemis 1991, Kao and Chang 1988 and Johnson et al. 1994). A recently developed methodology (Preston White 1999) concerns a Non-stationary Poisson process using Bivariate Thinning (NPBT). Arrivals are assumed to be non-stationary Poisson, with a piecewise-constant arrival rate independently regulated by hour and by day. Preston White (1999) showed that the non-stationary poisson process with bivariate thinning is an adequate method to simulate arrivals at a consumer store. The standard thinning technique was originally developed by Lewis and Shedler (1979) for generating non-stationary Poisson arrivals. It is shown that the NPBT method is appropriate for the simulation of arrivals in a traffic context as well. This methodology will be applied to the dataset of observed flows. The data are standardized (i.e. the mean is subtracted from the dataset values and subsequently divided by their standard deviation). Observations with a standardized value exceeding 2.5 will be removed (Hair et al. 1998).

Let x_{ij} be the observed flow data for hour i ($i = 1 \dots 24$) and day j ($j = 1 \dots 7$). Since the pattern of weekdays and weekend-days is typically different (e.g. weekend-days do not have an pronounced morning and evening peak period; considerably more traffic is present during weekdays; etc.), the NPBT method will be applied separately on both types of days (weekdays and weekend-days). Below, the procedure for weekdays is described (weekend-days are treated in a similar way). The

mean $\bar{x}_{ij}$ of each hour ($i = 1...24$) per day ($j = 1...5$) is calculated from the dataset. For each hour of the day and each day of the week we have:

$$\bar{x}_i = \frac{1}{5} \sum_{j=1}^5 \bar{x}_{ij}, \forall i = 1...24 \quad (3)$$

$$\bar{x}_j = \frac{1}{24} \sum_{i=1}^{24} \bar{x}_{ij}, \dots \forall j = 1...5 \quad (4)$$

Next, hourly and daily thinning factors are computed:

$$\eta_i = \frac{\bar{x}_i}{\max_i(\bar{x}_i)}, \forall i = 1...24 \quad (5)$$

$$\delta_j = \frac{\bar{x}_j}{\max_j(\bar{x}_j)}, \dots \forall j = 1...5 \quad (6)$$

The hour block with the highest mean is combined with the day block with the highest mean:

$$\lambda_{\max} = \bar{x}_{ij} \text{ for } (i, j) \text{ such that } \eta_i \delta_j = 1 \quad (7)$$

In the simulation, potential arrivals are thus generated with a mean equal to the minimum average inter-arrival rate, determined from the average arrival count for the hour/day time block with unit thinning factors. Candidate arrivals are then thinned using a bivariate acceptance probability equal to the product of the corresponding hourly and daily thinning factors. More specifically, the arrivals are generated using an exponential inter-arrival rate with mean $\frac{1}{\lambda_{\max}}$ and consecutively thinned with the bivariate acceptance probability $p_{ij} = \eta_i \delta_j \forall i, j$.

With the aim of assessing the ratio of service, the empirically recorded maximum number of vehicles is assumed to be a proxy for the service rate. However, since internal as well as external elements impact upon the ratio of service (e.g. accidents and rain), a traffic service rate cannot be considered as a constant quantity. The service rate is defined as the product of free flow speed v_f with the maximum traffic density k_j (Heidemann 1996). Because internal and external elements can influence free flow speed, the service rate will be changed as well due to the latter variability.

With the intention of constructing a distribution of the maximum number of vehicles, nonparametric bootstrap samples are taken. The key idea is to re-sample from the original data to create replicate datasets, from which the variability of the quantities of interest (in our case the maximum number of vehicles) can be assessed without long-winded and error-prone analytical calculations (Davison and Hinkley 1998). Let Y be a vector which contains all the flow observations. Let then

$Y_1^*, \dots, Y_{1000}^*$ be 1000 bootstrap samples (i.e. each Y^* independently sampled at random with replacement from Y).

$$Y_{max}^* = \{\max(Y_1^*), \dots, \max(Y_{1000}^*)\} \quad (8)$$

The empirical discrete distribution vector Y_{max}^* will now be used as a proxy for the distribution of the service ratio. In the simulation (executed in ARENA) arrivals are therefore generated with the NPBT method and the service rates are taken at random out of the bootstrap vector Y_{max}^* . In order to test the adequacy of the NPBT method, the Kruskal-Wallis test is applied, verifying the hypothesis H_0 whether all samples are drawn from the same population. This test (Kruskal and Wallis 1952) is the nonparametric version of one way ANOVA and is a straightforward generalization of the Wilcoxon test for two independent samples. If there are K independent samples of sizes $n_1, n_2, \dots, n_K$, all the samples are combined into one large sample of size $n = \sum n_i$, the result is sorted from smallest to largest and ranks are assigned (the average rank is assigned to any observation in a group of tied observations), and then the average $\bar{R}_i$ of the ranks of the observations in the i -th sample is found. The test statistic is then:

$$H = \frac{12}{n(n+1)} \sum \frac{\bar{R}_i^2}{n_i} - 3(n+1) \quad (9)$$

and the null hypothesis that all K distributions are the same is rejected if $H > \chi_{K-1}^2$ (with χ_{K-1}^2 the critical value from a χ^2 distribution with $(K-1)$ degrees of freedom). The p -value indicates the probability that one would obtain a test statistic which is more extreme than the observed one when the H_0 is true. The rule is that one rejects H_0 if $p < \alpha$. A sufficiently large amount of 30 simulations versus 30 empirical observations of flow data of one week from the dataset is compared by means of this test. The realization of this procedure reveals a p -value of 0.9922. Two substantial conclusions can be drawn from the former statistic: (i) the empirical observations are generated by one underlying process and (ii) the hypothesis that both the observed and simulated flow data originate from the same underlying distribution cannot be rejected at the 99 percent significance level.

In total 16 800 (i.e. 10 replications of a simulation of 10 weeks, with each week containing 168 hours) coordinates in the speed-flow space are obtained. To cope with the problem of the initial transient phase inherent to simulation, the general technique of Welch (Law and Kelton 2000) is applied. The number of vehicles on Monday between 0 and 1 a.m. is taken as the random variable of interest, and 10 replications of the simulation (each of length 10) are made (Law and Kelton (2000) propose a minimum number of replications of 5). It appears that no warmup period is necessary. No significant number of cars in the system on Monday between 0 and 1 a.m. is emanating from the preceding hour. This is due to the fact that (i) the unit of time is one hour and (ii) the simulation starts at 0 a.m. (if the simulation

would start at 7 a.m. (i.e. morning rush) an initial transient phase is more likely to occur). The simulation can therefore be considered as a terminating system, which implies the independence of the different weeks.

Since in this paper a functional speed-flow relationship is of more convenience than a large amount of simulated coordinates in the in speed-flow space, the speed-flow relationship is estimated using a Ordinary Least Squares (OLS) procedure with $v = \alpha + \beta q + \gamma q^2$ as functional form. Table 3 gives the estimated coefficients and their respective significance levels. The $R^2_{adjusted}$ is very satisfying: 99.5 percent of the variation of speed is explained by the variation of the flow specification. From the presented p -values (between brackets) the coefficients appear to be significantly different from zero.

Table 1. OLS Regression of the quadratic specification of the speed-flow relationship $v = \alpha + \beta q + \gamma q^2$

Coefficient	Estimation
α	117.751 (0.000)
β	-0.00844 (0.000)
γ	-0.00000158 (0.000)
$R^2_{adjusted}$	0.995

4 Methodology of comparison

The comparison shows to what extent the outcome of different queueing models is similar to the simulated results. Several queueing models are taken into consideration: $M/M/1$, $M/G/1$ and $GI/G/1$. Furthermore, for each of the mentioned queueing models different previously reported and new state-dependent functions are implemented. The comparison is done by means of the Theil inequality coefficient (Theil 1966), which takes into account for the disproportionate weight of large errors in other tools (e.g. Mean Square Error) is used. (Barceló 2001). The Theil coefficient $Theil_{vq, vobs}$ of the queueing speed v_q and the observed speed v_{obs} is defined as:

$$Theil_{vq, vobs} = \frac{\sqrt{\frac{\sum (v_q - v_{obs})^2}{n}}}{\sqrt{\frac{\sum v_q^2}{n}} + \sqrt{\frac{\sum v_{obs}^2}{n}}} \quad (10)$$

A low value of $Theil_{vq, vobs}$ means that the speeds obtained from the queueing models are close to the observed speeds. An extra advantage of using the Theil

inequality coefficient is the decomposition into the three proportions of inequality. These proportions are useful as a means of breaking down the error down into three different characteristic sources: bias (error due to the difference in means), variance (error due the difference in variance) and covariance (error due to randomness). The bias, variance and covariance proportions all add up to one. The best results are those for which the bias and variance proportions are small or where the covariance proportion is the largest (Holden et al. 1990). For more information, the reader is referred to the mentioned references and to Van Woensel and Vandaele (2005).

For each queueing model, the speed-flow diagram is constructed. Consequently, for every flow, the queueing speed is compared with the simulated speed for the same flow. The latter speed will be approximated by the next equation based on the simulation. Speed v_s has units kilometer per hour and q is in vehicles per hour.

$$v_s = 117.751 - 844.012 \times 10^{-5}q - 158.002 \times 10^{-8}q^2 \quad (11)$$

A final remark about the methodology of comparison is that since the simulation described in this paper is macroscopic: all individual vehicles are aggregated and described as flows (Van Woensel 2003), the slope of the speed-flow relationship will be typically negative. Therefore, only the descending part of the queueing models will be used to be compared with the simulation results. For each queueing model the parameter setting that minimizes the Theil coefficient is chosen.

An inherent feature of the queueing models used, concerns the limited flow range: the models are confronted with a maximum flow for which an exact speed can be calculated (due to the restricted setting of the parameters). It is therefore feasible that although a certain flow occurred in the simulation, this flow cannot generate a speed-flow profile in some queueing models due to the limiting behavior of the parameters. Consequently, in the former situation the Theil inequality coefficient cannot be calculated, and such limited flow models will accordingly be considered as irrelevant for purposes of our study.

5 State dependency functions

Based on the work of Jain and MacGregor Smith (1997), Vandaele et al. (2000) developed a state-dependent $GI/G/1$ model: instead of using a fixed service rate, the service rate is a function of the traffic flow. Vehicles are served at a certain rate dependent upon the number of vehicles in the system. In the state-dependent $GI/G/1$ model the nominal speed v_f is assumed to be a function of the traffic flow or $v_f = f(q)$. Concerning the state-dependent functions, Jain and MacGregor Smith (1997) presented two possible specifications: the linear and the exponential function. In this subsection, the application of state-dependent functions will be extended to $M/M/1$ and $M/G/1$ queueing models. Since the linear and exponential functions might be too simple as a representation of the state-dependent property of the nominal speed, the state-dependent specification will be extended to other alternatives.

Table 2. The different state-dependent functions

State-dependent function	Equation
Linear	$(v_f)_q = v_f \left[\frac{q_{\max} + 1 - q}{q_{\max}} \right]$
Exponential	$(v_f)_q = v_f e^{-\alpha \frac{q}{q_{\max}}}, \text{ with } \alpha > 0$
Elliptic	$(v_f)_q = \sqrt{\left(1 - \frac{q^2}{q_{\max}^2}\right)} v_f$
Gaussian	$(v_f)_q = v_f e^{(-\alpha q^2/q_{\max}^2)}, \text{ with } \alpha > 0$
Quadratic	$(v_f)_q = v_f + \alpha q^2 + \beta q$

A state-dependent function is subject to three self-explanatory conditions. First, the function should be decreasing since an increase in the number of vehicles provokes a decreasing free flow speed. Second, a saturated traffic situation leads to a free flow speed equal to zero. Third, if a single vehicle is present, the free flow speed equals the free flow speed used in the non state-dependent queueing models. The former conditions are written as follows:

$$\begin{aligned}
 \forall q : \frac{\partial f(q)}{\partial q} &< 0 \\
 f(q_{\max}) &\cong 0 \\
 f(0) &\cong v_f
 \end{aligned} \tag{12}$$

Since it is a priori quasi impossible to determine analytically a correct specification, we put forward functions which can easily be modified by a change of the parameter(s): the quadratic state-dependent function and the Gaussian state-dependent function. Since the exponential function (Jain and MacGregor Smith 1997) has an outspoken convex nature, an elliptic state-dependent function (which is concave) will be subject to our study as well. The state-dependent functions that are subject to evaluation in this paper are presented in Table 2. The functions mentioned in Table 2 are (except for the quadratic function) obviously in accordance with the state-dependent conditions described in equation (12). In order to set the quadratic function to a useful state-dependent function, additional parameter

restrictions are imposed:

$$\begin{aligned}\alpha &\in \left[-\frac{v_f}{q_{\max}^2}, \frac{v_f}{q_{\max}^2} \right] \\ \beta &= \frac{-\alpha q_{\max}^2 - v_f}{q_{\max}}\end{aligned}\tag{13}$$

The proof for restrictions (13) will not be discussed here in order to save space, and can be found in the technical report by Wuyts et al. (2004).

6 Results and insights

In this section, the results of the evaluation in terms of the Theil inequality coefficient is presented. The section ends with different insights based on the results.

6.1 Results

The best Theil inequality coefficient for every queueing model with matching parameter setting is shown in Table 3. A first observation concerns the good performance of the state-dependent queueing models: in general the state-dependent queueing models outperform the non state-dependent models (worse results are only observed in the linear state-dependent case). Secondly, all the $GI/G/1$ models (whether state-dependent or not) are of no real practical use since (i) the Theil inequality coefficients are relatively high and (ii) the ideal value of k_j is mostly beyond an acceptable value. A remark should be made upon the coefficient of variation of the service time in the $M/G/1$ case with linear state-dependency: it appears that the mentioned coefficient does not significantly influence the Theil coefficient for this specific parameter setting (i.e. $k_j \rightarrow \infty$). Furthermore, the Theil coefficient is similar for both the $M/G/1$ model with linear state-dependency and the $M/M/1$ model with linear state-dependency.

The Theil decomposition in respectively the bias proportion, the variance proportion and the covariance proportion is given in Table 4.

In order to verify which model performs best, the set of queueing models is further reduced to those with an acceptable difference for the k_j values (each time, the k_j value that results in the best Theil coefficient is compared with $k_{j,t.v.}$, i.e. the value for the maximum density that is given by the respective queueing model). The maximum allowed difference is set to 40%. In this way only 9 queueing models remain: $M/M/1$, $M/M/1$ (exponential), $M/M/1$ (elliptic), $M/M/1$ (Gaussian), $M/G/1$, $M/G/1$ (exponential), $M/G/1$ (elliptic), $M/G/1$ (Gaussian) and $GI/G/1$ (elliptic).

A next step concerns the Theil inequality coefficient and its decomposition: ideally the covariance proportion (i.e. the unsystematic errors) should be equal to

Table 3. The best Theil coefficients with corresponding parameter settings

Queueing Model	<i>Theil</i>	c_a	c_s	k_j	v_f	α	$k_{j,t.v.}^{***}$	$\frac{k_j - k_{j,t.v.}}{k_{j,t.v.}}$
<i>M/M/1</i>	0.0517	-	-	174	107	-	226	-23.0%
<i>M/M/1, linear</i>	0.0556	-	-	∞	137	-	176	$+\infty$
<i>M/M/1, exponential</i>	0.0112	-	-	228	120	0.50	201	+13.4%
<i>M/M/1, elliptic</i>	0.0395	-	-	247	118	-	205	+20.5%
<i>M/M/1, Gaussian</i>	0.0169	-	-	248	116	0.73	208	+19.2%
<i>M/M/1, quadratic</i>	0.0273	-	-	394	116	$-0.32 \times 10^{-5**}$	208	+89.4%
<i>M/G/1</i>	0.0510	-	1.10	178	110	-	161	+10.6%
<i>M/G/1, linear</i>	0.0556	-	<i>NI*</i>	∞	137	-	129	$+\infty$
<i>M/G/1, exponential</i>	0.0130	-	0.40	194	121	0.63	155	+25.2%
<i>M/G/1, elliptic</i>	0.0231	-	0.90	250	111	-	207	+20.8
<i>M/G/1, Gaussian</i>	0.0097	-	0.15	220	113	1.00	157	+40.1%
<i>M/G/1, quadratic</i>	0.0315	-	0.05	280	119	$-0.33 \times 10^{-5**}$	148	+89.2%
<i>GI/G/1</i>	0.0693	0.90	0.50	135	105	-	57	+136.8%
<i>GI/G/1, linear</i>	0.0592	0.10	0.10	163	140	-	43	+279.1%
<i>GI/G/1, exponential</i>	0.0347	0.10	0.10	80	123	0.82	49	+63.3%
<i>GI/G/1, elliptic</i>	0.0630	0.13	0.13	82	102	-	59	+39.0%
<i>GI/G/1, Gaussian</i>	0.0359	0.10	0.10	97	120	1.30	50	+94.0%
<i>GI/G/1, quadratic</i>	0.0432	0.10	0.10	106	120	$-0.33 \times 10^{-5**}$	50	+112.0%

*The variable has no influence on the value of the Theil inequality coefficient

**The minimum allowed value for α

*** $k_{j,t.v.}$ is the theoretical value for k_j for a certain queueing model

one. If those models are deleted that have a covariance proportion significantly different from one, only one model remains: the *M/G/1* with Gaussian state-dependency.

6.2 Insights

Since the *M/G/1* queueing model with Gaussian state-dependency has both the lowest Theil inequality coefficient and the highest covariance proportion, this model can be considered as the most accurate approximation of real-life situation.

Based on the above results, one can conclude that models with Poisson arrivals perform better than the ones with general arrivals. This conclusion is most likely partially due to the assumed exponential inter-arrival process in the simulation. However, the Kruskal-Wallis test shows the appropriateness of the exponential distribution to approximate the real-life inter-arrival process. Consequently, the choice for this distribution cannot be considered as arbitrary.

Compared to Van Woensel and Vandaele (2005), the results are less prone to errors due to outliers. This can be seen in the reduced overall Theil coefficient in the tables. When looking at the decomposition, it seems that in the current paper the variance component, has a higher weight compared to the ones reported

Table 4. Theil decomposition for the best parameter values

	<i>Theil</i>	<i>Bias</i>	<i>Var</i>	<i>Cov</i>
<i>M/M/1</i>	0.0517	0.0703	0.8969	0.0329
<i>M/M/1, linear</i>	0.0556	0.0633	0.8430	0.0937
<i>M/M/1, exponential</i>	0.0112	0.0214	0.3447	0.6339
<i>M/M/1, elliptic</i>	0.0395	0.8077	0.0193	0.1730
<i>M/M/1, Gaussian</i>	0.0169	0.6831	0.0370	0.2799
<i>M/M/1, quadratic</i>	0.0273	0.0197	0.6480	0.3323
<i>M/G/1</i>	0.0510	0.2128	0.7555	0.0317
<i>M/G/1, linear</i>	0.0556	0.0633	0.8430	0.0937
<i>M/G/1, exponential</i>	0.0130	0.0235	0.2505	0.7260
<i>M/G/1, elliptic</i>	0.0231	0.1248	0.3549	0.5203
<i>M/G/1, Gaussian</i>	0.0097	0.0163	0.0613	0.9224
<i>M/G/1, quadratic</i>	0.0315	0.3651	0.3619	0.2732
<i>GI/G/1</i>	0.0693	0.1031	0.8744	0.0225
<i>GI/G/1, linear</i>	0.0592	0.0622	0.8537	0.0841
<i>GI/G/1, exponential</i>	0.0347	0.0530	0.5830	0.3640
<i>GI/G/1, elliptic</i>	0.0630	0.0295	0.9071	0.0634
<i>GI/G/1, Gaussian</i>	0.0359	0.9272	0.0284	0.0444
<i>GI/G/1, quadratic</i>	0.0432	0.8710	0.0003	0.1287

in Van Woensel and Vandaele (2005). Apparently, the queueing models seem to generate estimates which are unbiased but not necessarily handles the variance component properly. Probably, this is also related to the assumptions regarding the input distributions for the simulation.

Finally, in their paper, Cruz et al. (2005) stated that one of the most universal and significant applications where $M/G/C/C$ state-dependent queueing networks occur, include vehicular traffic flows. This statement is confirmed by the results in the current paper, as the state-dependent queueing models describe the traffic process best.

7 Conclusion

In this paper, it is shown that the queueing models can adequately be used to represent traffic flows. Next to this new state-dependency functions outperforming the ones reported in the literature, were developed.

State-dependent queueing models were evaluated in terms of their fit with a traffic flow simulation. First, a simulation methodology based on the work of Preston White (1999) was developed to estimate an accurate speed-flow relationship when only flow data are available and which is less prone to outliers. Testing the different queueing models and implementing the previously reported and some new state-dependent functions revealed the poor performance of the $GI/G/1$ model and the inaccuracy of the linear state-dependent function. Overall, one queueing model

appeared to outperform all models in terms of the Theil inequality coefficient: the $M/G/1$ queueing model with Gaussian state-dependency.

Future research splits up in two directions: the first research direction involves the simulation methodology and the second one tackles the queueing approach to traffic flows. The simulation methodology proved to be a powerful tool to evaluate and validate the parameters of the queueing models (and of an analytical model in general). A crucial factor in this validation was the Theil coefficient which allowed for a detailed quantitative comparison of the queueing results with simulation output. Based on the results presented in this paper, it can be claimed that the queueing approach is a representative model of the traffic process. As such, this opens the door for interesting new applications where analytical queueing models can advance the research. One could think here about incorporating the traffic flow function into Vehicle Routing Problems (dynamic routing problems) or into Shortest Path Problems (e.g. useful in car navigation tools).

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