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THE EFFECTIVENESS OF TEMPORARY
MARGINAL COST SUBSIDIES

BY

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I. INTRODUCTION

An important part of the instruments of industrial policy in European countries involve some form of subsidisation. The economic relevance of these interventions needs little argument, witness for example the budgetary impact of direct subsidies in some countries and the constant policy debates on ways to improve on the efficiency and the effectiveness of the various modes of direct and indirect subsidisation. Industrial policies rely on subsidies in an attempt either to soften the social and economic impact of changed economic conditions or to try to speed up and possibly direct the adjustments considered necessary. For policies aimed at accelerating a specific dimension of economic performance some economists have argued for subsidizing the corresponding (specific) marginal costs of the desired changes. Marginal employment subsidies (Kesselman et. al. (1977), Layard and Nickell (1980)), for example, only support the employment of additional workers in order to accelerate increases in employment, that presumably would

take place anyway if economic conditions were to change favorably. Subsidies stimulating energy conservation or innovative activities, as well as many other schemes, likewise can be interpreted as supporting incremental changes in an attempt to speed up an improvement in these specific dimensions of economic performance that would realize in any case in the future if market incentives then were to give the appropriate signals.

Subsidies trying to accelerate changes in economic performance through lowering the corresponding incremental costs raise several issues from an economic point of view. They implicitly or explicitly assume that the current market signals result in a deficient speed or direction of industrial adjustment processes. Whether this is a legitimate view depends of course, among other things, on the evaluation of the economic and social costs of the pace of adaptation. In addition the subsidisation instruments may not be the most efficient vehicles to achieve the desired objectives even if the aims can be justified economically. Nontrivial incentive and implementation problems may accompany these interventions, while the effectiveness of various instruments may differ as well.

A number of these issues have at least partly been looked for some specific marginal cost subsidy schemes although there appears to be only limited insights available as to their potential effectiveness. This paper's focus is only on this specific issue: to what extent can temporary marginal cost subsidy programs be expected to accelerate a desired economic performance? Comparative static- and dynamic analysis ha-

ve been presented for marginal employment subsidies (e.g. Kesselman et. al. (1977) and Holmlund (1978))¹, assuming a permanent nature of the subsidies and ignoring the change in future economic conditions. Only Philips (1978) presented an analysis that is able to capture the potential accelerative effects of programs with a known and short length for a firm minimizing the cost of a transition to a new equilibrium level that is dictated by the anticipated new economic conditions. Since the subsidy program only lowers part of the increased marginal cost, it is in the interest of firms to postpone as long as possible a subsidized change in their economic performance and hence, within the Philips framework, the program fails to achieve its accelerative objectives. All of these analyses neglected a number of essential aspects. Subsidy schemes try to accelerate changes in economic performance that would be undertaken if economic conditions changed and a study of their effectiveness should therefore incorporate explicitly the possibility of a changing economic environment. In addition, profit maximization would appear to be a more natural objective to study the implications of subsidized marginal costs on economic performance. Finally, the program length of the subsidy scheme may or may not be known to the subject firms. Even if a program is announced for a short temporary period it may or may not be extended for a certain time. Or a program in existence may suddenly terminate say because of a change in economic policy as the result of a change in the relevant political or economic environment.

This paper present a theoretical analysis of the interaction between the timing characteristics of the change in economic conditions

and the change in the opportunities for obtaining marginal cost subsidies.² Firms may be certain or uncertain about the ending and implementation dates of a marginal cost subsidy program although they are uncertain about the timing of future changes in economic circumstances. In case of uncertainty, however, they are not ignorant but have stochastic beliefs as to the timing of events. The attention will focus on the existence and the magnitude of an acceleration effect and on the influence of the characteristics of the program on this decision. In particular it will be found that the anticipation of a change in economic conditions may or may not improve the effectiveness and situations will be identified for which the accelerative effects are likely to be better for programs with certain than with uncertain duration. The analytical framework is detailed in a following section after which the existence and extent of the accelerative effects during and near the end of the program are taken up.

II. THE FRAMEWORK

A profit maximizing firm is envisaged that has discretion over a certain variable x , the level of which is adapted to changing economic conditions related, for example, to the level of demand or relative factor prices. A government subsidy program attempts to accelerate and strengthen the modifications of the economic performance variable x , by temporarily lowering the corresponding incremental costs of these changes.³ The firm is uncertain about the time at which the economic

conditions will change and there may be certainty or uncertainty about the time at which the government may implement or terminate the marginal cost subsidy scheme. The decision problem of the firm following the implementation of such a program will be detailed first, after that attention will be given to the firm anticipating the introduction of programs in the future.

Suppose that the government implements a marginal cost subsidy program at a time when firms anticipate certain relevant economic conditions to change at an uncertain date in the future. The program subsidizes a fraction s , $0 < s < 1$, of the incremental costs caused by an increase in x , in an attempt to provide incentives for the firms to accelerate and possibly strengthen the improvement in economic performance that is to result from the anticipated new economic circumstances. Marginal employment subsidies, for example, may subsidize, temporally the increase in the wage bill that accompanies increases in employment from a certain base level, in an attempt to accelerate and stimulate the increase in employment that is to result from an anticipated improvement in economic activity and firm demand. Likewise the government may increase the incremental savings that result from decreases in x . Subsidies for energy conversion, for example, can be viewed as attempting to speed up and strengthen the change in the use of certain energy inputs by increasing the savings that result from such a change which presumably would take place (although at a different cost) if the relative prices of the inputs were to increase sufficiently. Other subsidy programs can be interpreted in a similar fashion as long as they attempt to "improve" on the incremental conse-

quences at the firm level.

To preserve a sharp focus the incremental cost of a change in x is posited as a constant equal to c .⁴ For a marginal cost subsidy program aimed at increasing x , the government provides a subsidy $sc(x-\bar{x}) = m(x-\bar{x})$ per unit of time during a subsidisation period of length τ , starting at the time when x deviates from $\bar{x}$. The rate s and period τ of subsidisation as well as the base value are known to the firm once the program is initiated. Presumably the government would set $\bar{x}$ to counteract anticipative reductions in x that would undermine the effectiveness of the program from the start (cfr. Philips (1978)). The present value of the subsidy granted for an increase in x is $(1-e^{-\rho\tau})sc(x-\bar{x})/\rho$ with ρ a (continuous compounding) discount rate. For a program attempting to decrease x , $m = -sc$, and the firm receives a current value of $(1-e^{-\rho\tau})sc(\bar{x}-x)/\rho$.

The firm is supposed to take its decision on x for at least a period of length Δ ($\Delta > 0$). At the end of the decision period Δ the firm may have formed different expectations about the future, say because of information gathered and interpreted in the meantime. The decision on x is reevaluated within that period if a change in economic conditions realizes or the subsidy program is terminated (or initiated). It is clear that the firms' current profits depend on whether or not the economic conditions have changed or the program has been stopped, while future discounted profits will differ according to which event realizes first. To clarify the notation let $T_s = 0$ denote the starting date of the program and let T indicate the time at which the exogenous

economic circumstances change and let T_e denote the ending date of the subsidy program. Let $X(x_1, T)$ and $Y(x_1, T_e)$ indicate the discounted profits at time T_s for respectively the change in economic circumstances occurring before or after termination of the program but within the decision period Δ , and let $Z(x_1, \Delta)$ stand for the discounted profits if neither of these events realizes within Δ . With further notation to be explained shortly:

$$\begin{aligned} X(x_1, T) = & (1 - e^{-\rho T})m_1(x_1 - \bar{x})/\rho + \int_0^T e^{-\rho t} \pi_1(x_1) dt \\ & + \int_T^\infty e^{-\rho t} \rho V_2^0(m_2, x_1) dt \end{aligned}$$

for $0 < T < T_e$, $T < \Delta$, and

$$\begin{aligned} Y(x_1, T_e) = & (1 - e^{-\rho T_e})m_1(x_1 - \bar{x})/\rho + \int_0^{T_e} e^{-\rho t} \pi_1(x_1) dt \\ & + \int_{T_e}^\infty e^{-\rho t} \rho W_1^0(m_1, x_1) dt \end{aligned}$$

(1)

for $T_e < T$, $T_e < \Delta$, and

$$\begin{aligned} Z(x_1, \Delta) = & (1 - e^{-\rho \Delta})m_1(x_1 - \bar{x})/\rho + \int_0^\Delta e^{-\rho t} \pi_1(x_1) dt \\ & + \int_\Delta^\infty e^{-\rho t} \rho V_1^0(m_1, x_1) dt \end{aligned}$$

for $\Delta < T_e$, $\Delta < T$.

For $t \leq T$ the firm obtains a current profit $\pi_1(x_1)$, with π_1 a concave function, while marginal costs are c_1 so that the current value of any subsidy obtained equals m_1 per unit of change of x_1 from the base value $\bar{x}$. The subscript one applies as long as the current economic conditions prevail. V indicates the value of the firm when the program remains in effect for at least some time in the future; the value from the last day of the program on is W . The maximum current profit at any time $t > T$, $T < T_e$, is therefore denoted by (the current value equivalent) $\rho V_2^0(m_2, x_1)$. The dot emphasizes that this value is dependent on the future optimal decisions on x , although the newly chosen levels themselves are suppressed. Since economic conditions change at time T , the firm may revise its decision at that time and the current subsidy obtained at that time will be based on the possibly new marginal cost c_2 and equal $m_2 = sc_2$ for any further deviation from the earlier chosen level of x_1 . The essence of the incremental nature of the subsidies is indeed that only increments are subsidized, hence an increase today will result in a subsidy for a period τ , but if a further increase is decided on in the future, the subsidy at that time will apply to the additional increase in x , again during a period τ . The future current value $\rho V_2^0(m_2, x_1)$ is therefore parametrically dependent on the earlier decision x_1 , because this decision affects future subsidy opportunities with new economic conditions. The magnitude of m_2 may or may not be different from m_1 : if T indicates a change in factor prices affecting the subsidized cost then clearly m_2 will differ from m_1 , but if T constitutes a change in demand m_1 may still equal m_2 (given, say, constant returns to scale).

The maximum current profits at any time $t \geq T_e$, $T_e < T$, are denoted by $\rho W_1^0(m_1, x_1)$. If the firm knows the termination date and only then, it may at the last day of the program decide to revise its decision if that date falls within the decision period. The subsidy received for a period τ from that time only will depend on the earlier chosen level of x_1 as well as on the current marginal cost c_1 , with $m_1 = sc_1$. In case the firm does not know the termination date it will, by definition, be taken by surprise and learn one morning that subsidy opportunities are gone. The maximum value of the firm from that time on will therefore no longer depend on earlier chosen level of x , but only on future decision and expectations as to future change in economic conditions and new government programs, perhaps including new similar subsidy programs. Let $U_1^0(m_1)$ be the maximum value of the firm from a time when no subsidy opportunities are present and economic conditions will have not changed (to be detailed at the end of this section). It follows that $\rho W_1^0(m_1, x_1) = (1 - e^{-\rho\tau})m_1(\tilde{x}_1 - \bar{x}^0) + U_1^0(m_1)$, for the situation in which T_e equals a known value t_e , with $\tilde{x}_1$ the level chosen at t_e and $\bar{x}^0$ the existing (optimal) level decided on before t_e . In case that T_e is random the previous discussion formally implies that $\rho W_1^0(m_1, x_1) = U_1^0(m_1)$.

Whether or not the firm views the program length T_e (or take-up period) as certain or uncertain may be based on the announced policy with respect to the program as well as on experience with similar ventures. The scenario $T_e = t_e$, with t_e a certain length may not be excluded altogether and perhaps is typical for subsidy schemes with explicit short-run stabilization aims. In other instances the government may ddibe-

rately treat the program length as variable or behave as if and shorten or lengthen the program in a response to political changes or change in economic conditions, for instance the realization or not of the economic circumstances that should improve economic performance. Combinations are of course also possible, with firms knowing the program to last for a certain period but being uncertain about the length of any extension period. In the present framework all of these possibilities can formally be analyzed as special cases of the situation with the firm viewing T_e as a random variable.

Suppose, then, that the firm views both the timing of the change in economic conditions T and the termination date T_e as random variables at the time $T_s = 0$ of a program implementation. The firm is not completely ignorant, however, and is able to subjectively assess the likelihood of both events. These assessments are formally summarized in the distribution functions $f_T(t; \theta)$ and $f_{T_e}(t_e; \theta)$, indicating the probability that respectively the change in economic conditions or the end of the program has occurred by a given time t . The firm may view both events as stochastically dependent. The termination date of a marginal cost subsidy program attempting to speed up changes in economic performance to be induced by an upturn in economic activity, for example, may be related to the realisation of a change in these economic conditions. To allow for such a dependence, the distribution functions are dependent on a joint parameter θ which itself is random with an assessed distribution function $dv_\theta(\theta)$, $0 \leq \theta \leq \infty$. Intuitively θ may be viewed as a state of nature variable the realisation of which may affect the change in the exogenous economic circumstances and the en-

ding of the program.⁵ Stochastic independence between T and T_e can be obtained with $\theta = 1$.

The value of the firm V at the time of program introduction equals the expected discounted profits from that time on. In Appendix A it is shown that:

$$\begin{aligned}
 V_1(x_1; m_1, \bar{x}) = & \int_0^\infty dv_\theta(\theta) \left\{ \int_0^\Delta [X(x_1, t)(1-f_{T_e}(t; \theta))f'_T(t; \theta) \right. \\
 & + Y(x_1, t)(1-f_T(t; \theta))f'_{T_e}(t; \theta)] dt \\
 & \left. + Z(x_1, \Delta)(1-f_{T_e}(\Delta; \theta))(1-f_T(\Delta; \theta)) \right\} \quad (2)
 \end{aligned}$$

$$\begin{aligned}
 = & (1-e^{-\rho\tau})m_1(x_1 - \bar{x})/\rho \\
 & + \int_0^\infty dv_\theta(\theta) \left\{ \int_0^\Delta e^{-\rho t} [\pi_1(x_1)(1-f_T(t; \theta))(1-f_{T_e}(t; \theta)) \right. \\
 & + V_2^0(m_2, x_1)(1-f_{T_e}(t; \theta))f'_T(t; \theta) \\
 & + W_1^0(m_1, x_1)(1-f_T(t; \theta))f'_{T_e}(t; \theta)] dt \\
 & \left. + e^{-\rho\Delta} V_1^0(m_1, x_1)(1-f_{T_e}(\Delta; \theta))(1-f_T(\Delta; \theta)) \right\} \quad (3)
 \end{aligned}$$

with f'_T and f'_{T_e} density functions and prime denoting differentiation.

The above expressions can readily be interpreted. The firm decides

on a level x_1 that will be kept constant until the next decision moment at time Δ , unless economic circumstances or the subsidy program were to end in the meantime in case of which the firm may change x again following these events. For a deviation from $\bar{x}$ at time zero the firm receives a current value of a subsidy equal to $(1-e^{-\rho\tau})m_1(x_1-\bar{x})/\rho$. Moreover it obtains $\pi_1(x_1)$ as long as neither the change in economic circumstances nor the end of the program has taken place within Δ , the probability of this being the case at any time t (for a given θ) is $(1-f_T)(1-f_{T_e})$. If the economic conditions change before the termination of the program it obtains at that time $V_2^0(m_2, x_1)$, the probability of this happening around a time t is $(1-f_{T_e})f'_T$. Likewise $(1-f_T)f'_{T_e}$ is the probability (for a given θ) of the program terminating around a time t given that no change in economic conditions has yet occurred and the maximum future discounted profits from that time on is $W_1^0(m_1, x_1)$. If the end of the decision period is reached without any event having taken place the firm will at that time again decide on x and the maximized value from that time on $V_1^0(m_1, x_1)$. The distribution and density functions describing the subjective beliefs of the firm as to the timing of the events T and T_e are dependent on a joint random parameter θ with distribution function $v_\theta(\theta)$.

If any of the events realizes or Δ is reached without them having taken place, the firm decides again on x for a period of maximum length Δ , although it may do so with different probability assessments. When the current program ends suddenly the maximum value of the firm from that time on will depend on the changed economic conditions and on the possibly introduction of similar subsidy programs. Suppose that the

firm decides on a level $\tilde{x}_1$ at a time (set equal to zero) when the subsidy program no longer is in effect and current marginal cost are c_1 . Let $f_{T_s}(t; \theta)$ and $\tilde{f}_T(t; \theta)$ be the assessments at that time of the likelihood of the implementation of a similar program⁶ and the change in economic respectively circumstances by a time t , with T_s denoting the random introduction time of a new subsidy scheme. Both assessments may again be dependent through a random parameter θ with distribution function $du_\theta(\theta)$. By analogy with the earlier discussion the expected discounted profits equal

$$\begin{aligned}
 U_1(\tilde{x}_1; m_1) = & \int_0^\infty du_\theta(\theta) \{ \int_0^\Delta e^{-\rho t} [\pi_1(\tilde{x}_1)(1-\tilde{f}_T(t; \theta))(1-f_{T_s}(t; \theta)) \\
 & + U_2^0(m_2)(1-f_{T_s}(t; \theta))\tilde{f}_T'(t; \theta) \\
 & + V_1^0(m_1, \tilde{x}_1)(1-\tilde{f}_T(t; \theta))f_{T_s}'(t; \theta)] dt \\
 & + e^{-\rho\Delta} U_1^0(m_1)(1-f_{T_s}(t; \theta))(1-\tilde{f}_T(t; \theta)) \} \quad (4)
 \end{aligned}$$

The maximum profits expected when the economic circumstances change but no program is yet introduced is $U_2^0(m_2)$, while the value of the firm at the time when such a program is implemented but no economic conditions have altered is $V_1^0(m_1, x_1)$ defined by equation (3). When the end of the current decision period Δ is reached, a similar problem then at time zero is solved with possibly new expectations on T and T_s , the discounted value of the maximum profits from Δ on is therefore $e^{-\rho\Delta} U_1^0(m_1)$.

The equations (3) and (4) allow to specify the expected discounted profits when the firm is only uncertain about the timing of the change in economic conditions but knows the time of ending or introduction of the subsidy program. These special cases will be taken up in due time in the following sections.

III. THE EXISTENCE OF AN ACCELERATION EFFECT

Once the marginal subsidy program has been implemented the firm may in any of the decision periods of length Δ change its economic performance. There is an acceleration effect in the present framework if and only if the variable x_1 is altered from a original value $\bar{x}$, before the change in economic conditions or the end program at respectively T and T_e realizes. Firms that are allowed to freely change x in anticipation of the introduction of the program will of course decrease (increase) this economic performance before the introduction time T_s of the program, to increase (decrease) it again at that date and take advantage of the subsidies. The government, however, may realize this incentive problem and define $\bar{x}$ accordingly. The program may, for example, stipulate that only changes from earlier values x at times when the program presumably could not have been anticipated yet x will be subsidized. Before clarifying the circumstances for which a speeding up effect may occur the solution to the firm's problem is detailed.

3.1. The optimal solution during the program

At a decision time when the program is in effect the firm chooses its level of economic performance, by solving:

$$\begin{aligned} \max_{x_1} V_1(x_1; m_1, \bar{x}) = & \left(\frac{1-e^{-\rho\tau}}{\rho} \right) m_1 (x_1 - \bar{x}) \int_0^{\infty} dv_{\theta} \left\{ \int_0^{\Delta} e^{-\rho t} \left[\pi_1(x_1) (1-f_T) (1-f_{T_e}) \right. \right. \\ & + V_2^0(m_2, x_1) (1-f_{T_e}) f_T' + U_1^0(m_1) (1-f_T) f_{T_e}' \left. \right] dt \\ & + e^{-\rho\Delta} \left[V_1^0(m_1, x_1) (1-f_T) (1-f_{T_e}) \right] \left. \right\} \end{aligned} \quad (5)$$

with $f_T(t)$ and $f_{T_e}(t)$ the likelihood of respectively the modification in economic environment and the ending of the program by a time t . Since the ending date of the program is uncertain in the formulation above, the maximized value of the firm from that date on with current economic conditions still prevailing is $U_1^0(m_1)$ defined by equation (4) of the previous section. For a firm that knows the program to end at a time Δ_e within the current decision period ($\Delta_e < \Delta$), the optimization problem can be written, noting the discussion following equation (3), as

$$\begin{aligned} \max_{x_1} V_1(x_1; m_1, \bar{x}) = & \left(\frac{1-e^{-\rho\tau}}{\rho} \right) m_1 (x_1 - \bar{x}) / \rho \\ & + \int_0^{\Delta_e} e^{-\rho t} \left[\pi_1(x_1) (1-F_T) + \rho V_2^0(m_2, x_1) F_T \right] dt \\ & + e^{-\rho\Delta_e} \left[V_2^0(m_2, x_1) F_T(\Delta_e) + W_1^0(m_1, x_1) (1-F_T(\Delta_e)) \right] \end{aligned} \quad (6)$$

with $F_T(t) = \int_0^\infty dv_\theta(\theta) f_T(t; \theta)$ the assessment at the decision time of the probability that the economic circumstances will have changed by a future time t . The optimal choice $\tilde{x}_1^0$ at time Δ_e will be analyzed in section five. The value of the firm at Δ_e when economic conditions have not yet changed by that time is $W_1^0(m_1, x_1) = (1 - e^{-\rho\tau})m_1(\tilde{x}_1^0 - x_1)/\rho + U_1^0(\tilde{x}_1^0, m_1)$. In case the firm knows that the program will not end within the period Δ , it solves

$$\begin{aligned} \max_{x_1} V_1(x_1; m_1, \bar{x}) = & \left(\frac{1 - e^{-\rho\tau}}{\rho} \right) m_1(x_1 - \bar{x})/\rho \\ & + \int_0^\Delta e^{-\rho t} [\pi_1(x_1)(1 - F_T) + \rho V_2^0(m_2, x_1)F_T] dt \\ & + e^{-\rho\Delta} [V_2^0(m_2, x_1)F_T(\Delta) + V_1^0(m_1, x_1)(1 - F_T(\Delta))] \end{aligned} \quad (7)$$

with V_1^0 and V_2^0 respectively the maximized value of the firm from the time economic circumstances change within Δ or from Δ on in case such a change did not occur.

For a program attempting to increase x the optimal choice x_1^0 solves $V_1'(x_1^0) \leq 0$, $V_1'(x_1^0)(x_1^0 - \bar{x}) = 0$ and $(x_1^0 - \bar{x}) \geq 0$. An acceleration effect occurs for $x_1^0 > \bar{x}$, while for $x_1^0 = \bar{x}$ a speeding up effect is absent in the current decision period. The concavity of π_1 ensures that these conditions are also sufficient. Clearly the inequalities need to be reversed for a subsidy program focusing on a decrease in x . Three equivalent expressions for V_1' computed from (5) for an uncertain program length, facilitate the verification of subsequent results:

$$\begin{aligned}
V_1'(x_1^0) &= \pi_1'(x_1^0) \int_0^\infty dv_\theta \int_0^\Delta e^{-\rho t} (1-f_T)(1-f_{T_e}) dt \\
&+ \left(\frac{1-e^{-\rho\tau}}{\rho} \right) m_1 \left[1-e^{-\rho\Delta} \int_0^\infty dv_\theta (1-f_T(\Delta))(1-f_{T_e}(\Delta)) \right] \\
&- \left(\frac{1-e^{-\rho\tau}}{\rho} \right) m_2 \int_0^\infty dv_\theta \int_0^\Delta e^{-\rho t} (1-f_{T_e}) f_T' dt \quad (8)
\end{aligned}$$

$$\begin{aligned}
&= \left[\pi_1'(x_1^0) + (1-e^{-\rho\tau})m_2 \right] \int_0^\infty dv_\theta \int_0^\Delta e^{-\rho t} (1-f_T)(1-f_{T_e}) dt \\
&+ \left[\left(\frac{1-e^{-\rho\tau}}{\rho} \right) (m_1-m_2) \right] \left[1-e^{-\rho\Delta} \int_0^\infty dv_\theta (1-f_T(\Delta))(1-f_{T_e}(\Delta)) \right] \\
&+ \left(\frac{1-e^{-\rho\tau}}{\rho} \right) m_2 \int_0^\infty dv_\theta \left[\int_0^\Delta e^{-\rho t} (1-f_T) f_T' dt + e^{-\rho\Delta} f_{T_e}(\Delta) \right] \quad (9)
\end{aligned}$$

$$\begin{aligned}
&= \left[\pi_1'(x_1^0) + (1-e^{-\rho\tau})m_2 \right] \int_0^\Delta e^{-\rho t} (1-f_T) dt \\
&+ \left[\left(\frac{1-e^{-\rho\tau}}{\rho} \right) (m_1-m_2) \right] \left[1-e^{-\rho\Delta} (1-f_T(\Delta)) \right] \\
&- \int_0^\infty dv_\theta \left\{ \int_0^\Delta e^{-\rho t} \left[\pi_1'(x_1^0) (1-f_T) - \frac{(1-e^{-\rho\tau})}{\rho} m_2 f_T' \right] f_{T_e} dt \right. \\
&\quad \left. - \left(\frac{1-e^{-\rho\tau}}{\rho} \right) m_1 e^{-\rho\Delta} (1-f_T(\Delta)) f_{T_e}(\Delta) \right\} \quad (10)
\end{aligned}$$

The last two equations basically only involve rewriting of (8), except that use has been made of the identity

$$\int_0^{\Delta} e^{-\rho t} (1-f_{T_e}) f_T' dt = \int_0^{\Delta} e^{-\rho t} [\rho(1-f_{T_e}) f_T + f_{T_e}' f_T] dt - e^{-\rho \Delta} f_T(\Delta) f_{T_e}(\Delta)$$

which can be verified by integration by parts. For a known program length, differentiation of (6) and (7) with $\partial V_i^0(m_i, x_i)/\partial x_i = -(1-e^{-\rho \tau})m_i/\rho$ for $i = 1$ or 2 and $\partial W_1^0(m_1, x_1)/\partial x_1 = -(1-e^{-\rho \tau})m_1/\rho$, results in:

$$\begin{aligned} V_1'(x_1^0) = & [\pi_1'(x_1^0) + (1-e^{-\rho \tau})m_2] \int_0^{\Delta} e^{-\rho t} (1-F_T) dt \\ & + [(\frac{1-e^{-\rho \tau}}{\rho}) (m_1 - m_2)] [1-e^{-\rho \Delta} (1-F_T(\Delta))] \end{aligned} \quad (11)$$

for a program lasting beyond Δ , while substituting $\Delta = \Delta_e$ in (11) yields the derivative applicable when the firm knows the program to end at a time Δ_e within a decision period.

Although all of the equations from (8) to (11) can be interpreted in terms of the consequences of changing x in an expected discounted sense it should suffice here to emphasize the basic information they encompass. Equation (8) for example, clearly reveals that marginal profitability of a change in economic performance is composed, in the present framework, of positive effects on current profits and subsidies (marginal "benefits") and of negative effects (marginal "costs"), because of lost future subsidisation opportunities. The firm receives a current marginal profit π_1' as long as neither the economic conditions nor the program has changed (since any one of these events may lead to a revision of its policy); the expected discounted value of this impact on profit is $\int_0^{\infty} dv_{\theta} \int_0^{\Delta} e^{-\rho t} (1-f_T)(1-f_{T_e}) dt$. An increase in the

economic performance variable x , results in a current value of subsidy $(1-e^{-\rho\tau})m_1/\rho$. If the end of the decision period Δ is reached without a change in the economic environment or the subsidy program, only a further increase from x_1 at that time will be subsidized, hence the current change in x , entails an opportunity cost with expected discounted value of $((1-e^{-\rho\tau})m_1/\rho)e^{-\rho\Delta} \int_0^\infty dv_\theta (1-f_T(\Delta))(1-f_{T_e}(\Delta))$. Similarly the economic conditions may change while the subsidy scheme is still in operation and the expected discounted value of the subsidy foregone at that time because of the change today is $-\int_0^\infty dv_\theta \int_0^\Delta e^{-\rho t} \left(\frac{1-e^{-\rho\tau}}{\rho} \right) m_2 (1-f_{T_e}) f'_T dt$.

In case the firm is certain that the subsidy program will end at a time Δ_e within Δ , the expected discounted value of the current profit implication until the next decision at either the time of a change in economic circumstances or the end of the program is $\pi_1' \int_0^{\Delta_e} e^{-\rho t} (1-f_T) dt$. The net subsidy implication with current economic conditions is $\left[\left(\frac{1-e^{-\rho\tau}}{\rho} \right) m_1 \right] [1-e^{-\rho\Delta_e} (1-f_T(\Delta_e))]$, while the expected discounted value of the subsidy foregone if economic circumstances were to change is $-\int_0^\infty dv_\theta \int_0^{\Delta_e} e^{-\rho t} \left(\frac{1-e^{-\rho\tau}}{\rho} \right) m_2 f'_T dt = - \left(\frac{1-e^{-\rho\tau}}{\rho} \right) m_2 \left[e^{-\rho\Delta_e} f_T(\Delta_e) + \rho \int_0^{\Delta_e} e^{-\rho t} (1-f_T) dt \right]$. The combination of these implications is summarized in expression (11) or expression (10) with $f_{T_e} \equiv 0$, if in both equations Δ is set equal to Δ_e . A similar reasoning can be made for a firm knowing that the program will not to end within the current decision period and schemes attempting to stimulate decreases rather increases in x_1 .

3.2. The occurrence of accelerative changes in economic performance.

An accelerated change in economic performance results when the firm changes x from an earlier level indicated by $\bar{x}$. Such an effect can be predicted with weaker conditions for a subsidy program with unknown termination date than for a scheme with certain ending characteristics, as the following results detail.

Proposition 1: With marginal costs expected to remain constant through time, a speeded up increase (decrease) in the economic performance variable x will occur if and only if $[\pi'_1(\bar{x}) + (1-e^{-\rho\tau})m_1] > (<0)$ for subsidy programs with known length and if $[\pi'_1(\bar{x}) + (1-e^{-\rho\tau})m_1] \geq 0$ (≤ 0) for schemes with unknown termination date. With marginal costs expected to decrease through time, the accelerative increase (decrease) will be stimulated by temporary subsidies if $[\pi'_1(\bar{x}) + (1-e^{-\rho\tau})m_2] \geq 0$ (≤ 0).

In case that acceleration does not materialize with non-decreasing marginal costs through time and a known program length, it may occur with a subsidy scheme with uncertain ending date.

With marginal costs expected to increase in the future from c_1 to c_2 and with $\pi'_1(\bar{x}) \leq 0$ (≥ 0) the accelerative increase (decrease) only appears if:

$$[1-e^{-\rho\Delta} \int_0^{\infty} dv_{\theta}(1-f_t(\Delta))(1-f_{T_e}(\Delta))] / [\int_0^{\infty} dv_{\theta} \int_0^{\Delta} e^{-\rho t}(1-f_{T_e})f'_{T_e} dt] > \frac{c_2}{c_1},$$

while this inequality together with $\pi'_1(\bar{x}) \geq 0$ (≤ 0) are sufficient to

ensure a speeded up increase (decrease).

The first paragraph of Proposition 1 follows immediately from the equations (11) and (9) for the situations of non-decreasing marginal costs. It is possible that in these cases the accelerative increase does not appear with $[\pi_1'(\bar{x}) + (1-e^{-\rho\tau})m_2] < 0$, thus with $\pi_1'(\bar{x}) < 0$. But if this inequality renders $V_1' < 0$ for a known program length, comparison of (11) and (10) shows that V_1' evaluated at $\bar{x}$ may well be positive for an unknown ending date implying a larger optimal value. A similar argument applies for a program stimulating a decrease ($m = -sc$). The reported inequalities for increasing marginal costs follows directly from equation (8) and the necessary conditions.

To interpret the conditions ensuring or necessary for an accelerative effect to occur, note that with non-increasing marginal costs the current value of the subsidy received minus the expected discounted value of future subsidy opportunities foregone is larger or equal than the expected discounted value of a current stream with value $(1-e^{-\rho\tau})m_2$ received until either the program ends or economic conditions change. This is most easily illustrated for a scheme that is known not to end within the period Δ . With the expected discounted value of the subsidy received minus the opportunities foregone on the left side of the equality sign, it is easily seen that

$$\left(\frac{1-e^{-\rho\tau}}{\rho} \right) [m_1(1-e^{-\rho\Delta}(1-F_T(\Delta))) - m_2 \int_0^{\Delta} e^{-\rho t} F_T' dt] \geq$$

$$\int_0^{\Delta} e^{-\rho t} (1-e^{-\rho\tau})m_2(1-F_T)dt \quad \text{as} \quad m_1 \geq m_2 > 0.$$

As long as the current profit implications of changing x (π_1') are larger than the minimum current value equivalent of the subsidies $(1-e^{-\rho\tau})m_2$, both applying until economic conditions change, it is worthwhile to increase x . Similar arguments can be made for an uncertain program length and subsidies trying to stimulate decreases in x . With increasing marginal cost the actual expected discounted value of the subsidies received minus the opportunities foregone need not necessarily be positive, since larger subsidies per unit of change can be realized if economic conditions change. Of course, for any acceleration effect to occur this actual value needs to be positive and this is precisely the necessary condition stated in the proposition.

With marginal costs non-decreasing in the future the accelerative increase (decrease) does appear if firm was operating with a level of economic performance that is not larger (smaller) than the level that maximizes current profits ($\pi_1'(\bar{x}) \geq 0$ (≤ 0)). For a firm operating with a larger (smaller) value then is dictated by current profit maximization the speeded up increase (decrease) may only occur for sufficiently large values of the period or rate of subsidisation, with further stimulative effects going out from an uncertain program ending. Future subsidy opportunities provide a disincentive to accelerative changes. If the firm adapts x today it is foregoing subsidies that would accompany further incremental changes in the future. This disincentive is largest for situations where changes in economic conditions will result in increased marginal costs and thus larger subsidies per unit of change in x . The following corollary clearly illustrates this and is obtained by substi-

tuting $F_T = 1 - e^{-at}$ and $F_{T_e} = 1 - e^{-bt}$ in the appropriate condition of the previous proposition:

Corollary 1.1: Suppose that a firm views the conditional probability (density function) of T and T_e as constant through time and equal to resp. a and b so that $E_T = 1/a$ and $E_{T_e} = 1/b$ equals the expected time of resp. a change in economic circumstances and the termination of the program. With marginal costs expected to increase and $\pi'_1(\bar{x}) \leq 0$ (≥ 0) an accelerative increase (decrease) in the economic performance variable will occur only if $E_T > (c_2/c_1 - 1)/(\rho + b)$, while this inequality and $\pi'_1(\bar{x}) \geq 0$ (≤ 0) is sufficient to accelerate an increase (decrease). These results also apply for a program with known ending date provided b is set equal to zero.

The accelerative effects only occur if the marginal costs are expected to increase in a sufficiently distant future. This expected date should be further away, the larger the increase in marginal cost and the smaller the discount rate or the conditional probability of a termination of the program in any time interval. The speeding up of economic performance with marginal costs increasing through time may be more likely to occur with an uncertain ending date than with certainty about this termination moment. For a program known to end within Δ or known not to end within this period it is only necessary that $E_T > (c_2/c_1 - 1)/\rho$, for exponential expectations. With more general expectations it is sufficient that the conditional probability $F'_T/(1 - F_T)$ is

not increasing through time in case the firm knows the program to end within Δ ⁷. The uncertainty about the program still being in effect when the higher marginal costs realize, incite the firm to discount more heavily the better future subsidy opportunities that would apply with these higher costs.

An increase in the economic performance variable may with current economic conditions only reduce current profitability ($\pi'_1(\bar{x}) < 0$). A closer look at these situations, that are perhaps the typical ones that some subsidy programs focus on, is possible by analyzing changes in x that only have consequences for the firm's costs.

Proposition 2: A marginal cost subsidy program will stimulate accelerative increases in x that currently only result in additional (partly subsidized) costs, if and only if

$$\int_0^{\infty} dv_{\theta} \left[1 - e^{-\rho\Delta} (1-f_T(\Delta))(1-f_{T_e}(\Delta)) - r\rho \int_0^{\Delta} e^{-\rho t} (1-f_T)(1-f_{T_e}) dt \right. \\ \left. - \frac{c_2}{c_1} \int_0^{\Delta} e^{-\rho t} (1-f_{T_e}) f'_T dt \right] > 0, \text{ with } r = 1/(1-e^{-\rho\tau})s.$$

Subsidized decreases in x without revenue implications will be accelerated if and only if

$$\int_0^{\infty} dv_{\theta} \left[1 - e^{-\rho\Delta} (1-f_T(\Delta))(1-f_{T_e}(\Delta)) + r\rho \int_0^{\Delta} e^{-\rho t} (1-f_T)(1-f_{T_e}) dt \right. \\ \left. - \frac{c_2}{c_1} \int_0^{\Delta} e^{-\rho t} (1-f_{T_e}) f'_T dt \right] > 0.$$

These specific results follow immediately from the necessary conditions and substitution of $\pi_1' = -c_1$ in (8) and rearranging terms.

The firm increases x to a new given level if the corresponding current marginal cost is smaller than the expected discounted value of subsidies received and subsidy opportunities foregone. A decrease in x_1 that saves on cost is likewise speeded up if future subsidy opportunities did not incite a postponement of this change. Again increasing marginal costs or the certainty of the program length are most likely to prevent acceleration to occur, as the following results emphasize :

Corrolary 2.1: A marginal cost subsidy program with known program length attempting to accelerate changes in economic performance that only result in additional (subsidized) costs with current economic conditions, will not succeed in this speeding up if marginal costs are expected to remain constant (Phlips (1978)). An accelerative increase in x will not (may) appear if marginal costs are expected to increase (decrease). A speeded up decrease in x will (may) result if the subsidized marginal costs are expected to decrease (increase).

This corrolary follows immediately from inspection of the conditions of the previous proposition by setting $f_{T_e} \equiv 0$, partially integrating the term containing f_T' and collecting terms. Exponential expectations are useful to clarify the underlying tendencies further:

Corrolary 2.2: Suppose that the firm views the conditional probability of T and T_e as constant through time and equal to resp. a and b so that $E_T = 1/a$ and $E_{T_e} = 1/b$. An accelerated increase in x that currently only yields additional costs will occur if and only if

$$E_{T_e} < \frac{1}{\rho(r-1)} \text{ for } c_1 = c_2 ;$$

$$E_T < \frac{(1-c_2/c_1)}{\rho(r-1)-b} \text{ for } c_1 > c_2 ;$$

$$E_T > \frac{c_2/c_1-1}{\rho(1+r)+b} \text{ for } c_1 < c_2 , \text{ with } r = [1/(1-e^{-\rho\tau})s] > 1$$

An accelerated subsidized decrease in x without revenue implications will appear with non-increasing marginal costs and also with marginal costs increasing through time if and only if

$$E_T > \frac{(c_2/c_1-1)}{\rho(1+r)+b} \text{ for } c_1 < c_2.$$

These results also apply for a program with known ending date provided b is set equal to zero.

An accelerated increase in x without revenue implications with current economic conditions and with marginal costs expected to remain unchanged will not occur with a know program length, as Philips (1978) already demonstrated. But with an uncertain ending of the program that is sufficiently likely, the speeding up will occur. With exponential expectations the expected termination date needs to be sufficiently near.

Likewise, with such expectations, if the decrease in marginal costs is expected to occur sufficiently soon it is better to take advantage of the program now. As the expected time of ending of the program decreases or the rate or period (τ) of subsidisation increases, the decrease in marginal cost with the change in economic circumstances may be expected in a more distant future without jeopardizing the accelerative effects. The speeding up will only occur with marginal costs expected to increase, if this is expected to happen in a sufficiently distant future especially if the program is unlikely to end soon, and the rate or period of subsidisation is large. More generous programs, thus will undermine the existence of an accelerative changes if the firms expect marginal costs to increase, since the future subsidy opportunities with higher marginal costs will look even more attractive and thus changes will be postponed.

IV. THE EXTENT OF AN ACCELERATION EFFECT

A change in the economic performance of the firm at any of the decision moments where a change in economic conditions is anticipated can further be characterized if the corresponding variable x_1^0 , constitutes an interior optimal solution solving $V_1'(x_1^0) = 0$. The impact on the effectiveness of the program length as well as the rate (s) and period (τ) of subsidisation will be analyzed next.

4.1. The effect of the program length.

Marginal cost subsidy programs may have a certain or an uncertain temporary duration. The extent of acceleration effects may depend on:

- (i) the likelihood of the termination of the program by any time: the firm either knows this will not occur in the decision period or may view the ending by any future time more or less likely (i)
 - (ii) the time period Δ_e after which the scheme will stop in the current decision period: the firm knows that the ending date is to occur after a time $\Delta_e < \Delta$;
 - (iii) whether or not the firm knows the ending date or is uncertain about it: i.e. the magnitude of the speeding effect may be different in the situation (i) than in the circumstances described by (ii) above.
- The following three propositions cover consecutively the cases (i) through (iii).

Proposition 3: The magnitude of an acceleration effect induced by a temporary marginal cost subsidy program increases as the likelihood of termination of the program by any time in the decision period increases (provided that with marginal costs expected to increase through time, $\pi'_1(x_1^0) \leq (> 0)$ for programs stimulating increases (decreases) in x).

To verify this result a distribution function $\bar{f}_{T_e}$ that stochastically dominates f_{T_e} in the first degree (Goovaerts et al. (1982)) is substituted in $V'_1(x_1^0)$. It is immediately clear from equation (10) that

$V'(x_1^0) = 0$ for f_{T_e} , then $V'(x_1^0) > 0$ (< 0) for $\bar{f}_{T_e} > f_{T_e}$, provided that $m > 0$ (< 0) and $\pi_1'(x_1^0) \leq 0$ (≥ 0). Equation (9) allows to verify that the inequality $\pi_1'(x_1^0) < 0$ (> 0) necessarily has to hold with marginal costs non-increasing through time. When these costs are increasing through time the results only appears clear with these inequalities satisfied.

The extent of an acceleration effect will thus be larger if the firm does not know whether the program will end within Δ ($f_{T_e} > 0$) than in a situation where it knows the program will not terminate within Δ ($f_{T_e} \equiv 0$). Moreover, as the termination by any time becomes more likely the future subsidy opportunities that can be expected to temper the extent of the acceleration effect, loose their importance and effectiveness increases. Suppose next that the firm knows that subsidy opportunities will cease after a known time period Δ_e within the decision period Δ . The influence of the length Δ_e on the effectiveness in this situation is summarized in the following proposition for which the supporting arguments are presented in Appendix B:

Proposition 4: The magnitude of an acceleration effect incited by a temporary marginal cost subsidy program is independent of the known time Δ_e after which the program ends within a decision period, if and only if either marginal costs or the conditional probability of a change in economic circumstances remain unchanged through time. In case that the subsidized marginal costs are expected to decrease in the future, a prolongation (shortening) of Δ_e increases the effectiveness if and only if the conditional probability of the change in mar-

marginal cost increases (decreases) as time goes on. With subsidized marginal costs expected to increase in the future, a prolongation (shortening) of Δ_e increases the effectiveness if and only if the conditional probability of the change in marginal cost decreases (increases) as time goes on.

The time at which the program is known to end has no effect on the magnitude of the economic performance if and only if either the firm has "exponential" expectations with a constant conditional probability of a change in economic circumstances around any time, or the marginal costs are not expected to change in the future. In both situations the subsidy opportunities remain essentially the same at any future time and how long these opportunities will present does not affect today's decisions. The period Δ_e will have an effect, however, on the firm anticipating a changing conditional probability of different marginal costs realising. Suppose, for example, a firm expects marginal costs to decrease in the future with an increasing conditional probability as time progresses, while the government attempts to stimulate larger values of x . As Δ_e enlarges it becomes intuitively more likely that the change in economic conditions will realize before the end date of the program. But if the change materializes only lower subsidies will be realized since they will apply to lower marginal cost. This providing an incentive to take more advantage of the subsidies before the lower cost realize. It can be shown that with a known program length a change in x is also strengthened (weakened) as the likelihood of a modification of economic conditions by any time increases and marginal

cost are expected to decrease (increase); and this provides further support for the reasoning made above.

A comparison of the effectiveness with knowledge or uncertainty about the ending date within the decision period is possible to a certain extent, as shown in Appendix C and summarized in:

Proposition 5: The magnitude of an acceleration effect stimulated by a program subsidizing marginal costs not increasing through time and ending after a time Δ_e within the decision period ($\Delta_e < \Delta$), is smaller than the effect that would result if the termination date were unknown, given that the conditional probability of a change in economic circumstances is not increasing through time.

The superior effectiveness of the program with uncertain length compared to schemes with a known termination date does not appear to hold in general for situations with anticipated increases in marginal cost. Given stochastic independence between the uncertain dates, for example, it appears that the uncertain program length will only prove to be more effective with a conditional probability of termination of the program sufficiently increasing (decreasing) through time for constant or increasing (decreasing) conditional probability of an augmentation in marginal costs.⁸ A combination of Proposition 3 and 4 and Lemma B.1 of Appendix B, allows to state:

Proposition 6: Suppose that the firm views the change in economic circumstances as stochastically independent from the termination of the

program with a constant conditional probability of occurrence of these events around any time. In these circumstances the magnitude of the acceleration effect in any decision period generated by a subsidy program with an uncertain ending date is larger than the effect stimulated by a scheme with known termination date.

Exponential stochastically independent expectations are sufficient for the known program length or the length of the decision period to have no effect on the acceleration effect.⁹ The relevance of this stronger result of course would seem to depend on whether or not it is reasonable to suppose that firms have formed the indicated expectations.¹⁰ In case the decision period Δ is relatively short this may be appropriate. The real limitation would seem to come from the assumption of stochastic independence of T and T_e . Indeed it can be shown that the length of the decision period is without influence on the magnitude of the acceleration effect with stochastically dependent expectation if the corresponding conditional probability density function $H_T = F_T' / (1 - F_T)$ and $H_{T_e} = F_{T_e}' / (1 - F_{T_e})$, with $F_T = \int_0^\infty dv_\theta f_T(t; \theta)$ and $F_{T_e} = \int_0^\infty dv_\theta f_{T_e}(t; \theta)$, are constant through time. Exponential expectations are, however, not sufficient to guarantee constant hazard rates H_T and H_{T_e} if the latter are resulting from stochastically dependent distributions. In Lemma B.2 of Appendix B it is shown in fact that $H_T(t) = a(t)\varphi \left[\int_0^t a(u)du \right]$, with $d\varphi/dt < 0$ for $f_T(t; \theta) = 1 - e^{-\int_0^t \theta a(u)du}$ and v_θ a distribution function with positive variance. A similar relationship can be specified for H_{T_e} . If the firm knows the distribution function of T and T_e to be exponential with constant hazard rate resp.

$a\theta$ and $b\theta$, but views both events as stochastically dependent through the uncertain parameter θ , the resulting conditional probability density functions will no longer be constant through time.¹⁰

4.2. The effect of the period (τ) and rate (s) of subsidisation.

The effects of the other parameters of the subsidy program on an interior solution are easily obtained. Implicit differentiation of resp. (9) and (8) yields:

$$\begin{aligned} \frac{\partial V_1'}{\partial \tau} &= \rho e^{-\rho\tau} m_2 \int_0^\infty dv_\theta \int_0^\Delta e^{-\rho t} (1-f_T)(1-f_{T_e}) dt \\ &\quad + e^{-\rho\tau} (m_1 - m_2) \left[1 - e^{-\rho\Delta} \int_0^\infty dv_\theta (1-f_T(\Delta))(1-f_{T_e}(\Delta)) \right] \\ &\quad + m_2 \left(\frac{1-e^{-\rho\tau}}{\rho} \right) \int_0^\infty dv_\theta \left[\int_0^\Delta e^{-\rho t} (1-f_T) f_{T_e}' dt + e^{-\rho\Delta} f_{T_e}(\Delta) \right] \end{aligned} \quad (12)$$

$$\begin{aligned} &= e^{-\rho\tau} \left[m_1 (1 - e^{-\rho\Delta} \int_0^\infty dv_\theta (1-f_T(\Delta))(1-f_{T_e}(\Delta))) \right. \\ &\quad \left. - m_2 \int_0^\infty dv_\theta \int_0^\Delta e^{-\rho t} (1-f_{T_e}) f_{T_e}' dt \right] \end{aligned} \quad (13)$$

and

$$\frac{\partial V_1'}{\partial s} = \left(\frac{e^{\rho\tau} - 1}{s\rho} \right) \frac{\partial V_1'}{\partial \tau}$$

From equation (12) it follows that $\partial V_1'/\partial \tau > 0$ (< 0) for $m > 0$ (< 0) and

$c_1 \geq c_2$. Effectiveness implies that the expression between square brackets in (13) is positive (negative) as $m > 0$ (< 0) and $c_1 < c_2$ (cfr. Proposition 2). The signs on the derivative of V_1' therefore also apply for $c_1 < c_2$. Noting the concavity of V_1 it is hence established that :

Proposition 7: The magnitude of an acceleration effect stimulated by a temporary marginal cost subsidy program, increases as the rate (s)- or period (τ) of subsidisation increases.

This result is to be expected and easily interpreted. It does not, however, indicate that enlarging τ or increasing s will necessarily ensure a greater accelerative effectiveness of the subsidy program. Suppose the firm expects marginal costs to increase while it attaches a sufficiently low probability to the likelihood that the subsidy scheme will have been terminated by the next planned decision.¹¹ According to Proposition 2 the firm will not change x in the current decision period, since higher subsidies can be realized when marginal costs have increased. An increase in τ or s only strengthens - and does not neutralize - the incentives to postpone the change in economic performance.¹² Acceleration effects that do occur, however, are larger as the subsidy program is more generous.

V. EFFECTIVENESS AT THE END OF THE PROGRAM

In case the firm knows the time of the ending of the program, it will near the end of the program face a somewhat different decision problem than at earlier times. If the ending time falls within a decision period the analysis of the previous section applies, and the shorter decision horizon may result in a change in economic performance (see Proposition 4). At the last day of the program the firm may revise its level of x since at that date the future subsidy opportunities may differ from the earlier ones. The firm either knows that a new subsidy program will start within or outside the decision period following the ending of the program, or it views the date of introduction of a new scheme as uncertain. The firm's corresponding problem and its solution is detailed next, after which the resulting effectiveness is contrasted with the accelerative effects that may have occurred during the program itself.

5.1. The optimal solution at the end of the program.

Suppose the firm reaches the end of the program at $T_e = t_e = 0$ with a level $\bar{x}_0$ decided on earlier. Current marginal costs are designated by c_1 , they may change at unknown date in the future to c_2 , the corresponding subsidies per unit of change in x are m_1 and m_2 . The firm decides at $t_e = 0$ on a level of x indicated by $\tilde{x}_1$, by solving the problem:

$$\max_{\tilde{x}_1} W_1(\tilde{x}_1; m_1) = (1 - e^{-\rho\tau})m_1(\tilde{x}_1 - \tilde{x}^0)/\rho + U_1(\tilde{x}_1; m_1) \quad (14)$$

U_1 is the value of the firm expecting the introduction of a marginal cost subsidy program (defined earlier) :

$$\begin{aligned} U_1(\tilde{x}_1; m_1) = & \int_0^\infty du_\theta(\theta) \{ \int_0^\Delta e^{-\rho t} [\pi_1(\tilde{x}_1)(1 - \tilde{f}_T(t; \theta))(1 - f_{Ts}(t; \theta)) \\ & + U_2^0(m_2)(1 - f_{Ts}(t; \theta))\tilde{f}'_T(t; \theta) \\ & + V_1^0(m_1, \tilde{x}_1)(1 - \tilde{f}_T(t; \theta))f'_{Ts}(t; \theta)] dt \\ & + e^{-\rho\Delta} U_1^0(m_1)(1 - f_{Ts}(t; \theta))(1 - \tilde{f}_T(t; \theta)) \} \end{aligned} \quad (4)$$

with $f_T(t)$ and $f_{Ts}(t)$ indicating the assessments on the likelihood of respectively a change in economic circumstances and implemented of a marginal cost subsidy program (with parameters s and τ)¹³ by any future time t .

In case the firm knows that no program will be introduced in the period Δ , U_1 specializes to

$$\begin{aligned} U_1(\tilde{x}_1; m_1) = & \int_0^\Delta e^{-\rho t} [\pi_1(\tilde{x}_1)(1 - \tilde{F}_T) + \rho U_2^0(m_2)\tilde{F}_T] dt \\ & + e^{-\rho\Delta} [U_2^0(m_2)\tilde{F}_T(\Delta) + V_1^0(m_1, \tilde{x}_1)(1 - \tilde{F}_T(\Delta))] \end{aligned} \quad (15)$$

with U_2^0 and V_1^0 respectively the maximized values of the firm from the

time economic conditions change while no program in effect and from the time a new scheme is implemented and $\tilde{F}_T = \int_0^\infty du_\theta \tilde{f}_T(t; \theta)$. The objective function for a known introduction time $t_s < \Delta$ can be found by substituting $t_s = \Delta$ in equation (15).

For a subsidy program attempting to increase (decrease) x , the optimal solution x_1^0 solves $W_1'(\tilde{x}_1^0) \leq 0$ ($\geq$), $W_1'(\tilde{x}_1^0)(\tilde{x}_1^0 - \bar{x}^0) = 0$ and $(\tilde{x}_1^0 - \bar{x}^0) \geq 0$ ($\leq$), with

$$\begin{aligned} W_1'(\tilde{x}_1^0; m_1) = & \left(\frac{1 - e^{-\rho\tau}}{\rho} \right) m_1 \left[1 - \int_0^\infty du_\theta \int_0^\Delta e^{-\rho t} (1 - \tilde{f}_T) f_{T_s}' dt \right] \\ & + \pi_1'(x_1^0) \int_0^\infty du_\theta \int_0^\Delta e^{-\rho t} (1 - \tilde{f}_T) (1 - f_{T_s}) dt \end{aligned} \quad (16)$$

given an uncertain introduction time. The marginal impacts on the value of the firm at time t_e with a known starting time beyond Δ is obtained by substituting $\int_0^\infty du_\theta \int_0^\Delta e^{-\rho t} (1 - \tilde{f}_T) f_{T_s}' dt = 0$ in (16), while the corresponding expression for $t_s < \Delta$ results from putting $\Delta = t_s$ and $\int_0^\infty du_\theta \int_0^{t_s} e^{-\rho t} (1 - \tilde{f}_T) f_{T_s}' dt = e^{-\rho t_s} \int_0^\infty du_\theta (1 - \tilde{f}_T(t_s))$ in the same expression.

5.2. Effectiveness at the end vis à vis effectiveness during the program.

An issue of interest is whether or not the effectiveness at the end of a program (with known ending date) is possibly different than at earlier times during the program. Both the existence and the ex-

tent of a change are reported on in the subsequent propositions.

Proposition 8: Changes in economic performance may occur at the ending date of a program with known length even if they did not previously occur or would not have occurred from a program with uncertain duration. Specifically if increasing x currently results only in additional costs it will be undertaken at t_e if and only if:

$$\int_0^{\infty} du_{\theta} [1 - r\rho \int_0^{\Delta} e^{-\rho t} (1 - \tilde{f}_T)(1 - f_{T_s}) dt - \int_0^{\Delta} e^{-\rho t} (1 - \tilde{f}_T) f'_{T_s} dt] > 0$$

with $r = 1/(1 - e^{-\rho T})$. A subsidized decrease in x without revenue implications will in any case increase the value of the firm at the ending date.

A comparison of equation (16) and Proposition 1 allows to substantiate these results. Setting $du_{\theta} = 1$, $\theta = 1$, $f_T = 1 - e^{-at}$ and $f_{T_s} = 1 - e^{-it}$ in the inequality of the previous proposition, integrating and rearranging terms, gives

Corollary 8.1: Suppose that the firm views the conditional probability of T and T_s as constant through time and equal to resp. a and i so that $E_T = 1/a$ and $E_{T_s} = 1/i$. An increase in x that currently only yields additional costs will appear at the last day of a program (with known ending) if and only if

$$\text{either } E_T < \frac{1}{\rho(r-1)},$$

or $E_T > \frac{1}{\rho(r-1)}$ and $\Delta < -\ln[\rho(r-1)-a]/(\rho+a+i)$.

with $r = 1/(1-e^{-\rho\tau})s$.

The increased likelihood of a change in economic performance appearing near the end of a program with a known ending date was already noticed by Philips (1978) for a more limited scenario. The necessary conditions that apply for the case of increases in x without beneficial revenue conditions can again be interpreted. The firm compares the expected discounted marginal cost of an increase in x , that will be incurred as long as neither economic conditions change nor a new program is introduced $(-c_1 \int_0^{\Delta} e^{-\rho t} (1-\tilde{f}_T)(1-f_{Ts}) dt)$ with the current value of the subsidy $c_1/r\rho$ and the subsidy foregone if a new program were introduced before economic conditions change $-(c_1/r\rho) \int_0^{\Delta} e^{-\rho t} (1-\tilde{f}_T) f'_{Ts} ds$.

If economic circumstances and possibly marginal costs change before a new program is introduced the level of economic performance will be changed at that time without any implications for subsidies. The possibility of higher marginal costs being subsidized no longer provides a disincentive and the change in x may appear more readily at time t_e than at earlier times during the program. In fact it is sufficient for the change in economic conditions to be likely or the introduction of a new program to be unlikely (or irrelevant in case the firm knows that the government will counteract anticipative actions) or the decision period to be relatively short for an increase in x to occur even if it only entails currently additional costs. To compare the magnitude of the

changes $\tilde{x}_1$ with changes x_1^0, x_1^0 solving $V_1'(x_1^0) = 0$ is substituted in $W_1'(x_1; m_1)$, to obtain

$$\text{sign } W_1' \Big|_{\tilde{x}_1 = x_1^0} = m_1 M + m_2 \frac{\int_0^\infty dv_\theta \int_0^{\Delta_i} e^{-\rho t} (1-f_{T_e}) f_{T_s}' dt}{\int_0^\infty dv_\theta \int_0^{\Delta} e^{-\rho t} (1-f_T) (1-f_{T_e}) dt} \quad (17)$$

with

$$M = \left[\frac{1 - \int_0^\infty du_\theta \int_0^{\Delta} e^{-\rho t} (1-\tilde{f}_T) f_{T_s}' dt}{\int_0^\infty du_\theta \int_0^{\Delta} e^{-\rho t} (1-\tilde{f}_T) (1-f_{T_s}) dt} \right] - \left[\frac{1 - e^{-\rho \Delta_i} \int_0^\infty dv_\theta (1-f_T(\Delta_i)) (1-f_{T_e}(\Delta_i))}{\int_0^\infty dv_\theta \int_0^{\Delta_i} e^{-\rho t} (1-f_T) (1-f_{T_e}) dt} \right] \quad (18)$$

and Δ_i and Δ the length of decision period respectively before and at the end of the program. The following result is based on these equations:

Proposition 9: Suppose that at the end of a marginal cost subsidy program with known length the firm views the introduction of a similar program at any date in the future sufficiently unlikely or as irrelevant, while the firm views the overall uncertainty as unchanged ($u_\theta = v_\theta$) and the likelihood of a change in economic conditions by any future time not lower than it was before. The magnitude of the change in economic performance at this ending date will be larger than the magnitude of a change during the program as well as larger than the change that would have been induced by a program of uncertain duration.

To verify these result from (17) and (18) note that the second term appearing after the negative sign is decreasing as f_{T_e} or Δ_i increases, while $M = 0$ for $f_{T_s}' \equiv 0$, $u_\theta = v_\theta$, $\Delta = \Delta_i$ and $f_{T_e} \equiv 0$. It is possible that $\Delta_i < \Delta$ (if the program is known to end before the end of the decision period) or that $f_{T_e}' > 0$ for programs with uncertain duration, but these parametric changes would render M positive as long as $u_\theta = v_\theta$ and $\tilde{f}_T \leq f_T$.

It appears that only the anticipation of a new program may possibly undermine the increased effectiveness at the ending date of a program with known length. This shows even more clearly if the firm knows that a new program will be introduced in the near future, say at a time $t_s < \Delta$. Additional changes in economic performance will in that case only occur with more limited circumstances: if marginal cost remain constant or if the assessed likelihood of a decrease in marginal costs is not lower than before and the conditional probability of a change constant or decreasing through time or if the assessed likelihood of an increase in marginal cost is lower than at earlier times and the corresponding conditional probability constant or increasing.¹⁴ These results again presuppose that the government does not recognize the anticipative actions. If it does and is able to effectively counteract these tendencies, e.g. by defining a reference value in a sufficiently distant past, the firm may view the introduction of a new program as irrelevant in which case the effectiveness at the end of the current subsidy program improves.

VI. CONCLUSIONS

Temporary marginal cost subsidy programs attempt to accelerate specific dimensions of economic performance by lowering the corresponding incremental costs or improving on the resulting cost savings. These instruments of industrial policy are employed in situations when firms anticipate a change in economic circumstances to materialize in the future while at the same time they recognize the temporary nature of the subsidy scheme. This paper clarified the potential effectiveness of the subsidy schemes to actually stimulate accelerated improvements in economic performance that would otherwise not appear as early, recognizing that firm's may have stochastic beliefs as to the timing of the change in the future relevant economic circumstances and as to the ending (or possible introduction) of the subsidy program. A framework was employed that allows firms to view the realisation of the future events as stochastically dependent and to revise the expectations at regular time intervals.

An accelerated change is most likely to be induced by a marginal cost subsidy program if it entails not only cost implications with the current economic conditions but other beneficial, for example revenue-, consequences as well and the change in the future economic outlook is not associated by the firm with an increase in marginal costs, while the termination date of the program is uncertain. The opportunity cost of current incremental changes is largest if the firm expects marginal costs to increase in the future, since higher future subsidies per unit

of change are foregone. The disincentive of future subsidy opportunities is weakened if the firm believes that the program may suddenly terminate. The occurrence of a speeded up increase in economic performance that only results in additional costs with the current economic situation, will be stimulated if the (risk neutral) firm believes that subsidy opportunities will suddenly disappear.

It may appear that it would be relatively easy for the government to "neutralize" the disincentives created by the expected increases in marginal costs since the subsidy scheme could stipulate, for example, either more generous provisions with regard to the rate or period of subsidisation or that if economic conditions and marginal costs were to change only the "old" incremental costs will be recognized for support. The previous analysis revealed that a more generous program may actually aggravate the problem in the sense that a speeding up effect may even be less likely to occur with marginal costs expected to increase, although it would strengthen the accelerative effects that do occur. A change in the subsidisation rate with the change in marginal cost would in turn create other problems, resulting from the possible reduced incentives for efficient firm operation and from the difficulty to monitor cost levels in the presence of asymmetric information between the firms and the government.

Once an accelerative effect occurs during the program it may in a number of situations be smaller for a firm that knows the timing characteristics of the program than for (a risk neutral) firm that is uncertain about the ending dates of the program. This result does not

necessarily hold, however, if the higher marginal costs may be subsidised in the future and if the firm views the change in economic circumstances as stochastically dependent with the ending date of the program. The improvement in economic performance occurring near the end of a scheme, say with a short take-up period, fails to be accelerative but may be larger than comparable changes induced during a program with known or uncertain ending time, particularly if the firm views the introduction of new similar subsidy schemes as unlikely or as irrelevant with the government being able to counteract anticipative modifications.

The anticipation of new economic circumstances that warrant an improvement in economic performance, may in itself provide a disincentive for accelerative changes, if subsidy opportunities will be better with these new economic circumstances. The accelerative effects may be stimulated by a random stop and go kind of a subsidy scheme. The optimal design of a temporary marginal cost subsidy program should however also be based on an analysis of the implications for welfare ¹⁵ and efficiency that should complement the results presented in this paper.

Footnotes

1. Marginal employment subsidies appear to be the most heavily researched, see e.g. Schmid (1979), Goossens and Vuchelen (1980), Chiavella and Steinherr (1982), and the references cited there.
2. See Nickell (1977), for the influence of uncertain government actions on investment and Lucas (1976), Kydland and Prescott (1977) among many others for the implications for the design of government policy.
3. These may be achieved by direct subsidies or indirect ways, e.g. tax credits. The term subsidy should be interpreted as encompassing all of these possibilities. The incentives to the firm from an increase in the corresponding costs through taxation could of course *mutatis mutandi* be analyzed in this framework as well.
4. Adjustment costs proportional to changes in x could be incorporated.
5. As an example suppose the firm's beliefs can be represented by exponential and gamma distributions, with $f_T = 1 - e^{-a\theta t}$ and $f_{T_e} = 1 - e^{-b\theta t}$ and $dv_\theta(\theta) = \theta^{\gamma-1} e^{-\theta} d\theta / \Gamma(\gamma)$ with $\gamma > 1$. The expected values are $E_T(T) = 1/a(\gamma-1)$ and $E_{T_e}(T_e) = 1/b(\gamma-1)$. As the mean γ of the random parameter θ increases, the firm expects an earlier change in economic conditions as well as a more rapid termination of the subsidy program.

6. Similar means here that the subsidisation rate and period are equal; although differences with respect to these parameters could easily be incorporated in the present framework.
7. For a program that is known to end at a time $\tilde{\Delta}$ and $c_2 > c_1$ the necessary condition of Proposition 1 implies

$$\frac{[1 - e^{-\rho\Delta} \int_0^\infty dv_\theta (1 - f_T(\Delta))(1 - f_{T_e}(\Delta))]}{\int_0^\infty dv_\theta \int_0^\Delta e^{-\rho t} (1 - f_{T_e}) f'_T dt} > \frac{[1 - e^{-\rho\tilde{\Delta}} (1 - F_T(\Delta))]}{\int_0^{\tilde{\Delta}} e^{-\rho t} f'_T dt} > 1$$

for $\tilde{\Delta} = \Delta$ and $f_{T_e} > 0$ and Δ the decision period for the program with uncertain ending. To verify that the inequality also holds for $\tilde{\Delta} = \Delta_e < \Delta$ with $H_T = F'_T / (1 - F_T)$ not increasing in t , denote the behind the first inequality sign by $L(\Delta)$. The sign $dL/d\Delta = \text{sign} [(\rho + H_T) \int_0^\Delta e^{-\rho t} F'_T dt - (1 - e^{-\rho\Delta} (1 - F_T(\Delta))) H_T]$. This last expression between square brackets is zero for $\Delta = 0$ and increases (is constant) (decreases) in Δ as H_T is decreasing (constant) (increasing) and $L(\Delta) > 1$.

8. See Lemma B.1 of Appendix B.
9. See Lemma B.1. Allowing for a freely variable x in any decision period and ignoring adjustment costs it could be shown that in these circumstances a constant policy would be optimal (see also Kamien and Muller (1983)).

10. See Kamien and Schwartz (1980) for a discussion and applications of a generalized hazard rate $\int_0^t a(u)du$. To illustrate the discussion suppose that θ is gamma distributed with mean γ . One verifies that $H_T(t) = a(t)\gamma/(1 + \int_0^t a(u)du)$ and hence $dH_T/dt < 0$ for $da(t)/dt \leq 0$.

11. More specifically,

$$\left[1 - e^{-\rho\Delta} \int_0^\infty dv_\theta (1 - f_T(\Delta))(1 - f_{T_e}(\Delta)) / \int_0^\infty dv_\theta \int_0^\Delta e^{-\rho t} (1 - f_{T_e}) f_T' dt \right] < \frac{c_2}{c_1}$$

with $c_2 > c_1$.

12. Substituting the inequality of the previous footnote in (13) results in $\partial V'(\bar{x})/\partial \tau < 0$ ($>$) as $m > 0$ (< 0).

13. Different values of these parameters would be incorporated but are neglected here to keep the analysis attractive.

14. The proof of these statement rests on an evaluation of (17) and (18)

with $\Delta = t_b < \Delta_i$. It can be verified that $\text{sign } W'|_{x_1} = (m_1 - m_2)M$ $\int_0^{t_b} e^{-\rho t} (1 - \tilde{F}_T) dt + m_2 [e^{-\rho t_b} \tilde{F}_T(t_b) + \int_0^{t_b} \rho e^{-\rho t} \tilde{F}_T dt]$ with M defined according to (18) as $M = N(t_b, \tilde{F}_T) - N(\Delta_i, F_T)$, $N = (1 - e^{-\rho t_b} (1 - \tilde{F}_T(t_b))) / \int_0^{t_b} e^{-\rho t} (1 - \tilde{F}_T) dt$ and $t_b < \Delta_i$. The claimed result is immediate for $m_1 = m_2$, while it continues to hold for $m_1 < m_2$ and $M \leq 0$ and for $m_1 > m_2$ and $M > 0$. The conditions stated allow to determine the appropriate sign, noting that N is increasing in F_T and $\text{sign } \partial N / \partial \Delta =$

$\text{sign } \partial H_T(\Delta)/\partial \Delta$, since $\text{sign } \partial N/\partial \Delta = \text{sign } K(\Delta)$, while $K(\Delta) =$
 $(\rho + H_T(\Delta)) \int_0^\Delta e^{-\rho t} (1 - F_T) dt - (1 - e^{-\rho \Delta} (1 - F_T(\Delta)))$, $K(0) = 0$, and
 $\text{sign } dK/d\Delta = \text{sign } \partial H_T(\Delta)/\partial \Delta$.

15. See e.g. Loury (1983) for a possible analytical framework.

Appendix A

The value of the firm $V_1(x_1; m_1, \bar{x})$ equals the expected value of future discounted profits. Let $g_{T, T_e}(t, t_e)$ denote the assessed joint density function on the random variables T and T_e , with

$$g_{T, T_e}(t, t_e) = \int_0^\infty f'_T(t; \theta) f'_{T_e}(t_e; \theta) dv_\theta(\theta) \quad (A.1)$$

By definition one obtains

$$V_1 = E_T E_{T_e}(D) = \int_0^\infty \int_0^\infty D(t, t_e) g_{T, T_e}(t, t_e) dt dt_e \quad (A.2)$$

with E denoting the expectation operation and

$$\begin{aligned} D(T, T_e) = & X(x_1, T) \delta(T_e - T, \Delta - T) + Y(x_1, T_e) \gamma(T - T_e, \Delta - T_e) + \\ & Z(x_1, \Delta) \gamma(T - \Delta, T_e - \Delta) \end{aligned} \quad (A.3)$$

where $\delta(q, p) = 1$ for $q \geq 0$ and $p \geq 0$ and $\delta \equiv 0$ otherwise, while $\gamma(q, p) = 1$ for $q > 0$ and $p > 0$ and $\gamma \equiv 0$ otherwise, and X, Y, Z defined by equation (1) in the main text.

Substituting (A.3) in (A.2) and noting (A.1), yields:

$$\begin{aligned} V_1 = & \int_0^\Delta \left[\int_0^{t_e} X(x_1, t) g_{T, T_e}(t, t_e) dt + \int_{t_e}^\infty Y(x_1, t_e) g_{T, T_e}(t, t_e) dt \right] dt_e \\ & + \int_\Delta^\infty \left[\int_0^\Delta X(x_1, t) g_{T, T_e}(t, t_e) dt + \int_\Delta^\infty Z(x_1, \Delta) g_{T, T_e}(t, t_e) dt \right] dt_e \end{aligned}$$

$$\begin{aligned}
&= \int_0^\infty dv_\theta \left\{ \int_0^\Delta f_{T_e}'(t_e) \left[\int_0^{t_e} X(x_1, t) f_T'(t) dt + \right. \right. \\
&\quad \left. \left. (1-f_{T_e}(t_e)) Y(x_1, t_e) \right] dt_e + \right. \\
&\quad \left. \int_\Delta^\infty f_{T_e}'(t_e) \left[\int_0^\Delta X(x_1, t) f_T'(t) dt + \int_\Delta^\infty Z(x_1, \Delta) f_T'(t) dt \right] dt_e \right\}
\end{aligned}$$

Suppressing the dependence on θ for notational convenience. Integrating by parts the terms that contain $f_{T_e}' dt_e$ and X , noting that $F_{T_e}(0) = 0$, changing variables and collecting terms, provides

$$\begin{aligned}
V_1 &= \int_0^\infty dv_\theta(\theta) \left\{ \int_0^\Delta \left[X(x_1, t) (1-f_{T_e}(t)) f_T'(t) + \right. \right. \\
&\quad \left. \left. Y(x_1, t) (1-f_T(t)) f_{T_e}'(t) \right] dt + \right. \\
&\quad \left. Z(x_1, \Delta) (1-f_T(\Delta)) (1-f_{T_e}(\Delta)) \right\} \tag{A.5}
\end{aligned}$$

which is equation (2) of the main text. Substitution from (1) in (A.5), integrating by parts the terms that contain both π_1 and a distribution function f_T' or f_{T_e}' , changing variables and collecting terms, finally results in (3), with $V_i^0 = V_i^0(m_i, x_1)$, $i=1$ or 2 , and $U_1^0 = U_1^0(m_1)$,

$$\begin{aligned}
V_1 &= \int_0^\infty dv_\theta \left\{ \int_0^\Delta e^{-\tau t} \left[\pi_1(x_1) (1-f_{T_e}(t)) (1-f_T(t)) + \right. \right. \\
&\quad \left. \left. V_2^0 (1-f_{T_e}(t)) f_T'(t) + U_1^0 (1-f_T(t)) f_{T_e}'(t) \right] dt + \right. \\
&\quad \left. e^{-\tau \Delta} V_1^0 (1-f_T(\Delta)) (1-f_{T_e}(\Delta)) \right\}
\end{aligned}$$

Appendix B

The following lemma is useful to determine the claims made in Proposition 4 :

Lemma B.1. : With stochastically independent distribution functions f_T and f_{T_e} on resp. the timing of the change in economic circumstances and the termination of the program and with $a(t) = f'_T / (1 - f_T)$ and $b(t) = f'_{T_e} / (1 - f_{T_e})$ the corresponding conditional probability density functions :

$$\text{sign } \frac{\partial x_1^0}{\partial \Delta} = \text{sign} \left[\frac{da}{d\Delta}(\Delta)(m_1 - m_2) + \frac{db}{d\Delta}(\Delta) \right]$$

Implicit differentiation of (8) and substituting again from (8) for the term containing $(m_1 - m_2)$ reveals:

$$\text{sign } \frac{\partial x_1^0}{\partial \Delta} = \text{sign } I(\Delta)$$

with

$$\begin{aligned} I(\Delta) &= \pi'_1(x_1^0) + \left(\frac{1 - e^{-\rho\Delta}}{\rho} \right) [(m_1 - m_2) a(\Delta) + m_1 (\rho + b(\Delta))] \\ &= \pi'_1(x_1^0) [1 - e^{-\rho\Delta} (1 - f_T)(1 - f_{T_e}) \\ &\quad - (\rho + a + b) \int_0^\Delta e^{-\rho t} (1 - f_T)(1 - f_{T_e}) dt] \\ &\quad + \left(\frac{1 - e^{-\rho\Delta}}{\rho} \right) m_2 [(\rho + a + b) \int_0^\Delta e^{-\rho t} (1 - f_{T_e}) f'_T dt \\ &\quad - a(1 - e^{-\rho\Delta} (1 - f_T)(1 - f_{T_e}))] \end{aligned}$$

All functions that are not appearing behind an integral sign in the last equation are evaluated at Δ . But $I(0) = 0$, noting $f_T(0) = 0 = f_{T_e}(0)$, while the sign $\frac{dI}{d\Delta} = \text{sign} \left[\frac{da}{d\Delta} (\Delta)(m_1 - m_2) + \frac{db}{d\Delta} (\Delta)m_1 \right]$ so that the claim of the lemma is established.

To obtain the results of Proposition 4 it is sufficient to put $b=0$, $\Delta = \Delta_e$ and $a = F'_T/(1-F_T)$ with $F_T = \int_0^\infty dv_\theta f_t(t; \theta)$ in the lemma above.

The discussion following Proposition 6 is based on the following result :

Lemma B.2. :

For $f_T(t; \theta) = 1 - e^{-\int_0^t \theta a(u) du}$ and $v_\theta(\theta)$ a distribution function,

$H_T(t) = F'_T(t)/(1-F_T(t))$ with $F(t) = \int_0^\infty f_T(t; \theta) dv_\theta(\theta)$ can be cast into

the form $H_T(t) = a(t) \varphi \left[\int_0^t a(u) du \right]$ where φ is decreasing.

Proof:

One has

$$1 - F(t) = \int_0^\infty e^{-\theta \int_0^t a(u) du} dv_\theta(\theta)$$

consequently

$$H_T(t) = -\frac{d}{dt} \ln \int_0^\infty e^{-\theta \int_0^t a(u) du} dv_\theta(\theta)$$

and hence

$$H_T(t) = a(t) \varphi \left(\int_0^t a(u) du \right)$$

where

$$\varphi(z) = \frac{\int_0^\infty e^{-\theta z} \theta \, dv_\theta(\theta)}{\int_0^\infty e^{-\theta z} \, dv_\theta(\theta)}$$

One still has to show that $\varphi(\cdot)$ denotes a decreasing function of z .

The derivative

$$\varphi'(z) = \frac{- \int_0^\infty e^{-\theta z} \theta^2 \, dv_\theta(\theta) \int_0^\infty e^{-\theta z} \, dv_\theta(\theta) + \left[\int_0^\infty e^{-\theta z} \theta \, dv_\theta(\theta) \right]^2}{\left[\int_0^\infty e^{-\theta z} \, dv_\theta(\theta) \right]^2}$$

or

$$\varphi'(z) = - \int_0^\infty \theta^2 \, dU(\theta) + \left[\int_0^\infty \theta \, dU(\theta) \right]^2$$

with

$$dU(\cdot) = \frac{e^{-\theta z} \, dv_\theta(\theta)}{\int_0^\infty e^{-\theta z} \, dv_\theta(\theta)}$$

The distribution U denotes the so-called Essler transform of the distribution $v_\theta(\theta)$.

Consequently $\varphi'(z) = - \text{variance of the distribution } U$, hence $\varphi'(z) \leq 0$.

Appendix C

To prove the Proposition 5, the derivative $\partial V'_1 / \partial x$ is evaluated at a value $\underline{x}$ that is optimal for a program ending at a time $\Delta e < \Delta$, and given $\partial H_T(\Delta) / \partial \Delta \geq 0$ and $c_1 \geq c_2$. From (11) it is seen that $\underline{x}$ solves:

$$\begin{aligned} & (\pi'_1(\underline{x}) + (1-e^{-\rho\tau})m_2) \int_0^{\Delta e} e^{-\rho t} (1-F_T) dt \\ & + \left(\frac{1-e^{-\rho\tau}}{\rho} \right) (m_1 - m_2) [1-e^{-\rho\Delta} e(1-F_T(\Delta_e))] = 0 \end{aligned}$$

Solving for the term containing $(m_1 - m_2)$ and substituting in $\partial V'_1 / \partial x = 0$ as defined by (10) establishes

$$\begin{aligned} \partial V'_1 / \partial x \Big|_{x=\underline{x}} &= [\pi'_1 + (1-e^{-\rho\tau})m_2] \{ [\int_0^\infty dv_\theta \int_0^\Delta e^{-\rho t} (1-f_T)(1-f_{T_e}) dt \\ &\quad - \left(\frac{\int_0^{\Delta e} e^{-\rho t} (1-F_T)}{1-e^{-\rho\Delta} e(1-F_T)} \right) (1-e^{-\rho\Delta} (1-F_T(\Delta)))] \} \\ &\quad + \left(\frac{1-e^{-\rho\tau}}{\rho} \right) m_2 \int_0^\infty dv_\theta \int_0^\Delta e^{-\rho t} [\rho f_{T_e} (1-f_T) + f_{T_e} f'_T] dt \end{aligned}$$

Denoting by A the expression between akolades, it is clear that $A \leq B$, with

$$B = \int_0^{\Delta} e^{-\rho t} (1-F_T) dt - \left(\frac{\int_0^{\Delta_e} e^{-\rho t} (1-F_T) dt}{1-e^{-\rho \Delta_e} (1-F_T(\Delta_e))} \right) (1-e^{-\rho \Delta} (1-F_T(\Delta)))$$

with $B|_{\Delta=\Delta_e} = 0$. Letting $C(\Delta) = \int_0^{\Delta} e^{-\rho t} (1-F_T) dt / (1-e^{-\rho \Delta} (1-F_T(\Delta)))$,

it is verified that the sign $dC/d\Delta = \text{sign} [1-e^{-\rho \Delta} (1-H_T(\Delta)) - (\rho+H_T)$

$\int_0^{\Delta} e^{-\rho t} (1-f_T) dt]$. This last expression is zero for $\Delta = 0$ and is negative (zero) (positive) as $\partial H_T / \partial \Delta$ is increasing (constant) (decreasing).

Hence for $\partial H_T(\Delta) / \partial \Delta \geq 0$, B must be negative for any $\Delta > \Delta_e$. Therefore

$A < 0$ and with $c_1 \geq c_2$ and $m = sc$ it must be that $[\pi'_1(\underline{x}) + (1-e^{-\rho \tau})m_2] \leq 0$,

so that $\partial V'_1 / \partial x |_{x=\underline{x}} > 0$. For $c_1 \geq c_2$ and $m = -sc$ all of the inequality signs

need to be reversed. The proof rests on the supposition that $\Delta < \infty$, for

$\Delta \rightarrow \infty$ the results can be established along somewhat different lines.

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