Strategy Development in Children with Mathematical Disabilities:

Insights from the Choice/No-Choice Method and the Chronological-Age/Ability-Level-Match Design

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Abstract

This study investigated the strategy characteristics and development of children with mathematical disabilities (MD) in the domain of simple addition and subtraction, in terms of Lemaire and Siegler's model of strategic change, using the choice/no-choice method and the combined chronological-age (CA)/ability-level (AL)-match design. Four groups of children, matched on either CA or mathematical AL, solved a series of 36 problems with the bridge over 10 in four conditions. In the choice condition, children could choose between retrieval, decomposition to 10, and counting on to solve each problem. In the retrieval, decomposition, and counting conditions, they had to answer the same 36 problems using retrieval, decomposition to 10, and counting on, respectively. The results revealed clear differences in the frequency, efficiency, and adaptiveness with which the CA-matched children applied the available strategies. In contrast, we observed no differences in strategy frequency, efficiency, and adaptiveness between the AL-matched children. These results support the hypothesis that the strategy development of children with MD is marked by a delay rather than a specific deficit. Moreover, this study further documents the value of the methodology used to study children's strategy use and development in the domain of simple arithmetic.

During the last decade, the number of studies on the strategy characteristics and development of children with mathematical disabilities (MD) has increased significantly. The majority of these studies has been conducted in the domain of simple arithmetic, in particular addition and subtraction up to 20, using models of typical arithmetical development as the guiding principle for data collection and data analysis (Geary, Hoard, & Hamson, 1999). One influential model is the *model of strategic change* proposed by Lemaire and Siegler (1995). This model distinguishes four dimensions along which developmental changes in strategy competence can occur. The first dimension, *strategy repertoire*, refers to the different strategies a student uses to solve a task. The second dimension, *strategy distribution*, involves the rela-

tive frequency with which each strategy is used. The third dimension, *strategy efficiency*, concerns the accuracy and speed of strategy execution. The fourth dimension, *strategy selection*, refers to the adaptiveness of individual strategy choices: Does the individual choose the most efficient strategy, that is, the strategy that leads the individual fastest to an accurate answer to the problem?

Studies on the characteristics of the strategies that first and second graders with MD use to solve simple additions and subtractions up to 20 (Bull & Johnston, 1997; Geary, 1990; Geary, Bow-Thomas, & Yao, 1992; Geary et al., 1999; Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2000) revealed that they apply the same repertoire of retrieval (e.g., "8 + 5 = 13, I know this by heart") and counting (e.g., counting on; "8 + 5 = (8), 9, 10, 11, 12, 13") strate-

gies as their typically achieving peers. However, first graders and second graders with MD seem to rely more frequently on immature counting strategies and less frequently on retrieval than their peers without MD. The students with MD also execute the counting and retrieval strategies less accurately than their peers without MD. Moreover, whereas first and second graders without MD adapt their strategy choices to the difficulty of the problems (i.e., they prefer counting strategies to solve difficult problems and apply retrieval most frequently on easier ones), children with MD make less adaptive strategy choices. At older ages (i.e., third to sixth graders; Geary & Brown, 1991; Jordan & Montani, 1997; Ostad, 1998), the differences in strategy accuracy and adaptiveness between children with MD and their peers without MD seem to disappear.

However, the differences in strategy frequency remain. Whereas typically achieving third to sixth graders retrieve the answer to the majority of simple additions and subtractions, their peers with MD rely primarily on counting.

The results of studies focusing on the development of addition and subtraction strategies in children with MD (Geary, Brown, & Samaranayake, 1991; Geary, Hamson, & Hoard, 2000; Gray, 1991; Ostad, 1997) are generally in line with those of the aforementioned studies on strategy characteristics. Both younger and older elementary school children with MD apply the same repertoire of retrieval and counting strategies on simple additions and subtractions as their peers without MD. As age and experience increase, typically achieving children execute the retrieval strategy ever more frequently and also more accurately and, consequently, rely ever less on counting strategies. In contrast, children with MD show almost no developmental changes in strategy frequency and accuracy. Even after extensive experience with additions and subtractions up to 20, children with MD keep solving most problems with counting strategies and keep applying the retrieval strategy infrequently. Although the accuracy of counting strategies clearly increases with age and experience, the accuracy of retrieval remains low.

Taking into account these findings on strategy characteristics and development in children with MD, Geary (1993, 1994) concluded that children with MD are, in comparison with their typically achieving peers, marked by a delay in the development of counting strategies (i.e., the counting strategies of children with MD develop slower than those of their peers without MD) as well as by a more fundamental retrieval deficit (i.e., the retrieval strategy does not develop more slowly, but qualitatively differently, in children with MD than in their peers without MD).

However, the aforementioned studies on the strategy characteristics and

development of children with MD suffer from two methodological weaknesses that jeopardize their results and conclusions. First, most researchers studied the efficiency and adaptiveness of strategy use by means of the so-called *choice method* (Siegler & Lemaire, 1997; see Note 1). This method involves offering all participants a set of items in only one condition, in which all participants can freely choose which strategy they will use to solve each item. However, the data about the efficiency of the strategies used in such a choice condition can be biased by selection effects. As Siegler and Lemaire (1997) argued convincingly, a strategy that is used mainly on the most difficult problems, or applied primarily by the least able participants, will seem less efficient than a strategy that is used almost exclusively to solve the easiest problems or applied most frequently by the most able participants. Thus, the choice method does not yield unbiased data about the efficiency of the different strategies and, consequently, makes it difficult if not impossible to assess the adaptiveness of individual strategy choices. As an alternative to this methodology, Siegler and Lemaire (1997) developed the *choice/no-choice method*. This method requires testing each participant under two types of conditions. In the choice condition, participants can use their preferential strategy on each problem. In the no-choice condition, the researcher forces them experimentally to solve all problems with one particular strategy. The obligatory use of one particular strategy on all problems in the no-choice condition makes selective assignments of strategies to problems or participants impossible and, thus, yields unbiased data about strategy efficiency. Moreover, comparison of the data about the efficiency of the different strategies in the no-choice conditions with the strategy choices made in the choice condition allows the experimenter to assess the adaptiveness of individual strategy choices accurately: Does the participant solve each problem in the choice condition with the

strategy that leads fastest to an accurate answer to this problem, as evidenced by the information obtained in the no-choice conditions?

A second weakness of the aforementioned studies is that participants were selected according to a chronological-age (CA)–match design, which means that the researcher selected children matched on chronological age but of different mathematical ability level (e.g., children with MD and their peers without MD; see Note 2). In line with the CA-match design, the strategy development of children with MD was further described by comparing the strategy characteristics of these children with the same strategy characteristics of their peers without MD. However, as discussed in detail in the domain of reading development (e.g., Backman, Mamen, & Ferguson, 1984; Bradley & Bryant, 1978; Ellis, McDougall, & Monk, 1996; Rack, Snowling, & Olson, 1992), a profound understanding of the strategy development of children with MD requires the combined use of a chronological-age–match and ability-level (AL)–match design. The combined CA/AL-match design allows the researcher to compare the strategy characteristics of children with MD with those of their peers without MD (CA-match comparison) as well as with those of younger children matched on mathematical ability level with the children with MD (AL-match comparison). If the strategy characteristics of children with MD differ from the strategy characteristics of the CA-matched children but not from those of the AL-matched children, then the strategy characteristics of children with MD simply reflect their mathematical ability level, suggesting that their strategy development is marked by a delay. If, in contrast, the strategy characteristics of children with MD differ from the strategy characteristics of both the CA-matched and the AL-matched children, we can rule out the possibility that the strategy characteristics of children with MD simply reflect their mathematical ability level, suggesting

that their strategy development is not marked by a delay but by a more fundamental deficit.

Taking into account these two methodological weaknesses of previous research on simple addition and subtraction strategies in children with MD, the aim of the present study was to investigate the strategy characteristics and development of these children using a research methodology that integrates the choice/no-choice method with the combined CA/AL-match design.

Method

Participants

In line with the CA/AL-match design, four groups of children participated in the study, namely (a) second graders with strong mathematical abilities, (b) second graders with weak mathematical abilities, matched on CA with the strong second graders, (c) fourth to sixth graders with moderate MD, matched on mathematical AL with the strong second graders, and (d) fourth to sixth graders with severe MD, matched on CA with the fourth to sixth graders with moderate MD, and matched on mathematical AL with the weak second graders.

The second graders with strong and weak mathematical abilities were selected from a group of 359 second graders from eight elementary schools for general education in Flanders. The

fourth to sixth graders with moderate and severe MD were selected from a group of 166 fourth to sixth graders with MD from 12 Flemish special schools for children with specific learning disabilities (LD; see Note 3). All second graders and all fourth to sixth graders with MD completed two standardized achievement tests, *Rekenen Begin Tweede Leerjaar* (Arithmetic for Beginning Second Graders; AB2; Dudal, 2000) and *Lexicale-Decisionproef* (Lexical Decision Test; LDT; Schippers & Corens, 2000), and the *Standard Progressive Matrices Set A and B* (SPM A+B; Raven, Court, & Raven, 1979). The AB2 is a standardized group test designed to assess the mathematical abilities of beginning second graders. It addresses children's number sense (30 items), arithmetical abilities (20 items), and measurement competencies (10 items). The LDT is a standardized paper-and-pencil test for assessing 6- to 12-year-olds' reading abilities. The LDT contains 90 Dutch words and 30 pseudowords. Children are asked to cross out as many pseudowords as possible within a time limit of 2 minutes. The SPM A+B is a test for measuring young children's general intellectual abilities. Children are shown two series of 12 patterns in which one piece is missing. Children are required to complete each pattern by choosing the only correct piece out of a row of six alternatives.

The second graders were selected on the basis of three criteria. With respect

to the *intelligence* criterion, children had to be of at least average intelligence (i.e., scoring 1 *SD* or less below the mean score of their age group on the SPM A+B). With respect to the *reading ability* criterion, they had to have at least average reading abilities (i.e., scoring 1 *SD* or less below the mean score of their age group on the LDT). Finally, with respect to the *mathematical ability* criterion, they had to have either strong mathematical abilities (i.e., scoring at or above the 75th percentile on the AB2) or weak mathematical abilities (i.e., scoring at or below the 25th percentile on the AB2). The two groups of children with MD were selected on the basis of the same intelligence criterion as the two groups of second graders. In other words, they had to have at least average intelligence. Because children who receive instruction in Flemish special schools for children with specific LD frequently have difficulties with both reading and mathematics (De Fever & Coppens, 1997), no reading ability criterion was included for these children. Finally, the selected children with MD were matched on mathematical ability level with the strong and weak second-graders: Only those children with MD with a raw score at or above the 75th percentile, respectively at or below the 25th percentile on the AB2, were included in the sample. Table 1 summarizes the children's demographics and test scores by group.

TABLE 1
Number, Age, Intelligence, and Reading and Mathematics Test Scores by Group

Group	<i>n</i>		Age (mos.)		SPM A+B		LDT		AB2	
	Boys	Girls	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Strong2	14	12	88.15	2.84	18.23	2.64	58.42	18.25	52.81	0.85
Weak2	08	17	88.52	4.66	15.52	1.83	42.32	11.85	31.20	6.16
MD-m	15	04	123.74	11.27	19.16	2.29	55.26	28.81	52.95	0.85
MD-s	09	10	125.00	10.50	17.37	2.34	45.37	26.72	33.53	4.29

Note. SPM A+B = *Standard Progressive Matrices Set A and B* (Raven et al., 1979); LDT = *Lexical Decision Test* (Schippers & Corens, 2000); AB2 = *Arithmetic for Beginning Second Graders* (Dudal, 2000). The maximum scores on the SPM A+B, LDT, and AB2 are respectively 24, 120, and 60. Strong2 = second graders with strong mathematical abilities; Weak2 = second graders with weak mathematical abilities; MD-m = fourth to sixth graders with moderate mathematical disabilities; MD-s = fourth to sixth graders with severe mathematical disabilities.

Table 1 shows that the age of the strong second graders did not differ significantly from the age of the weak second graders at the time of testing, $S_W = 638.000$, $p = .8201$ (see Note 4), and that the former scored higher on the AB2 than the latter, $S_W = 325.000$, $p < .0001$. The strong and weak second graders thus had matching CA but differed in mathematical AL. Whereas all second graders met the criterion of having at least average intellectual and reading abilities, the strong second graders scored higher on the SPM A+B, $S_W = 439.500$, $p < .0001$, and the LDT, $S_W = 461.000$, $p = .0002$, than the weak second graders.

Table 1 further reveals that the age of the fourth to sixth graders with moderate MD did not differ significantly from that of the fourth to sixth graders with severe MD, $S_W = 353.500$, $p = .6193$, and that the former scored higher on the AB2 than the latter, $S_W = 551.000$, $p < .0001$. The children with moderate and severe MD thus were matched on CA but differed in mathematical AL. Despite the fact that the children with moderate and severe MD all had at least average intellectual abilities, the children with moderate MD scored higher on the SPM A+B than the children with severe MD, $S_W = 448.500$, $p = .0107$. The two groups' scores on the LDT were not significantly different, $S_W = 411.000$, $p = .2367$.

The fourth to sixth graders with moderate MD were older than the strong second graders at the time of testing, $S_W = 684.000$, $p < .0001$, but did not differ in their scores on the AB2, $S_W = 460.000$, $p = .5725$. The fourth to sixth graders with moderate MD and the strong second graders thus had matched mathematical AL but differed in CA. The two groups of children did not differ in their scores on the SPM A+B, $S_W = 492.500$, $p = .1987$, and the LDT, $S_W = 389.000$, $p = .2697$.

Finally, Table 1 reveals that the fourth to sixth graders with severe MD were older than the weak second graders at the time of testing, $S_W = 665.000$, $p < .0001$, but did not differ in their scores on the AB2, $S_W = 477.500$,

$p = .2344$. The fourth to sixth graders with severe MD and the weak second graders thus were matched on mathematical AL but differed in CA. The fourth to sixth graders with severe MD scored higher on the SPM A+B than the weak second graders, $S_W = 550.500$, $p = .0013$, but their scores on the LDT did not differ, $S_W = 390.000$, $p = .3738$.

The second graders were tested in November 2001, and the fourth to sixth graders with MD in March 2002. At the respective times of testing, all children had learned how to solve additions and subtractions with the bridge over 10. The second graders were taught how to solve such problems at the end of first grade (May–June 2001) and had intensively practiced these problems during the first months of second grade (September–October 2001). The children with MD had intensively practiced problems with the bridge over 10 for at least 4 months, but the amount of practice differed from child to child. A careful analysis of the textbook used and a structured interview with the teachers of the participating children revealed that the teachers of the second graders and the teachers of the children with MD had heavily focused on mastery of the strategy of decomposition to 10 (e.g., $8 + 5 = 8 + 2 + 3 = 10 + 3 = 13$). Other decomposition strategies (e.g., the tie strategy; $6 + 7 = 6 + 6 + 1 = 12 + 1 = 13$) had not been explicitly and systematically taught to the children.

Materials

All children solved a series of 40 additions and subtractions with the bridge over 10 (e.g., $8 + 5$ or $13 - 5$; by respectively adding or subtracting the second integer, children have to pass the number 10). The series consisted of 4 practice items (2 additions and 2 subtractions) and 36 experimental items (18 additions and 18 subtractions). The additions were constructed from all possible pairwise combinations of the integers 2 to 9; they included all combinations with the largest addend first ($n = 16$) and all tie sums ($n = 4$). The

subtractions were exactly the reverse of the additions. The additions and subtractions were ordered on the basis of the following three constraints:

1. the additions and subtractions are offered separately (i.e., each child starts with the additions and solves the subtractions afterward or vice versa);
2. the repetition of neither integer nor answer is permitted across two consecutive trials; and
3. the answer to each experimental item cannot be deduced easily from the previous item.

Four lists of problems were constructed. All four lists consisted of the same 36 addition and subtraction problems, but the order of the problems differed.

Conditions

All children individually completed the 40 items in four conditions. In the *choice condition*, children could solve each problem with retrieval, decomposition to 10, or counting on with steps of one unit. They could choose among the three strategies by means of pictures that represented the strategies visually as shown in the Appendix. Problems were offered on A4 or 8.5×11 -inch paper, one problem per page. Children were instructed to solve all problems as fast and as accurately as possible with one of the three visualized strategies. No verbal statement of the strategy or of the answer was required. Children were asked to complete directly the box that corresponded to the strategy chosen. If a child used retrieval, he or she had to write down the retrieved answer in the first box. If a child applied decomposition to 10, he or she had to write down both the way he or she split the second integer and the answer in the second box. In the case of counting on, the child was asked to point to or cross out the counted fingers while counting and to write down the answer in the third box.

In the three no-choice conditions, children were forced to use either retrieval, decomposition to 10, or counting on by means of the instruction and the presentation of the problems. In the *retrieval condition*, all problems were offered for maximum 2 seconds, which made it almost impossible for the children to use a strategy other than retrieval. In the *decomposition condition*, only the picture that visualized decomposition to 10 was offered to the children. As in the choice condition, children had to write down both the way they split the second integer and the answer directly (without verbally stating it first). In the *counting condition*, children received only the picture that visualized counting on. In line with the choice condition, children had to point to or cross out the counted fingers while counting and write down their answer afterward.

Procedure

As we said before, all children were tested individually in each condition. In each condition, a different list (of the same 36 problems, but in different order) was used. Thus, all children solved all four lists in the course of the whole experiment, and every list was seen by an almost equal number of children in each condition. All children started with the choice condition, so that strategy choices in the choice condition would not be influenced by recency effects from having solved all problems with one particular strategy in the no-choice conditions. The order of the no-choice conditions was counterbalanced as much as possible over the children within each group. The consecutive conditions were separated for each child by at least one day in time. The experimenter registered the reaction time and the answer per child and per problem in each condition.

Results

The results are presented for the four dimensions of Lemaire and Siegler's

(1995) model of strategic change. Taking into account the aim and design of the study, we compare the strategy repertoire, frequency, efficiency, and adaptiveness of both the CA-matched and the AL-matched groups. More specifically, we first compare the strategy characteristics of the strong second graders with those of the weak second graders (CA-match comparison) and the strategy characteristics of the fourth to sixth graders with moderate MD with those of the fourth to sixth graders with severe MD (CA-match comparison). Second, the strategy characteristics of the fourth to sixth graders with moderate MD and the strong second graders are compared as well as those of the fourth to sixth graders with severe MD and the weak second graders (AL-match comparisons). If the strategy development of children with MD is characterized merely by a delay, we expect differences in strategy use between the CA-matched groups but not between the AL-matched groups. If, in contrast, the strategy development of children with MD is marked by a deficit, we expect differences in strategy use both between the CA-matched and between the AL-matched groups.

We executed multilevel analyses to determine the frequency, accuracy, speed, and adaptiveness of the strategies, using hierarchical linear models with two levels: Items (Level 1) are "nested" within children (Level 2; see Note 5). We analyzed the data about strategy frequency and speed by means of the so-called "proc mixed" of the SAS computer program (Littell, Milliken, & Stroup, 1996), which assumes the data to be normally distributed. Taking into account the binomial distribution of the accuracy and adaptiveness data, we analyzed these data by means of the so-called "glimmix macro" of the same statistical program.

Initial analyses indicated that neither the sequence of the problems in the list nor the order of the three no-choice conditions influenced children's task performances (i.e., accuracy and speed of problem solving). Therefore,

we grouped the data across lists and order of conditions in all further analyses. Taking into account the group differences in intellectual and reading abilities, children's scores on the SPM A+B and LDT were used as a covariate in all analyses. All reported results are statistically significant at an alpha level of .05, unless otherwise noted.

Strategy Repertoire

The four groups of children used retrieval, decomposition to 10, and counting on at least once to solve the set of 36 items in the choice condition. Moreover, each group applied the three strategies at least once on both the additions and the subtractions.

Most children used multiple strategies to solve the problems in the choice condition. Eight strong second graders, three weak second graders, eight children with moderate MD, and two children with severe MD solved all problems with only one strategy. The other children (i.e., 18 strong second graders, 22 weak second graders, 11 children with moderate MD, and 17 children with severe MD) applied two or three strategies. We observed no differences in strategy repertoire between the strong and weak second graders, $\chi^2(2, N = 51) = 2.9836, p = .2250$. However, the repertoire of strategies used differed between the children with moderate and those with severe MD, $\chi^2(2, N = 38) = 9.0971, p = .0106$. More children with severe MD ($n = 8$) than with moderate MD ($n = 1$) applied all three types of strategies to solve the problems in the choice condition. Finally, we found no differences in strategy repertoire between the AL-matched groups, that is, between the children with moderate MD (MD-m) and the strong second graders (Strong2), $\chi^2(2, N = 45) = 2.0179, p = .3646$, and between the children with severe MD (MD-s) and the weak second graders (Weak2), $\chi^2(2, N = 44) = 0.4777, p = .7876$.

Strategy Distribution

We predicted the relative frequency of strategy use in the choice condition by

means of the variables of strategy (retrieval, decomposition to 10, counting on), group (Strong2, Weak2, MD-m, MD-s), and problem type (addition, subtraction). The strategy variable was added to the model as a random coefficient, which means that the influence of this variable on the relative frequency of strategy use may differ from child to child. Table 2 summarizes the relative frequency of retrieval, decomposition to 10, and counting on strategies in the choice condition by group and by problem type (least-squares means).

First, the three strategies were applied with different frequencies in the choice condition, $F(2, 171) = 21.68, p < .0001$. The children used retrieval most frequently, and they solved more problems with decomposition to 10 than with counting on.

Second, we observed group differences in the frequency of strategy use in the choice condition, $F(6, 260) = 8.35, p < .0001$. The frequency of strategy use differed clearly between the CA-matched groups. The strong second graders solved more problems with retrieval and less problems with counting on than their weak peers. Both second-grade groups applied the decomposition strategy with the same frequency. Likewise, the fourth to sixth graders with moderate MD used retrieval more frequently than their

peers with severe MD. The frequency of decomposition to 10 and counting on did not differ between the MD groups. With respect to the AL-matched groups, we found no differences in the frequency of retrieval, decomposition to 10, and counting on between the fourth to sixth graders with moderate MD and the strong second graders. Also, the frequency of retrieval and decomposition to 10 did not differ between the fourth to sixth graders with severe MD and the weak second graders. However, the MD-s group used counting on less than the weak second graders.

Finally, the frequency of retrieval, decomposition to 10, and counting on differed between problem types, $F(2, 260) = 11.43, p < .0001$. The children applied retrieval more frequently and decomposition to 10 less frequently on the additions than on the subtractions. The frequency of counting on was the same for the two problem types.

Strategy Efficiency

We derived the accuracy and speed of retrieval, decomposition to 10, and counting on from the score and the reaction time, respectively, in the corresponding no-choice conditions. In the retrieval condition, all problems that were answered out of time (i.e., not within 2 seconds) were scored as in-

correct. The accuracy of strategy execution in the no-choice conditions was predicted by means of the variables of strategy (retrieval, decomposition to 10, counting on), group (Strong2, Weak2, MD-m, MD-s), and problem type (addition, subtraction). We added the strategy variable as a random coefficient to the model. Because we experimentally forced the children to answer all problems within 2 seconds in the retrieval condition, we did not include the reaction time data from this condition in the analysis. The speed of strategy execution was thus predicted by means of the variables of strategy (decomposition to 10, counting on), group (Strong2, Weak2, MD-m, MD-s), and problem type (addition, subtraction), with strategy and problem type as random coefficients. Table 3 describes the accuracy and speed of strategy execution in the retrieval, decomposition, and counting condition for each group and each problem type (least-squares means).

The children did not solve the problems with the same accuracy in the different no-choice conditions, $F(2, 170) = 328.60, p < .0001$. They solved the problems most accurately in the decomposition condition. As shown in Table 3, the obligatory use of decomposition to 10 resulted almost always in a correct answer. The accuracy rate was lowest in the retrieval condition. The low accuracy rate in the retrieval condition was due to the fact that most problems were answered out of time, rather than to their being answered inaccurately (86% of the answers retrieved within 2 seconds were correct).

Furthermore, the accuracy of strategy execution in the no-choice conditions differed between the four groups of children, $F(6, 9244) = 2.97, p = .0067$. We observed clear differences in strategy accuracy between the CA-matched groups. The strong second graders executed retrieval and decomposition to 10 more accurately than their weak peers. For counting on, no significant difference in accuracy was found between both groups. Likewise, the fourth to sixth graders with moderate MD ex-

TABLE 2
Relative Frequency of Strategies in the Choice Condition by Group and Problem Type (Least-Squares Means)

Variable	Retrieval	Decomposition to 10	Counting on
Group			
Strong2	68.68	20.78	10.54
Weak2	31.77	25.67	42.56
MD-m	61.11	33.33	5.56
MD-s	37.93	41.00	21.07
All	49.87	30.20	19.93
Problem type			
Addition	54.29	25.37	20.34
Subtraction	45.46	35.03	19.51

Note. Frequency is expressed in percentages. Strong2 = second graders with strong mathematical abilities; Weak2 = second graders with weak mathematical abilities; MD-m = fourth to sixth graders with moderate mathematical disabilities; MD-s = fourth to sixth graders with severe mathematical disabilities.

ecuted retrieval and decomposition to 10 more accurately than their peers with severe MD, whereas the accuracy of counting on was the same for both groups of children. In contrast to the findings for the CA-matched groups, we found no differences in strategy accuracy between the AL-matched groups. The fourth to sixth graders with moderate MD solved the problems as accurately as the strong second graders in the retrieval, decomposition, and counting conditions. Similarly, the fourth to sixth graders with severe MD applied retrieval, decomposition to 10, and counting on in the respective no-choice conditions with the same accuracy as the weak second graders.

Finally, the accuracy of strategy execution in the no-choice conditions did not differ between the addition and subtraction problems, $F(2, 9244) = 0.45$, $p = .6406$.

With respect to the speed of strategy execution in the no-choice conditions, we found that the children did not execute decomposition to 10 and counting on equally fast, $F(1, 85) = 4.51$, $p = .0366$. Problems were solved faster with decomposition to 10 than with counting on.

Moreover, we observed group differences in strategy speed in the decomposition and counting condition, $F(3, 6046) = 5.41$, $p = .0010$. The speed of strategy execution differed between the CA-matched groups. The strong second graders solved the problems in the decomposition condition faster than their weak peers. The two groups did not differ in speed in the counting condition. Likewise, the fourth to sixth graders with moderate MD applied decomposition to 10 faster than their peers with severe MD, whereas there were no differences in the speed of counting on between the two groups of children. With respect to the AL-matched groups, we found no differences in the speed of strategy execution in the decomposition and counting conditions between the fourth to sixth graders with moderate MD and the strong second graders. The

TABLE 3
Accuracy and Speed of Strategy Execution in the No-Choice Conditions by Group and Problem Type (Least-Squares Means)

Variable	Retrieval		Decomposition to 10		Counting on	
	Accuracy	Speed ^a	Accuracy	Speed	Accuracy	Speed
Group						
Strong2	0.23	—	0.99	7.71	0.96	11.27
Weak2	0.12	—	0.88	14.81	0.92	13.12
MD-m	0.20	—	0.99	5.97	0.94	9.63
MD-s	0.08	—	0.94	10.74	0.92	10.35
All	0.15	—	0.97	9.81	0.94	11.09
Problem type						
Addition	0.17	—	0.97	10.20	0.95	9.85
Subtraction	0.13	—	0.96	9.41	0.92	12.33

Note. Accuracy is expressed in proportion correct, speed in seconds. Strong2 = second graders with strong mathematical abilities; Weak2 = second graders with weak mathematical abilities; MD-m = fourth to sixth graders with moderate mathematical disabilities; MD-s = fourth to sixth graders with severe mathematical disabilities.

^aBecause the children were forced to answer all problems within 2 seconds in the retrieval condition, we did not include the reaction time data from this condition in the analysis.

fourth to sixth graders with severe MD solved the problems as fast as the weak second graders in the counting condition, but the weak second graders answered the problems more slowly than the children with severe MD in the decomposition condition.

Finally, the speed of strategy execution in the decomposition and counting conditions differed between the problem types, $F(1, 6046) = 59.38$, $p < .0001$. Remarkably, the children solved the additions more slowly than the subtractions in the decomposition condition. However, in the counting condition, the additions were answered faster than the subtractions.

In addition to the analyses of the accuracy and speed data reported previously, we analyzed the accuracy and speed data gathered in the choice condition. As illustrated by Siegler and Lemaire (1997), comparison of the data about each strategy's accuracy and speed gathered in the choice condition with the accuracy and speed data obtained in the no-choice conditions makes it possible to test the value of the accuracy and speed data obtained in the choice condition. The accuracy of strategy execution in the choice condition was predicted by means of the

variables of strategy (retrieval, decomposition to 10, counting on), group (Strong2, Weak2, MD-m, MD-s), and problem type (addition, subtraction). The strategy variable was added as a random coefficient to the model. The speed of strategy execution in the choice condition was predicted by means of the same variables, with strategy and problem type as random coefficients.

These analyses revealed differences in the strategy efficiency data obtained in the choice and the no-choice conditions. For instance, whereas clear group differences were observed in the accuracy of retrieval, decomposition to 10, and counting on in the respective no-choice conditions, the accuracy of strategy execution in the choice condition did not differ between the four groups of children, $F(6, 2990) = 1.59$, $p = .1464$. As suggested by Siegler and Lemaire (1997), the most plausible explanation for these contrasting results between the choice and no-choice conditions is the selectivity of children's strategy assignment in the choice condition. Indeed, the data from the no-choice conditions revealed that the strong second graders and the children with moderate MD were able to ap-

ply retrieval and decomposition to 10 more accurately than the weak second graders and the children with severe MD, whereas the latter two groups of children executed counting on as accurately as their mathematically stronger peers. In the choice condition, the strong second graders and the children with moderate MD solved about 90% of the problems with retrieval and decomposition to 10 (which they mastered better than their mathematically weaker peers, as indicated by the data from the no-choice conditions); the weak second graders and the children with severe MD relied primarily on counting on (the only strategy they mastered as well as their mathematically strong peers, as evidenced by the data from the no-choice conditions) and decomposition to 10 (which they could apply very accurately—although not as accurately as their mathematically stronger peers—in the decomposition condition). Thus, the absence of group differences in strategy accuracy in the choice condition was probably due to the fact that each group solved the problems in the choice condition most frequently with the kind of strategy they mastered best.

Strategy Selection

Using the same approach as Torbeyns, Verschaffel, and Ghesquière (2002, in press), we determined the adaptive nature of individual strategy choices in the choice condition by comparing for each child and for each problem the accuracy of strategy execution in the retrieval, decomposition, and counting condition with the strategy a particular child had chosen to solve a given problem in the choice condition (see Note 6). A strategy choice was scored as adaptive if the child solved the problem in the choice condition with a strategy that belonged to at least the same developmental level of strategy use as the highest level available to this child, as indicated by his or her task performance on the same problem in the no-choice conditions. More specifically, a strategy choice was scored as adaptive

if the child used in the choice condition,

1. retrieval to solve a problem he or she could answer accurately with this strategy in the retrieval condition;
2. decomposition to 10 to solve a problem he or she could not answer accurately with retrieval in the retrieval condition but could answer accurately with decomposition to 10 in the decomposition condition; if the child used retrieval to answer such a problem in the choice condition, this choice was also scored as an adaptive one, provided it resulted in a correct answer;
3. counting on to solve a problem he or she could not answer accurately with retrieval nor with decomposition to 10 in the respective no-choice conditions, but could answer accurately with counting on in the counting condition; if the child used decomposition to 10 or retrieval to answer such a problem in the choice condition, this choice was scored as an adaptive one if it led to an accurate answer (see Note 7).

We next predicted the adaptiveness of individual strategy choices in the choice condition by means of the variables of group (Strong2, Weak2, MD-m, MD-s) and problem type (addition, subtraction), with problem type as random coefficient. This analysis revealed that the four groups of children did not choose equally adaptively between the strategies in the choice condition, $F(3, 2962) = 4.56, p = .0034$. We observed differences in the adaptiveness of strategy choices between the CA-matched groups. The strong second graders made more adaptive strategy choices than their weak peers, $M = 0.90$ and $M = 0.69$, respectively. Likewise, the fourth to sixth graders with moderate MD chose more adaptively between the three types of strategies than their peers with severe MD, $M = 0.94$ and $M = 0.78$, respectively. In contrast, we

found no differences in the adaptiveness of strategy choices between the AL-matched groups. The fourth to sixth graders with moderate MD chose as adaptively as the strong second graders, and the fourth to sixth graders with severe MD made equally adaptive strategy choices as the weak second graders.

The adaptiveness of individual strategy choices did not differ between the problem types, $F(1, 85) = 0.27, p = .6059$.

Discussion

This study aimed at investigating the strategy characteristics and strategy development of children with MD in the domain of simple addition and subtraction, using a research methodology that combined two innovative features, namely the choice/no-choice method and the CA/AL-match design. More specifically, we compared the repertoire, frequency, efficiency, and adaptiveness of the strategies that four groups of children, matched on either chronological age (CA) or mathematical ability level (AL), applied on problems with the bridge over 10 by means of the choice/no-choice method.

From a theoretical point of view, our results support the hypothesis that children with MD are, in comparison with their typically achieving peers, delayed in the development of simple addition and subtraction strategies—a hypothesis for which we already provided empirical evidence in our previous work (Torbeyns et al., in press). In line with previous studies on the strategy characteristics of children with MD (for a review, see Geary, 1993), the comparison of the strategies of the CA-matched children revealed no difference in strategy repertoire between the mathematically stronger children (i.e., the strong second graders and the fourth to sixth graders with moderate MD) and their mathematically weaker peers (i.e., the weak second graders and the fourth to sixth graders with severe MD, respectively). However, the frequency, accuracy, speed, and adaptiveness with which the mathe-

matically stronger and mathematically weaker children applied the retrieval, decomposition, and counting strategies differed, which is again in line with the results of previous research. The strong second graders used retrieval more often and counting on less often than their weak age peers. Furthermore, the strong second graders made less retrieval errors than their weak age peers. The strong second graders executed the decomposition strategy with greater accuracy and speed than the weak second graders and made more adaptive strategy choices. Likewise, the fourth to sixth graders with moderate MD applied retrieval more frequently and more accurately than their peers with severe MD, executed decomposition to 10 more accurately and faster, and chose more adaptively between the retrieval, decomposition, and counting strategies.

In contrast, the AL-match comparison, between the fourth to sixth graders with moderate MD and the strong second graders and between the fourth to sixth graders with severe MD and the weak second graders, revealed that the groups involved in the AL-match comparison used the same repertoire of strategies to solve the problems in the choice condition and applied these strategies with the same frequency, accuracy, speed, and adaptiveness. It should be noted, however, that the weak second graders solved more problems with counting on than the children with severe MD and executed decomposition to 10 more slowly. These differences between the weak second graders and the fourth to sixth graders with severe MD are probably the result of differences in their amount of experience with the task. Indeed, most children with severe MD had learned how to solve problems with the bridge over 10 at the end of the first grade and the beginning of the second grade, when they were still in a general education elementary school. After their referral to special education, they had been taught problems with the bridge over 10 for a second time and had again intensively practiced these problems for

several months. In contrast, the weak second graders had learned how to solve problems with the bridge over 10 only at the end of first grade and had only practiced them during the first months of second grade.

Taken together, our results reveal clear differences in strategy characteristics between the CA-matched children but not between the AL-matched children. This indicates that the strategy characteristics of children with MD simply reflect their acquired mathematical ability level and, thus, suggests that their strategy development is delayed in comparison with the development of their typically achieving peers. However, as discussed in detail by Francis, Shaywitz, Stuebing, Shaywitz, and Fletcher (1996), there is more than one version of the delay hypothesis. The first version, the so-called "weak" version, hypothesizes that the strategy competencies of children at the lowest ability level develop more slowly than those of their typically achieving peers. The second version, the so-called "strong" version, not only assumes that children at the lowest ability level differ from their typically achieving peers in the rate of development of strategy skills; it also assumes that there are no differences between children at the lowest ability level and their typically achieving peers in the end point of the development—in other words, that over time the former completely "catch up" to the latter. Our study addressed the first, weak version of the delay hypothesis. The evaluation of the second, strong version requires a longitudinal research design, in which the development of children's mathematical and strategy competencies is assessed frequently throughout a longer period of time (e.g., from Grade 1 to Grade 6). Future studies using a longitudinal design are needed to test whether children with MD ultimately catch up to their typically achieving peers, as age and experience in the domain of simple arithmetic increase.

Contrary to the results of previous studies (Geary, 1993), we found no evidence for a specific retrieval deficit in

children with MD. These contrasting results could be due to differences in the intellectual, mathematical, and reading capacities of the children who participated in the studies. In previous studies, children with MD were selected on the basis of a mathematical ability criterion (e.g., scoring below the 30th, 35th, or 46th percentile on a standardized mathematics achievement test), sometimes combined with an intelligence criterion (e.g., scoring in the low-average to high-average IQ range; Geary et al., 1999; Geary et al., 2000) or a reading ability criterion (Geary et al., 1999; Geary et al., 2000; Hanich et al., 2001; Jordan & Hanich, 2000; Jordan & Montani, 1997). On the basis of the reading ability criterion, researchers have made a clear distinction between children with only MD (e.g., scoring below the 30th or 35th percentile on a standardized mathematics achievement test but above the 30th or 35th percentile on a standardized reading achievement test) and children with both MD and reading disabilities (RD; e.g., scoring below the 30th or 35th percentile on both a standardized mathematics and a standardized reading achievement test). Although we gathered information about all children's intellectual, mathematical, and reading abilities, we formulated only an intelligence and mathematical ability criterion. All children with MD had to be of at least average intelligence and had to be at least 2 years behind in their mathematical development in comparison with their typically achieving peers. However, the information we gathered about their reading abilities revealed that the children with MD had reading problems too. They scored about 2 years below their typically achieving peers on the reading achievement test. Notwithstanding the differences in the materials and criteria used to select children with MD, it seems reasonable to assume that the intellectual, mathematical, and reading abilities of the children with MD who participated in our study were comparable to those of the children with both MD and RD from previous studies.

Another possible explanation of our results refers to differences in the presentation format of the items. As mentioned before, the strategy characteristics and development of children with MD were thus far studied by means of the so-called choice method (for exceptions, see Geary et al., 2000; Hanich et al., 2001; Jordan & Montani, 1997). Children solved a series of problems in only one (choice) condition, in which they could choose freely which strategy they would apply to each problem; immediately after solving each problem, children were asked to report verbally which strategy they had used. In contrast, we offered all children a series of problems in one choice condition and three no-choice conditions. In the choice condition, children could choose between three strategies—retrieval, decomposition to 10, and counting on—by means of pictures that represented the strategies visually. It could be argued, however, that the children with MD who participated in our study used a rich diversity of decomposition and counting strategies to solve the problems in the choice condition and filled in the box that visualized the retrieval strategy each time they used a strategy that was not visualized by means of the pictures (e.g., the tie strategy or verbal counting), leading to an overestimation of the frequency with which the children with MD applied retrieval. However, neither our observational nor our reaction time data support this argument. In trials classified as retrieval, no overt problem-solving behavior (e.g., moving lips silently) was observed by the experimenter. Moreover, the children with moderate and severe MD needed on average 2.64 and 3.74 seconds, respectively, to retrieve the answer to the problems in the choice condition, which is in line with the research literature about the speed of retrieval (Baroody, 1999; Siegler, 1996).

Another possible explanation for the contrasting findings on the development of retrieval in children with MD involves the use of the CA/AL-match design. As noted before, in previous

studies, the development of strategies in children with MD was described on the basis of only age-related comparisons (i.e., by comparing the strategy characteristics of children with MD with those of their CA-matched, typically achieving peers. In contrast, we tried to deepen our insight in the strategy development of children with MD by means of both age- and ability-related comparisons. Taking into account the value of the design used in this study, future studies using the same combined CA/AL-match research methodology are needed to validate and refine our findings.

From a methodological viewpoint, our study documents the value of the choice/no-choice method and the combined CA/AL-match design to study young children's strategy use and strategy development in the domain of simple arithmetic.

The choice/no-choice method proved necessary to gather unbiased data about the efficiency of the retrieval, decomposition, and counting strategies on problems with the bridge over 10, and to accurately assess the adaptive nature of individual strategy choices. As demonstrated previously, the comparison of the efficiency data obtained in the choice condition with those from the no-choice conditions revealed that the data from the choice condition were biased by selection effects. Our findings thus provide evidence for Siegler and Lemaire's (1997) critical remarks about the choice method that has been applied in almost all previous studies in the domain of simple arithmetic (for a review, see Siegler, 1996). However, our study also revealed some methodological constraints on the applicability of the choice/no-choice method, which require continued attentive concern in future studies using this method (see also Torbeyns et al., 2002).

A first requirement for the application of the choice/no-choice method is that the experimenter designs one no-choice condition for each of the available strategies in the choice condition. Taking into account the rich diversity

of strategies that people use on most cognitive tasks, one has to somehow limit the number of strategies available in the choice condition. In other words, within the choice/no-choice paradigm, the choice condition cannot be a genuine free-choice condition but only a restricted-choice condition as far as strategy choice is concerned (Lemaire & Lecacheur, 2001a, 2001b, 2002; Luwel, Verschaffel, Onghena, & De Corte, 2003; Siegler & Lemaire, 1997). Consequently, the choice/no-choice paradigm does not allow the study of the strategy choice process in its full complexity, which, in turn, limits the ecological validity of the obtained strategy choice data.

A second requirement for the application of the choice/no-choice method is that the experimenter is able to control the strategies that are applied in the different conditions. The paper-and-pencil task format was used to control the strategies that the children used in the choice, decomposition, and counting conditions. However, the paper-and-pencil task format could not completely rule out the possibility that some children first used their own preferred strategy to determine the answer to the problems in the decomposition and counting conditions (e.g., silently, without moving lips, counting the answer to the problems in the decomposition condition), and afterward applied the strategy prescribed by the experimenter (viz., the decomposition strategy). This was almost if not completely impossible in the retrieval condition, in which we did not only explicitly instruct the children to apply the retrieval strategy, but also included a strict time limit to prevent the use of strategies other than retrieval as much as possible (Baroody, 1999; Payne, Bettman, & Johnson, 1988).

With respect to the value of the combined CA/AL-match design, our study shows that the CA/AL-match design can be applied successfully to investigate children's mathematical development and can shed new light on the study of strategy development in children with MD. As summarized earlier,

the comparison of the strategy characteristics of the CA-matched children revealed clear differences in strategy frequency, accuracy, speed, and adaptiveness between children with different mathematical abilities. These differences in strategy characteristics can be interpreted as mere reflections of the differences in the acquired mathematical ability level of the children, as indicated by the additional ability-related comparisons. Our results thus illustrate the ability of the CA/AL-match design to deepen our understanding of the developmental characteristics of children with MD (Backman et al., 1984; Ellis et al., 1996). However, the CA/AL-match design has its limits too. First, the selection of well-matched groups of children is a rather labor-intensive and time-consuming task. For the present study, we needed four groups of children with at least average intellectual and reading abilities, matched on either CA or mathematical AL. Of the original sample of 525 children, only 89 children passed the selection and matching criteria. Moreover, to make the selection of a sufficiently large number of children with MD possible, we had to relinquish the reading ability criterion for these children. Second, although children's strategy characteristics and development seem to be strongly related to their acquired ability level in the mathematical domain, the influence of other variables, such as the quality of instruction and the amount of experience with the task, cannot be denied (Geary, 1995; Siegler, 1996). Because the CA/AL-match design does not control for the influence of such variables, one should take them into serious consideration when interpreting the data obtained by means of this design (Velutino & Scanlon, 1989).

From a practical point of view, the materials and procedures used in this study proved valuable tools to diagnose the specific pitfalls in individual children's conceptual and procedural knowledge. The forced application of one particular strategy to all problems with the bridge over 10 yielded fine-

grained information about the concepts and procedures that an individual child did or did not master yet. For instance, the choice condition allowed the children at the lowest ability level to apply decomposition to 10 only on those problems they could solve efficiently by means of this strategy and to count or retrieve the answer to the other problems. The forced application of the decomposition strategy in the decomposition condition revealed, however, that some children were not able to split the second integer correctly without using their fingers on the majority of problems (i.e., had problems with the fast and accurate decomposition of the second integer), whereas others decomposed the second integer fast and accurately without external aids but did not know whether they had to add or subtract the decomposed parts on subtraction problems (e.g., $16 - 9 = 16 - 6 - 3$ or $16 - 9 = 16 - 6 + 3$). This kind of fine-grained information about each child's task- and domain-specific strengths and weaknesses makes it possible to adapt subsequent remediation and intervention techniques to his or her specific educational needs.

Furthermore, because our results suggest that the strategies of children with MD develop more slowly than those of their typically achieving peers, and because the level of mastery of elementary arithmetical skills constrains a child's further mathematical development (Geary, 1993), remedial teachers should not hesitate to try to facilitate the development of addition and subtraction strategies in children with MD from an early age. Early and frequent intervention directed toward facilitating the development of these children's procedural skills might reduce the difference in the rate of development between children with MD and their typically achieving peers and, thus, mitigate the persistence of the poor mathematical skills of children with MD.

Finally, although the results of our study enhance our understanding of the strategy characteristics and devel-

opment of children with MD, they leave some questions unanswered and also raise new questions. A first question refers to the end point of the development of simple addition and subtraction strategies in children with MD. As mentioned before, future studies should longitudinally follow the strategy development of children with MD to determine if they ever catch up to their typically achieving peers. Second, our study focused on the development of elementary arithmetical strategies in children with MD and did not take into account the development of supportive cognitive systems, such as working memory (e.g., Geary et al., 1999; Geary et al., 2000; McLean & Hitch, 1999). To deeply understand the mathematical development of children with MD, future studies should address the characteristics and development of these supportive cognitive systems in children with MD and investigate how they constrain the children's mathematical development. Third, our study analyzed the strategy competence of only one group of children with MD, namely, children with at least average intellectual abilities who have problems with the development of both their mathematical and reading skills. Future research should focus on the developmental characteristics of other groups of children with MD, such as children with below-average intellectual abilities or children with specific MD and no RD. Taking into account the strengths of the research methodology used in this study, it seems an important aim for future research to evaluate each of the aforementioned issues on the strategy characteristics and development of children of different mathematical abilities using the choice/no-choice method and the CA/AL-match paradigm.

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AUTHORS' NOTES

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NOTES

1. It should be noted that Geary et al. (2000), Hanich et al. (2001), and Jordan and Montani (1997) applied the choice/no-choice method partially by offering their participants problems in a choice condition and a forced retrieval condition.
2. McLean and Hitch (1999) studied the working memory impairments of children with MD by comparing their working memory characteristics with those both of children matched on chronological age and of children matched on mathematical ability level. Furthermore, Keeler and Swanson (2001) examined the relationship between working memory, declarative strategy knowledge, and mathematical achievement in children with and without MD, using the same methodology.
3. In Flanders, children with specific LD receive instruction in schools for special education of the so-called Type 8. These schools offer the same learning content to the children with LD as schools for general education and try to achieve the same educational standards. However, in schools for special education of Type 8, children are instructed in small groups of the same domain-specific ability level, by means of methods and materials that are adapted to the special characteristics and educational needs of these children.
4. Taking into account the nonnormal distribution of the data, we used the nonparametric Wilcoxon two-sample test.

5. In the choice condition, children used a strategy other than retrieval, decomposition to 10, or counting on in 0.91% of the trials. In the decomposition condition, children did not use the required decomposition strategy in 2.93% of the trials. These trials (i.e., 0.91% and 2.93% of the trials in the choice and the decomposition condition, respectively) were excluded from the analyses.
6. Taking into account Lemaire and Siegler's (1995) definition of an adaptive strategy choice as choosing the strategy that leads the individual fastest to an accurate answer to the problem, we intended to describe the adaptiveness of individual strategy choices in the choice condition on the basis of both accuracy and speed data gathered in the no-choice conditions. However, because the children answered the majority of problems out of time in the retrieval condition, we could not apply the information about the speed of strategy execution in our adaptiveness criterion. Consequently, we had to rely on the accuracy data only.
7. If a child either applied a strategy other than the visualized strategies in the choice condition or answered a problem inaccurately in each no-choice condition, the trial was not scored. In this way, 2.00% of the trials were excluded from the analysis.

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APPENDIX

Presentation Format of the Items in the Choice Condition

$$8 + 3 = _$$

$8 + 3 = _$	$ \begin{array}{r} 8 + 3 = _ \\ \diagup \quad \diagdown \\ _ \quad _ \end{array} $	$8 + 3 = _$
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In the choice condition, children could choose among retrieval, decomposition to 10, and counting on strategies to solve the

additions and subtractions. If children retrieved the answer to a problem in the choice condition, they had to write down the retrieved answer in the first box (behind the = symbol). In case of decomposition to 10, children had to fill in the second box. More specifically, they were asked to write down the way they split the second integer (below the second integer) as well as the answer (behind the = symbol). If children applied counting on, they were asked to use the third box. They were asked to point to or cross out the counted fingers (visualized on the hands in the third picture) while counting and to write down the answer (behind the = sign).

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