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**Probability Models for Duration :
the Data don't Tell the Whole Story**

by

Marc VANHUELE

Marnik G. DEKIMPE

Sunil SHARMA & Donald G. MORRISON

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Marc Vanhuele

University of California, Los Angeles

Marnik G. Dekimpe

Catholic University Leuven

Sunil Sharma, Donald G. Morrison

University of California, Los Angeles

send correspondence to Donald G. Morrison, Anderson Graduate School of
Management, UCLA, 405 Hilgard Ave, Los Angeles CA 90024.

Abstract

Probability models for duration have been applied to a wide range of individual-level and organizational phenomena. Seemingly similar models may, however, produce different results. Building on previous work by Morrison & Schmittlein (1980) and Schmittlein and Morrison (1983), we propose a hierarchy of models and show how an analysis of the results *across* models helps explain why different studies came to different conclusions. This analysis also leads to substantive insights that even the most complete model alone does not provide.

Introduction

Popular-press articles often refer to the seventh year of a marriage as the critical one in terms of the probability of divorce. A recent USA Today article, summarizing divorce statistics of over 50 countries, concludes that "there may be some truth to the seven-year itch, but the scratching starts in the fourth year" (Hellmich, 1990). Although methodological details are not discussed, nothing in the article suggests that the researcher went beyond interpreting simple frequency tables of official divorce statistics. Such crude analysis cannot shed light on, and may even mask, the true effect of time on marital dissolution. Because of the lack of accuracy and detail in aggregate statistics, academic researchers have used panel and survey data to unravel the dynamics of divorce. The Current Population Survey of 1985 has been the prime source for recent work in this area (e.g. Heaton, 1990, 1991; Martin & Bumpass, 1989; Morgan & Rindfuss, 1985).

The evolution in the empirical divorce rate (also called the empirical hazard of marital dissolution) for a sub-sample of this survey is shown in Figure 1. This statistic is calculated as the number of couples in the sample that separated in a given duration interval divided by the total number that were at "risk" in that interval. For instance, of all couples observed for at least three years, about 4.8% separated during the third year of their marriage. This picture of the time dependence of divorce differs from the one described in the previous paragraph in that the divorce rate peaks in the third year, but then drops below its initial level after the fourth year. Except for the slight increases in

years eight and ten, the longer the partners stay together after the seemingly critical third year, the longer their expected *additional* married life.

 Insert Figure 1 here

This pattern seems to confirm the "adjustment theory" of divorce (Heaton, Albrecht & Martin, 1985), which states that factors responsible for divorce have a relatively greater impact in the early years of the marriage. Past a critical point (say the third year) the relationship stabilizes as the partners adjust to their situation. The data also seem to corroborate an explanation based on economic theory (Becker, Landes & Michael, 1977). Knowledge about the characteristics of a spouse is initially incomplete and uncertain. During the first few years after the wedding, as the couple acquires more information about the (mis)match, the probability of divorce is the highest. On the other hand, investments in marriage-specific assets (e.g. non-transferrable skills, children) eventually accumulate over time, and this raises both the expected gains from the relationship and the expected costs of dissolution. According to Figure 1, the latter effect is reflected in the decreasing probability of divorce over time after the initial learning period of three years (see also Booth, Johnson & White, 1986).

Some years ago, this journal published two articles that warned against the kind of inferences we just made (Morrison & Schmittlein, 1980; Schmittlein & Morrison, 1983). The aggregate pattern in the hazard rate need not reflect the true underlying individual-level behavior. For instance, even if every single marriage in the sample has a constant

divorce rate over time, heterogeneity in the individual rates would cause the aggregate hazard to decrease. The observed inertia can therefore be a spurious effect due to differences in the individual rates, and proper analysis of the data may reject the behavioral interpretations suggested above. Schmittlein and Morrison (1983) show that the pattern observed in Figure 1 could arise from two scenarios:

- a homogeneous population in which each marriage is characterized by an identical non-monotonic pattern in its divorce probabilities¹, or
- a heterogeneous population (each marriage has a different mean divorce probability) where the individual hazards are increasing over time.

They present a detailed theoretical analysis of these effects in a probability-mixture framework that allows the separation of individual time dependence from the spurious effects of heterogeneity (see also Lancaster, 1979; Lancaster & Nickell, 1980). Their starting point is a probabilistic representation of the individual process. A constant hazard rate corresponds to an exponential duration distribution. A Weibull distribution is used for monotonically increasing and decreasing rates. By letting the rate parameter of these distributions vary according to a Gamma distribution, a broad range of heterogeneity patterns across the population is captured. The parameters of the resulting mixture distributions are shown to reflect the degree of heterogeneity and the trend in the individual hazard rate.

This article extends Morrison and Schmittlein's analysis in two important ways:

- the inclusion of covariates, and
- the modeling of non-monotonic time dependence at the individual level.

In many duration studies, information on some characteristics of the individuals being analyzed is available. By introducing these traits as covariates in a probability-mixture model, one can assess their impact on the hazard rate. At the same time, the mixing distribution can still take care of the remaining heterogeneity caused by differences in unobserved factors.

A second extension concerns the time dependence of individual marriages. The models in Schmittlein and Morrison (1983) only allow for constant and monotonically increasing or decreasing rates. We utilize a nonparametric extension that can capture any pattern in the individual hazard rates (see Kalbfleisch & Prentice, 1978; Kiefer, 1988; Meyer, 1990). This is especially important when analyzing marriage durations: the previously discussed adjustment theory suggests a non-monotonic pattern, and it is quite possible that different factors become important at different points in the marriage (as when the children leave home), which may cause a multi-modal hazard function.

Most duration models in the divorce literature use parametric hazard specifications that, if misspecified, lead to inconsistent estimates of both the covariate effects and the time dependence of the hazard (for a discussion see Meyer, 1990). An advantage of our non-parametric specification of the baseline hazard is that it allows consistent estimates of the effect of explanatory variables for arbitrary true underlying hazards. Further, the approach is flexible enough to allow for unobserved heterogeneity without any increase in computational complexity. Finally, our data on marriage durations are grouped into annual intervals. We explicitly model the grouping in the data which, if not taken into account, may also lead to inconsistent estimates (see

Sharma, 1992).

This paper presents a systematic comparison and critical evaluation of a hierarchy of alternative models. The aim is to disentangle the effect of time from that of the available covariates while accounting for the impact of unobservables. Probability models of duration have been applied in a variety of contexts to explain event durations and transition probabilities of both individuals (e.g. unemployment; family transitions such as marriage, birth, divorce and remarriage; purchase timing) and organizations (e.g. strikes, company failures, wars). Our analyses lead to a better understanding of how the specification of these probability duration models may actually drive the empirical results. Illustrating this on divorce data is appealing: not only have many different behavioral theories about the phenomenon been proposed, formal modeling approaches have also resulted in conflicting empirical findings, even when applied to the same data set. Our model comparisons explain these contradictory findings in previous studies. We show how an inspection of the results across different models gives better and more complete insights into the duration phenomenon of interest.

The paper is organized as follows. The second section describes the data and contains a discussion about the effects of the covariates. The next section explains in detail the empirical models used. The following section discusses the estimation results. The final section presents some concluding remarks.

Data Description and Hypotheses

Sample Selection

Its large sample size coupled with detailed information about marital histories has made the June 1985 Current Population Survey a popular source for research on divorce (e.g. Heaton, 1990, 1991; Martin & Bumpass, 1989; Morgan & Rindfuss, 1985). In the survey, women fifteen years of age or older ($n=42,178$), married at least once, were asked for their marital and fertility histories. In this article we only consider first marriages. The data are retrospective in nature and may therefore be subject to recall error. To minimize this effect, the sample was restricted to women married during or after 1970. Also, the analysis only considers black and white respondents to allow a comparison with previous studies. This leaves us with a total of 15,258 observations.

At the moment of the interview, the respondent was either living with her first husband or was separated, divorced, or widowed. First marriages that were intact in June 1985 or that ended by death of the partner are right censored, i.e. for these marriages the available information on their duration is that the match lasted *at least* a certain number of years. It is important that these right-censored observations are taken into account. Ignoring right-censored marriages, as was for instance done in Maneker and Rankin (1985) and Rankin and Maneker (1985), would not only result in discarding much of the data (75% in our case), but also lead to biased estimates of both the true divorce rate and the impact of the covariates of interest (Heckman & Singer, 1984a, 1984b).

The date when a husband and wife started to live separately is used as the

dissolution date, regardless of whether a legal divorce took place subsequently.²

Variations in the length of the legal process and the fact that people can terminate their marriage by separation without a later divorce decree makes the divorce date a less valid measure.³

Covariates

Most studies on divorce focus on how failure rates vary across marriages with differing characteristics (see White, 1990 for a review). The covariates used in this study are:

- marriage cohort,
- race, age at marriage, and education level of the bride, and
- the number of children and the timing of their respective births.

We give a brief summary of previous findings on each of these covariates and explain how they were coded for our analysis.

Race. A consistent finding in divorce studies is that black Americans have a higher divorce rate than white Americans, even if one controls for factors such as socio-economic status, age at marriage, and fertility (see e.g. Teachman, 1982; Lehrer, 1988).

Marriage cohort. This factor is introduced as a proxy variable for the socio-economic determinants of divorce. Changing gender roles and cultural values, the diminishing importance of the family as a social institution, improved contraceptive methods, legal shifts, etc. all contribute to differences in the cohorts' divorce rates (Michael, 1988; White, 1990). Observations were divided into three cohorts based on the marriage dates (see Table 1).

Age at marriage. Early marriages have shorter expected durations (see e.g. Lehrer, 1988; Martin & Bumpass, 1989). Two explanations for the negative effect on marital stability of an early marriage are the poor role performance by youngsters (Booth & Edwards, 1985) and the reduced length of the search process for the "ideal partner" (Lehrer, 1988). To allow for the nonlinear effects that are typically found for age at marriage, this continuous variable was recoded in four age categories.

Education. This variable stands for a number of factors ranging from interpersonal skills to the economic situation of the family. Higher education is normally associated with a lower separation rate (see Martin & Bumpass, 1989). On the other hand, one could argue that higher education gives the woman more opportunities to make it on her own when her marriage is unsatisfactory, what could lead to a negative relationship between education and marital stability (Heaton, 1991). Hence, the impact of education may not be clear a priori. Again, the continuous variable is discretized to allow for a non-linear effect.

Children. Previous studies (e.g. Heaton, 1990) found that the likelihood of separation or divorce declines after the birth of each of the first three children. Each subsequent child has a smaller effect, but the fourth child has no extra impact. Following Heaton, we not only keep track of the number of children in each marriage but also of when they were born, and therefore these covariates are time-dependent.

Premarital child. Marriages that start off with a child are less stable. Several factors may be responsible for this effect. The marriage may have been contracted to legitimize the birth and therefore not benefit from a firm bond and good match between

the partners. The combined strains of having to adjust to marital life and raising the child also play a role. These factors may be compounded by the fact that the husband sometimes is not the biological father (see Morgan & Rindfuss, 1985; Martin & Bumpass, 1989).

Table 1 presents the different categories for each characteristic and gives the percentage of observations in each group.

Insert Table 1 here

Time Dependence. Different studies give support to at least four different relationships between the probability of getting a divorce and the length of the marriage:

- monotonically decreasing (Gonul, 1989; Heaton, 1991), implying that the longer one is married, the less likely the marriage will end in the coming period,
- monotonically increasing (Peters, 1988; Gonul, 1989),
- first increasing and then decreasing (Becker et al., 1977; Heaton, 1990, 1991).
- no dependence (Peters, 1988)⁴.

A procedure to model the time dependence of marriage dissolutions should be flexible enough to represent a variety of time dependencies. Almost all divorce studies with a time-varying dissolution rate assume a Weibull baseline hazard (e.g. Gonul, 1989; Peters, 1988; Teachman, 1982)⁵. The Weibull hazard, however, either increases or decreases monotonically. To detect the possible existence of more complex patterns in the evolution of the hazard rate, we use a nonparametric method. The time dependence

of the dissolution probabilities is captured by adding time dummy variables to the vector of covariates. This procedure is discussed in more detail in a following section. The basic idea, however, is that we assume that the hazard remains constant within each year, but can vary across different years. There were not enough observations with completed durations to obtain stable coefficient estimates for the dummy variables associated with the longest durations in the sample. For instance, we had no separations in 1985 (before June, the interview date) for couples married in 1970. Years 13 to 15 were therefore combined in the 13+ category.

Statistical Models for Marriage Durations

First, we estimate the exponential baseline model that implies that all marriages have the same constant rate of dissolution. This model does not include covariates, and does not make adjustments for unobserved heterogeneity. Then, this simple model is extended to include (time-varying) explanatory variables. Next, we add time-varying dummy variables to allow for a wide variety of time-dependent divorce patterns. Finally, we show how unobserved heterogeneity can be incorporated into our analysis.

Let T denote the random duration of the marriage with probability density function $f(t, \lambda)$, cumulative distribution function $F(t, \lambda)$ and hazard function $h(t, \lambda)$. For the exponential model these are given by:

$$\begin{aligned}
f(t;\lambda) &= \lambda e^{-\lambda t}, & t > 0; \lambda > 0 \\
F(t;\lambda) &= 1 - e^{-\lambda t}, \\
h(t;\lambda) &= \lambda.
\end{aligned} \tag{1}$$

To account for the discrete nature of our data, let the (annual) grouping intervals be $[t_{k-1}, t_k)$, $k=1,2,\dots,m+1$, $t_0=0$, $t_{m+1}=\infty$, with divorce in the interval $[t_{k-1}, t_k)$ recorded as t_k . *Note that these intervals do not refer to calendar time, but to the time from the wedding date for each marriage.* The contribution to the likelihood function of a marriage that lasted t_i years is $S(t_{i-1};\lambda) - S(t_i;\lambda)$, where the survivor function $S(t_i;\lambda)=1-F(t_i;\lambda)$ denotes the probability that the marriage lasts *at least* t_i periods. The contribution to the likelihood of an observation *censored* in interval $[t_{i-1}, t_i)$ is given by $S(t_{i-1};\lambda)$, so we assume that censoring takes place at the beginning of the interval. Clearly some such assumption is required given the grouping of the data. The likelihood contribution of the i -th marriage can therefore be written as

$$L_i(t_i|\lambda) = [e^{-\lambda(t_i-1)} - e^{-\lambda t_i}]^{1-d_i} [e^{-\lambda(t_i-1)}]^{d_i}, \tag{2}$$

where t_i = the observed number of years for the i -th marriage (i.e. until the divorce, or until the censoring date),

$$\begin{aligned}
d_i &= 1 && \text{if the marriage is censored,} \\
&= 0 && \text{if the marriage ends in a divorce or separation.}
\end{aligned}$$

The log-likelihood for N marriages (which are all assumed to have the same mean divorce rate λ) given by

$$LL = \sum_{i=1}^N [(1 - d_i) \log(e^{-\lambda(t_i-1)} - e^{-\lambda t_i}) - d_i \lambda(t_i-1)], \quad (3)$$

is maximized to estimate λ .

Including Covariates

To incorporate covariates, we use the Cox (1972) formulation and write the divorce rate of marriage i in period t as⁶

$$\lambda_i(t) = \lambda_0 e^{b' X_i(t)}, \quad (4)$$

where λ_0 = the baseline divorce rate,

$X_i(t)$ = a vector of (possibly) time-varying covariates.

λ_0 is the divorce rate of the base group, that consists of those marriages for which all covariates are zero (remember that the explanatory variables in our study are coded as 0-1 dummy variables). A description of the base group is given in Table 1. The interpretation of the b -coefficients is straightforward. Positive coefficients imply that an increase in the value of the covariate augments the hazard of marriage dissolution. Specifically, when the j -th covariate changes by one unit, the conditional divorce probability changes by $[\exp(b_j)-1] * 100$ percent.

Time dependence is implicitly incorporated in Eq. (4) through the time-varying covariates for the number of children. We allow for the differential impact of each child by introducing four time-varying dummy variables, that are initially set to zero. When the first child is born, its dummy variable (CHILD1 in our notation) takes the value one; it is

equal to this value until the second child is born. At that point in time, dummy CHILD2 becomes one and CHILD1 becomes zero. This is illustrated in Table 2 for a woman who gets a child in years 2, 4 and 7.

 Insert Table 2 here

This procedure is applied to account for the effect of children, up to the fourth child.

The survivor function associated with Eq. (4) is (Ross, 1980)

$$S_i(t) = e^{-\theta_i(t)}, \quad (5)$$

where

$$\theta_i(t) = \int_0^t \lambda_i(u) du. \quad (6)$$

$\theta_i(t)$ is referred to as the integrated hazard. We assume that the covariates remain constant *within* each given period (i.e. a year in our case), but they can change from period to period. Given this simplifying assumption, Eq. (6) can be rewritten as

$$\begin{aligned}
\theta_i(t) &= \int_0^1 \lambda_i(1) du + \int_1^2 \lambda_i(2) du + \dots + \int_{t-1}^t \lambda_i(t) du \\
&= \lambda_i(1) + \lambda_i(2) + \dots + \lambda_i(t) \\
&= \lambda_0 e^{b X_i(1)} + \lambda_0 e^{b X_i(2)} + \dots + \lambda_0 e^{b X_i(t)} \\
&= \lambda_0 B_i(t), \text{ where } B_i(t) = \sum_{j=1}^t e^{b X_i(j)}
\end{aligned} \tag{7}$$

The log-likelihood for N observations is (see Appendix A):

$$LL = \sum_{i=1}^N \left\{ (1 - d_i) \log [e^{-\lambda_0 B_i(t_i-1)} - e^{-\lambda_0 B_i(t_i)}] - d_i \lambda_0 B_i(t_i-1) \right\}, \tag{8}$$

It is maximized to get estimates of both the baseline divorce rate (λ_0) and the covariate coefficients (b).

Time Dependence

The time dependence of the dissolution probabilities can be assessed by adding a set of time-varying dummy variables $D_i(t)$ to the set of explanatory variables in Eq. (4):

$$\lambda_i(t) = \lambda_0 e^{b X_i(t)} e^{c D_i(t)} \tag{9}$$

A time-varying dummy variable is used for each year of the marriage, *except* for the first year which is used as base year. These dummy variables are created in much the same way as illustrated in Table 2 for the time-varying effect of children. For example, the dummy for the third year of the marriage is always zero, except in year three when it takes a value of one. No dummy variable is used for the first year, because the estimation of both c_1 and λ_0 would result in identification problems. λ_0 should therefore

be interpreted as the divorce rate of the base group in year one. Positive c -coefficients for the other years indicate an increased divorce probability as compared to the first year of the marriage. Positive c -coefficients for the other years indicate an increased divorce probability as compared to the first year of the marriage.

This approach does not require any distributional assumptions on the baseline hazard and we therefore refer to it as the non-parametric baseline model. The only assumption we make is that within a grouping interval the hazard rate remains constant. Intuitively, this procedure can be interpreted as a piece-wise approximation to the underlying continuous baseline hazard, and allows us to estimate and test a wide array of time-dependent divorce patterns:

- the exponential (or constant-hazard) assumption requires that the coefficients of the time-varying dummy variables (c_i) be equal to zero for all $i \geq 2$. This can easily be tested with a likelihood-ratio test.
- the appropriateness of a Weibull baseline hazard as used in Schmittlein and Morrison (1983) can be assessed by checking whether the coefficients are monotonically increasing or decreasing.
- finally, multi-modal patterns can easily be accommodated without added complexity.

The log-likelihood function is still given by Eq. (8), but $B_i(t)$ is now defined as

$$B_i(t) = \sum_{j=1}^r e^{[b X_i(t) + c D_i(t)]} \quad (10)$$

It is easily shown that the resulting likelihood function is a reparameterization of those in

Prentice and Gloeckler (1978) or Han and Hausman (1990).⁷ The implementation of our procedure, however, is very simple as the time dummy variables can be treated in the same way as all the other covariates. Also, the c -coefficients have a straightforward interpretation, in that they reflect the increase or decrease in (conditional) divorce probability relative to the first year of marriage.

Accounting for Unobserved Heterogeneity

It is clear that some of the factors known to influence a marriage's probability of success may not be measurable (e.g. certain personality traits). Also, data on some measurable variables may not be available. Our data set gives no information on the husband it is therefore not possible to study the interplay of the husband's and wife's characteristics (e.g. their age difference) in determining the success of a match. It is well known that not accounting for unobserved heterogeneity may yield a spurious negative duration dependence, as well as estimated coefficients of the explanatory variables that are biased and inconsistent (see Heckman & Singer, 1984a, 1984b, and the discussion in Lancaster, 1990) .

A procedure to account for these unobservable factors is outlined in terms of the model with covariates and time dummy variables for each interval. Other specifications (i.e. without covariates and/or time-varying dummy variables) can then be derived by restricting the appropriate coefficients. Unobserved heterogeneity in the mean divorce rate is accommodated by allowing λ_0 to vary across the population according to a certain (time-invariant) distribution. In other words, marriages may have a different mean divorce probability, even when their observable characteristics are the same.

The likelihood contributions derived above are conditional in the sense that they depend on a specific value of λ_0 . If this parameter varies across individual marriages, the unconditional likelihood contribution for the i -th marriage, which is the relevant one for data analysis under unobserved heterogeneity, is obtained by weighing the conditional likelihood by the relative occurrence of its λ_0 -value. This is specified as

$$L_i(t_i) = \int_0^\infty L_i(t_i|\lambda_0) g(\lambda_0) d\lambda_0, \quad (11)$$

where $g(\cdot)$ is called the (unobservable) mixing distribution. Like Morrison and Schmittlein (1980) and Schmittlein and Morrison (1983), we use the gamma mixing distribution, whose p.d.f. is given by

$$g(\lambda_0; r, a) = \left[\frac{a}{\Gamma(r)} \right] [a \lambda_0]^{r-1} [e^{-a\lambda_0}] \quad \lambda_0, r, a > 0, \quad (12)$$

where $\Gamma(\cdot)$ is the gamma function (see also Meyer, 1990). The gamma mixing distribution, besides being quite flexible, gives a closed-form solution for the log-likelihood function (see Appendix B):

$$LL = \sum_{i=1}^N \log \left\{ (1+d_i) \frac{a^r}{[B_i(t_i-1)+a]^r} - \frac{a^r}{[B_i(t_i-1)+(1-d_i)e^{bX_i(t_i)+cD_i(t_i)+a}]^r} \right\} \quad (13)$$

The mean and variance of the gamma distribution are given by

$$\begin{aligned}
E(\lambda_0) &= \frac{r}{a}, \\
Var(\lambda_0) &= \frac{r}{a^2}.
\end{aligned}
\tag{14}$$

The coefficient of variation "CV", which can be interpreted as measuring the degree of heterogeneity of λ_0 across marriages, therefore reduces to

$$CV = \frac{SD(\lambda_0)}{E(\lambda_0)} = r^{-1/2} \tag{15}$$

giving the r -parameter a straightforward interpretation: a low r -value indicates a large amount of heterogeneity, whereas high values are obtained for (relatively) homogenous populations.

The gamma distribution is quite flexible, and can take on a variety of forms: for $r < 1$, a J shape is obtained, while for $r > 1$ a unimodal form results. However, the gamma distribution cannot take on multi-modal forms. The latter could be captured when using a non-parametric operationalization for the unobserved-heterogeneity component. This approach was adopted in Heaton (1990, 1991), who modeled the baseline hazard parametrically and the unobserved-heterogeneity component nonparametrically. Recent research indicates that the specification of the heterogeneity component may not be as crucial as a flexible specification of the baseline hazard (see Lancaster, 1990; Han & Hausman, 1990; Manton, Vaupel & Stallard, 1986; Ridder, 1986 and Sharma, 1992). Put differently, while it is crucial to model unobserved heterogeneity, the way in which this is done is of less importance, provided that the parametric

specification which is adopted is reasonably flexible. Based on these findings, and given our interest in disentangling the time dependence of marriage dissolutions, we decided to model the baseline hazard nonparametrically (which maximizes the flexibility of this part of the model), and to model the unobserved heterogeneity component parametrically by means of a gamma mixing distribution.

The relationships between the different possible models are summarized in Figure 2.

 Insert Figure 2 here

These models vary along three dimensions:

- the baseline hazard is either constant or can take on any pattern over time; the corresponding models are labeled as *EXP* for exponential and *NONPAR* for nonparametric time dependence,
- covariates are included (*COV*) or excluded,
- unobserved heterogeneity is modeled through a Gamma distribution (*G*), or is unaccounted for.

Empirical Results

We use the most general model, i.e. NONPAR-COV-G, to assess the nature of the time dependence and the effect of the explanatory variables. This model has a non-parametric baseline hazard and allows for both observed and unobserved heterogeneity. The estimates are presented in column 5 of Table 3. The hazard rate of marital dissolution for the base group (marriages between 1970 and 1975, of white women under 18, who did not finish high school, and had no children at the time of the interview) during the first year is 0.075 ($= 0.901/11.969$). This value is simply the mean of the Gamma mixing distribution. The effects of the *covariates* confirm our hypotheses and are similar to what other researchers have found.

Insert Table 3 here

The (conditional) divorce probability of a *black* woman with all other characteristics identical to those of the base group, is 76 percent ($100 * [\exp(0.566) - 1]$) higher than for a white woman. The more recent the marriage, the higher the likelihood of disruption: again using the base group as reference point, people in *cohort* 2 and 3 have a 41 and 90 percent higher hazard rate. *Age at marriage* shows the expected non-linear effect. The separation rate for 18 to 22 year old brides is only 51 percent of that of the youngest group. Marrying between 22 and 30 years of age reduces the disruption probability by 71 percent, but marrying after 30 does not bring a further reduction.

With respect to the woman's *education*, we see a lower separation rate for women

with 13 or more years of education, and an even lower rate for women with at least sixteen years of education (i.e. those who finished college). The mere completion of high school does not have a significant impact. Heaton (1990) also found an unstable parameter for this category (12 years of education), which may be due to the small sample size for that category.

Finally, marriages that begin with a child are more likely to end in separation than the base group. The birth and presence of children after the marriage, however, bring stability to the relationship. Families with one child have only about 68 percent the disruption rate of childless families. For a second child, this drops to 45 percent and for a third to 39 percent of the base rate. The fourth child does not decrease the hazard any further.

The effect of *time* on a couple's divorce probability (as revealed by the coefficients of the time dummy variables) is quite different from that obtained from the aggregate data (see Figure 3). The hazard of individual families belonging to the base group (i.e. childless families) increases with time until the tenth year and then decreases in the eleventh and twelfth year.⁸ Also, the baseline hazard for each of the years two to thirteen is higher than that in the first year --all coefficients on the time dummy variables are positive and significant.

Insert Figure 3 here

These findings are in sharp contrast with the aggregate (empirical) hazard which suggests

a positive duration dependence until year three followed by a negative duration dependence. Our analyses suggest that the observed decline in the empirical hazard after year three is spurious, and an artifact of the sorting effect.

Even though the *baseline* hazard is increasing until year 10, after which it starts to fall, other patterns may be obtained for the hazard function of specific marriages. Indeed, the form of the hazard function is heavily influenced by the birth and presence of children. This is illustrated in Figure 4, where we depict the hazard for four different families. The first family belongs to the base group, and its hazard function equals the estimated baseline hazard. The second family has similar characteristics as the first one, except that it has children in the fourth, sixth and seventh year of the marriage. The last two families have the modal value on each of the time-invariant covariates. One of these modal families remains childless, while the other has four children, born between the second and fifth year of the marriage.

 Insert Figure 4 here

Even though the base-group families start with a 25% higher hazard, the arrival of their first child during year four immediately reduces the hazard to 85% of that of a childless "modal" family. Note that we use a proportional hazard model. The absolute drop in the hazard in year four for the base-group family is therefore greater than that caused by the first birth in the "modal" family during year 2, but proportionally the reductions in the hazard rate are the same. These four examples illustrate that children are an important

stabilizing factor in marital relations. Also, their arrival may induce a non-monotonic hazard at the individual level, and the time when the hazard reaches a maximum varies with the timing and number of children.

The previous discussion focused on the NONPAR-COV-G model, since this specification allows for a flexible baseline hazard, and incorporates both observed- and unobserved- heterogeneity components. In what follows, we argue that additional insights into the underlying dynamics of probability duration models can be obtained from comparing these results with the findings emerging from the other, more restricted, model specifications.

Model Comparisons

Covariates. The coefficients of the covariates are remarkably robust across different model specifications, and the qualitative impact of the covariates is clearly the same in all the models. However, NONPAR-COV-G yields covariate effects that are larger and statistically more significant compared to NONPAR-COV. Parameterizing unobserved heterogeneity apparently eliminates (some of) the attenuating effects of omitted variables, model misspecification and measurement error. No such increase in the effect of the covariates is observed for the models based on the exponential baseline (compare EXP-COV with EXP-COV-G). In this case, the gamma mixing distribution does not pick up a lot of unobserved heterogeneity (notice in this respect the large r value), and therefore its inclusion/omission should not affect the other parameters in the model. We explain this phenomenon in more detail in a next section.

Time Dependence. Figure 5 plots the estimated baseline hazard for five models: NONPAR, NONPAR-COV, NONPAR-G, NONPAR-COV-G and a version of NONPAR-COV which does not contain the dummy variables for children (NONPAR-COV-NC). NONPAR is just the Kaplan-Meier estimator of the (aggregate) empirical hazard (depicted in Figure 1), and is used here to compare the aggregate with the individual hazard rates.

 Insert Figure 5 about here

In interpreting Figure 5, one should keep in mind that not all curves refer to the same group of marriages. Curves for models without covariates represent the separation pattern for an *average* marriage in the total sample. Curves for models with covariates are for that subgroup in the sample defined as the base group. Changing this reference group would preserve the pattern of time dependence but would make it more or less pronounced, depending on whether the reference group becomes more or less risk prone (remember that all effects, also those of time, are proportional; see Eq. 9). However, inspection of Figure 5 shows that much more is going on than a mere change of reference base. As one corrects for observed and unobserved heterogeneity, the spurious negative duration dependence generated by the sorting effect decreases and we begin to see a hazard that increases until the tenth year and then starts to decrease⁹. The lowest curve (i.e. the strongest negative time dependence) corresponds to the model that includes neither covariates nor an unobserved-heterogeneity component (NONPAR).

The top curve which shows an inverted U pattern is associated with the NONPAR-COV-G model that explicitly models the impact of observable covariates and that also accounts for unobserved heterogeneity. The intermediary curves correspond to models that capture only one of the heterogeneity components: i.e. observed (NONPAR-COV) or unobserved (NONPAR-G).

A similar conclusion was obtained by Gonul (1989), who used a Weibull model to specify the baseline hazard. When unobserved heterogeneity was not controlled for, a monotonically decreasing hazard function was obtained. After controlling for unobserved heterogeneity, a positive time dependence was found. Similarly, Peters (1989) finds a positive time dependence when working with a Weibull specification for the baseline hazard, after controlling for several observable covariates. Peters (1989), however, makes a case for a more flexible specification of the baseline hazard, since she expected a non-monotonic pattern. Our nonparametric specification offers this flexibility, but nevertheless a positive time dependence is found for the first ten years of the marriage. Of course, if sufficient data were available on longer marriages, it is entirely possible that the hazard would start increasing again, especially around the time when children leave home.

The previous discussion clearly illustrates the interrelationship between the obtained baseline hazard and the other components of the model, i.e. whether unobserved heterogeneity is accounted for, and whether observable covariates are included in the model. This interrelationship is analyzed in more detail in the following paragraph.

Heterogeneity. The r parameter of the gamma mixing models reflects the degree of heterogeneity in separation rates across marriages. High values indicate low heterogeneity in the divorce rates and mean marriage durations. The estimated r -values indicate a considerable degree of heterogeneity in NONPAR-COV-G and EXP-G, and extreme homogeneity in EXP-COV-G. A priori we expected a low r value for all models, because they all omit factors with a significant impact on divorce (e.g. the characteristics of the husband, and the interaction of the characteristics of the partners). A possible explanation for these large variations in r lies in the interrelationship between this parameter, the estimated baseline hazard and the coefficients of the covariates.

Our models basically try to explain the sorting effect that is responsible for the empirical hazard rate of Figure 1. This is done with three different instruments:

- the inclusion of explanatory variables,
- the parameterization of unobservable heterogeneity,
- the formulation of a flexible baseline hazard.

In EXP-G, the gamma mixing distribution is the only "tool" to fit the downward curve. A small r value is indeed obtained, reflecting a large degree of unobserved heterogeneity. In the EXP-COV model, the covariates pick up as much as possible of the variability in the separation rates and in this way fit the downward overall pattern. The covariates in EXP-COV and EXP-COV-G are apparently sufficient to capture the downward-sloping part of the curve, and r becomes high to reflect the redundancy of the gamma mixing distribution. *Thus, the nominal value of r is no longer an absolute indicator of the amount of heterogeneity, and a high r value should not be interpreted as an indication*

that all important factors have been included in the model.

Similarly, the time dummy variables pick up both individual-level time dependence and unobserved heterogeneity. The model that includes time dummy variables and all available covariates except children (NONPAR-COV-NC) elucidates this interaction between duration dependence and unobserved heterogeneity. The last column of Table 3 shows that the negative effect of children on marital dissolution is now captured by the time dummy variables whose coefficients become smaller¹⁰ and even negative for larger durations (i.e. when families have more children). In other words, the estimated baseline hazard reflects not only pure time dependence, but also reflects the impact of omitted variables (i.e. the dummy variables for the presence/absence of children).

A similar phenomenon is expected to occur in the most general NONPAR-COV-G model. When the included covariates and the gamma mixing distribution cannot fully capture all heterogeneity in the population's mean divorce rate, then part of the latter will be reflected in the estimated baseline hazard, which will be lower than the true baseline that would be obtained in the absence of any sorting effect. This situation may arise because of a variety of factors. First, the gamma distribution, although quite flexible, cannot have a multi-modal shape. Second, the unobserved heterogeneity was assumed to be time-invariant: the current operationalization does not allow for changes over time in the underlying distribution. Third, the same heterogeneity distribution is assumed to underlie every single sub-group or stratum. It may well be that the amount of unobserved heterogeneity is, for instance, much smaller for women with a premarital child than for the group of women who do not have children when marrying. Violations

of any of these assumptions imply that the gamma-mixing distribution will be unable to capture all unobserved heterogeneity, and that part of it will be picked up by the time-varying dummy variables. This has two important implications:

- the magnitude of the individual-level time dependence cannot be quantified; only qualitative insights on the direction of this dependence can be obtained. Indeed, the positive time dependence in the NONPAR-COV-G model is certainly not due to the sorting effect. However, we cannot determine whether the "true" baseline hazard is even steeper.
- the r parameter is no longer the sole indicator of the amount of unobserved heterogeneity. Instead, it is the interplay of r and the twelve time dummy variables that now describes the amount of heterogeneity.

In other words, the parameters of the mixing distribution and of the time-varying dummies can be interpreted as two complementary forces. Note in this respect that the more ways a model is given to capture unobserved heterogeneity, the steeper the increase in the hazard rate of the nonparametric models becomes. This is indeed what the dynamics we just explained predict, since the coefficients of the time dummy variables have to account for an ever smaller fraction of the downward trend in the aggregate hazard rate. Note also that if a downward sloping baseline hazard had been obtained, we could not know whether this would reflect a "true" negative time dependence, or would be due to heterogeneity that was not accounted for by the covariates and/or the mixing distribution. Hence, firmer qualitative inferences on the nature of the "true" time dependence are possible when an upward than when a

downward sloping baseline hazard is obtained.

Conclusion

A general model for the analysis of duration phenomena has been presented that extends previous work by Morrison and Schmittlein (1980) and Schmittlein and Morrison (1983) in two ways. First, it allows for non-monotonic time dependencies at the individual level, and second, it explicitly incorporates covariates.

We illustrated our methodology using marriage duration data, and developed an explanation for why previous studies found different individual-level time dependencies. Our main conclusions are not only applicable to marriage durations, but should also be of interest to researchers studying phenomena such as organizational mortality, the length of (un)employment spells, strikes, and auditor-client relationships, In all these areas, non-monotonic time dependencies cannot be excluded a priori (see e.g. Fichman and Levinthal 1991), necessitating a flexible base-line specification, and a reliable quantification of the impact of mediating variables on the probability of the event is crucial in understanding the phenomenon.

We presented a hierarchy of model specifications, and argued that additional insights in the underlying dynamics can be obtained by comparing their respective parameter estimates. We also advanced the following caveats for the interpretation of the parameter estimates in probability models for duration:

- the estimated base-line hazard reflects both the time dependence at the individual level and unobserved heterogeneity not captured by the other

components of the model (the gamma mixing distribution and the observable covariates).

- the magnitude of the r parameter of the gamma mixing distribution can no longer be interpreted as a measure of the amount of unobserved heterogeneity, as could be done in models without covariates.

At first sight, we seem to present a rather grim picture of the usefulness of these models. One should however be careful not to overlook their merits. First, they allow to quantify the impact of covariates, which will often be the main research objective. Regardless of the model, the sign of the coefficients in our results *never* change and the magnitudes stay very similar. Thus we have robust estimates for the effects of our covariates. An advantage of our non-parametric specification of the baseline hazard is that it allows for consistent estimates of the effect of the covariates for arbitrary true underlying hazards. Second, previous research suggests that unobserved heterogeneity should be accounted for to obtain reliable estimates of the covariates, but that the way in which this is done is of less importance than flexibility in the baseline hazard. Our methodology offers maximum flexibility in the baseline specification, and the use of the gamma mixing distribution reduces the sorting effect without adding considerable complexity to the model specification. Finally, qualitative insights in the nature of the true time dependence can still be obtained as long as the estimated baseline hazard is not monotonically decreasing. Indeed, the spurious sorting effect can never cause an observed increase in the baseline hazard.

We therefore conclude that the hazard models discussed in this paper can be very valuable to researchers in the field of organizational behavior and human decision making, provided that one is aware of the aforementioned caveats. In that case, one can say that even though the data do not tell the whole story, they may still tell most of it.

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Appendix A

The Log-Likelihood Function for a Model with Covariates

The contribution to the likelihood function of the i -th marriage is, analogous to the exposition for the exponential base model, given by

$$L_i(t_i|\lambda_0) = [S_i(t_i-1) - S_i(t_i)]^{1-d_i} [S_i(t_i-1)]^{d_i}, \quad (\text{A.1})$$

where $S_i(\cdot)$ indicates the survivor function, and where d_i is a zero-one dummy to indicate censoring. Combining equations (5) and (7), the survivor function becomes

$$S_i(t) = e^{-\lambda_0 \sum_{j=1}^t e^{b' x_{ij}(j)}} = e^{-\lambda_0 B_i(t)}. \quad (\text{A.2})$$

The log-likelihood function for N marriages can therefore be written as

$$LL = \sum_{i=1}^N \{(1-d_i) \log [e^{-\lambda_0 B_i(t_i-1)} - e^{-\lambda_0 B_i(t_i)}] - d_i \lambda_0 B_i(t_i-1)\}. \quad (\text{A.3})$$

Appendix B

The Likelihood Function when Accounting for Unobserved Heterogeneity

The conditional likelihood of marriage i when both covariates and time-varying dummy variables are included is given by (A.3), in which $B_i(t_i)$ is now defined as

$$B_i(t_i) = \sum_{j=1}^r e^{[b X_i(j) + c D_i(j)]}. \quad (\text{B.1})$$

Noting that

$$B_i(t_i) = B_i(t_i-1) + e^{[b X_i(t_i) + c D_i(t_i)]}, \quad (\text{B.2})$$

the conditional likelihood for marriage i can be written as

$$L_i(t_i | \lambda_0) = e^{-\lambda_0 B_i(t_i-1)} \{1 - e^{-\lambda_0 e^{[b X_i(t_i) + c D_i(t_i)]}}\}^{1-d_i}. \quad (\text{B.3})$$

The unconditional likelihood is obtained by weighing the conditional likelihood by the relative occurrence of its λ_0 -value, which is implemented as follows:

$$\begin{aligned}
L_i(t_i) &= \int_0^\infty L_i(t_i | \lambda_0) g(\lambda_0) d\lambda_0, \\
&= \int_0^\infty e^{-\lambda_0 B_i(t_i-1)} [1 - e^{-\lambda_0 e^{b X_i(t_i) + c D_i(t_i)}}]^{1-d_i} g(\lambda_0) d\lambda_0.
\end{aligned} \tag{B.4}$$

Since

$$[1 - e^{-\lambda_0 e^{b X_i(t_i) + c D_i(t_i)}}]^{1-d_i} = (1 + d_i) - [e^{-\lambda_0 e^{b X_i(t_i) + c D_i(t_i)}}]^{1-d_i},$$

(B.4) becomes

$$\int_0^\infty e^{-\lambda_0 B_i(t_i-1)} (1 + d_i) g(\lambda_0) d\lambda_0 - \int_0^\infty e^{-\lambda_0 B_i(t_i-1)} [e^{-\lambda_0 e^{b X_i(t_i) + c D_i(t_i)}}]^{1-d_i} g(\lambda_0) d\lambda_0. \tag{B.5}$$

Using the gamma mixing distribution (Eq. (12)) for $g(\lambda_0)$, the first part of (B.5) can be written as

$$\begin{aligned}
&\int_0^\infty e^{-\lambda_0 B_i(t_i-1)} (1+d_i) \frac{a^r}{\Gamma(r)} e^{-a \lambda_0} \lambda_0^{r-1} d\lambda_0 \\
&= (1+d_i) \frac{a^r}{\Gamma(r)} \int_0^\infty \frac{e^{-\lambda_0 [B_i(t_i-1)+a]} [B_i(t_i-1)+a]^r \lambda_0^{r-1} d\lambda_0}{[B_i(t_i-1)+a]^r} \\
&= (1 + d_i) \frac{a^r \Gamma(r)}{\Gamma(r) [B_i(t_i-1) + a]^r} \\
&= (1 + d_i) \left[\frac{a}{B_i(t_i-1) + a} \right]^r.
\end{aligned} \tag{B.6}$$

Similarly, the second part of (B.5) becomes

$$\begin{aligned}
 & \int_0^\infty e^{-\lambda_0 B_i(t_i-1)} [e^{-\lambda_0 e^{bX_i(t_i) + cD_i(t_i)}}]^{1-d_i} g(\lambda_0) d\lambda_0 \\
 &= \int_0^\infty e^{-\lambda_0 [B_i(t_i-1) + (1-d_i)e^{bX_i(t_i) + cD_i(t_i)} + a]} \frac{a^r}{\Gamma(r)} \lambda_0^{r-1} d\lambda_0 \\
 &= \frac{a^r \Gamma(r)}{\Gamma(r) [B_i(t_i-1) + (1-d_i)e^{bX_i(t_i) + cD_i(t_i)} + a]^r} .
 \end{aligned} \tag{B.7}$$

Combining (B.6) and (B.7), the expression for the log-likelihood function of N marriages is given by

$$LL = \sum_1^N \log \left\{ (1+d_i) \left[\frac{a}{B_i(t_i-1) + a} \right]^r - \left[\frac{a}{B_i(t_i-1) + (1-d_i)e^{bX_i(t_i) + cD_i(t_i)} + a} \right]^r \right\} . \tag{B.8}$$

Footnotes

1. Note that this is unlikely, given all the previous research on the causes of different individual divorce rates.
2. The divorce date was used in those cases where the actual separation date was missing.
3. A potential drawback of this procedure is the overestimation of marital instability when some separations are followed by reconciliation after the end of the observation period (June 1985). This bias should, however, be smaller than the one arising from using the legal divorce date (Lehrer 1988).
4. Peters (1988) and Heaton (1991) compare different data sets, and Gonul (1989) discusses different models.
5. An exception is Heaton (1990), who uses a Box-Cox form for the baseline hazard. This formulation was also used by Flinn & Heckman (1982) and allows for an inverted U-shape.
6. Other functions could be used to evaluate the impact of covariates. The exponential function, besides being quite flexible, ensures the non-negativity of the hazard rate.
7. See Sharma (1992).
8. Note that the increase in the end merely reflects the fact that the hazard rate of divorce in the interval defined from 13 years till "death do them apart" is larger than the hazards in the twelfth year and eleventh year of marriage.
9. Note that in Figure 3 model NONPAR is just the Kaplan-Meier estimator (Kaplan & Meier, 1958) of the (aggregate) empirical hazard (depicted in Figure 1).
10. Interestingly, we see that also the other coefficients (i.e. for the remaining covariates) become smaller and less significant, indicating a smaller signal-to-noise ratio when omitting some important variables from the model.

Table 1. Description of Covariate Levels.

Variable	Categories ¹	Frequencies (n=15,258)
Race	White*	91%
	Black	9%
Marriage Cohort	1970-74*	34%
	1975-79	32%
	1980-85	35%
Age at Marriage	under 18*	12%
	18-22	56%
	23-30	28%
	over 30	5%
Education in Years	0-11*	3%
	12	1%
	13-15	52%
	16 and more	44%
Premarital Child	No*	88%
	Yes	12%
Number of Children	0*	27%
	1	26%
	2	30%
	3	11%
	4 and more	5%

¹ * defines the base case in the hazard-rate analysis.

Table 2. Example of Variable Coding for Children (Time-Varying Covariate).

Births are during year 2, 4, and 7 of the marriage.

Year	Child 1	Child 2	Child 3	Child 4
1	0	0	0	0
2	1	0	0	0
3	1	0	0	0
4	0	1	0	0
5	0	1	0	0
6	0	1	0	0
7	0	0	1	0
Etc.				

Table 3. Parameter Estimates. Base Rate Hazard and Time Dependence.

	NONPAR	EXP - COV	NONPAR - COV	NONPAR -COV- G	EXP -COV- G	NONPAR -COV- NC
λ_0	0.045	0.080	0.065	-	-	0.055
a	-	-	-	11.969	32753	-
r	-	-	-	0.901	2634.17	-
D2	0.117**	-	0.202*	0.289**	-	0.143
D3	0.171**	-	0.346**	0.518**	-	0.227*
D4	0.036	-	0.296**	0.548**	-	0.125
D5	-0.072	-	0.280**	0.598**	-	0.063
D6	-0.093	-	0.330**	0.710**	-	0.069
D7	-0.157**	-	0.327*	0.764**	-	0.026
D8	-0.111	-	0.424**	0.922**	-	0.096
D9	-0.244**	-	0.344	0.891**	-	-0.005
D10	-0.125	-	0.529	1.121**	-	0.160
D11	-0.520**	-	0.168	0.803**	-	-0.216
D12	-0.669**	-	0.027	0.696**	-	-0.364**
D13 ⁺	-0.422**	-	0.300	1.01**	-	-0.113

EXP: $\lambda_0 = 0.04$, LL = -16,122

EXP-G: a = 18.906, r = 0.975, LL = -16,063

** significant at the 5% level

* significant at the 10% level

Table 3. Parameter Estimates (Continued). Effects of Covariates.

	NONPAR	EXP - COV	NONPAR - COV	NONPAR -COV- G	EXP -COV- G	NONPAR -COV- NC
Race	-	0.416**	0.426**	0.566**	0.416**	0.403
Cohort 2	-	0.272**	0.285**	0.343**	0.272**	0.281*
Cohort 3	-	0.488**	0.575**	0.641**	0.488**	0.563*
Age (18-22)	-	-0.541**	-0.555**	-0.718**	-0.542**	-0.522
Age (23-30)	-	-0.989**	-1.000**	-1.255**	-0.991**	-0.960
Age (>30)	-	-0.929**	-0.955**	-1.213**	-0.930**	-0.846
Edu (=12)	-	0.122*	0.139	0.131	0.124	0.108
Edu (13-15)	-	-0.113**	-0.109	-0.152**	-0.111**	-0.085
Edu (≥ 16)	-	-0.336**	-0.344**	-0.435**	-0.366	-0.254
Prem. Child	-	0.692**	0.747**	0.948**	0.690**	0.471
Child 1	-	-0.255**	-0.318**	-0.386**	-0.255**	-
Child 2	-	-0.534**	-0.655**	-0.801**	-0.534**	-
Child 3	-	-0.621**	-0.743**	-0.946**	-0.620**	-
Child 4+	-	-0.596**	-0.726**	-0.950**	-0.597**	-
LL	-16,063	-15,638	-15,606	-15,638	-15,595	-15,697

** significant at the 5% level

* significant at the 10% level

List of Figures

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Figure 1. Empirical Hazard Rate

Figure 2. Relationships between Different Models

Figure 3. Time Dependence
Baseline Hazard NONPAR-COV-G Model

Fig 4. Time Dependence in NONPAR-COV-G

(Legend on next page)

Figure 4. Legend

- ✱ base group (marriages between 1970 and 1975, of white women under 18, who did not finish high school, and had no children at the time of the interview).
- base group, but children in years 4, 6, and 7.
- "modal" group (marriages between 1980 and 1985, of white women between 18 and 22, with between 13 and 15 years of education, and no children at the time of the interview).
- ✚ "modal" group, but children in years 2, 3, 4, and 5.

Figure 5. Time Dependence

Comparison of Nonparametric Models