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A COMPARATIVE STUDY OF LOT SIZING PROCEDURES FOR
MULTI-STAGE ASSEMBLY SYSTEMS*

by

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This paper focuses on heuristic procedures for multi-stage assembly systems. Level-by-Level and Period-by-Period approaches are compared based on a large set of simulation experiments. The Period-by-Period approach we propose is of practical importance because the method considers the cost structure and the product structure of the system simultaneously.

I. INTRODUCTION

In this paper we consider the dynamic lot sizing problem for a single item, multi-level production system of the assembly type. The objective is to determine over a finite horizon order quantities and inventory levels at each stage of the production process so as to satisfy all demand requirements at minimum cost. Literature traditionnally distinguishes between several types of production-inventory systems. A multi-facility production process is called an assembly system if each stage or facility in the process requires inputs from a number of immediate predecessor stages while supplying, in turn, at most one immediate successor. (see figure 1a). In an assembly structure we end up with one final product. The particular structure has been studied by VEINOTT [14], CROWSTON and WAGNER [3], McLAREN [12], S. GRAVES [5] and many others. An arborescence structure, on the other hand, is characterized as follows: each production stage either has a unique immediate predecessor or no predecessors at all, whereas the number of successors is unlimited (see figure 1b). Such systems have been considered, among others, by VEINOTT [14] and KALYMON [5]. The last confi-

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guration is a serial structure where each facility has at most one immediate predecessor and one immediate successor stage (see figure 1c). The serial structure has been analyzed, among others, by ZANGWILL [16], LOVE [10] LAMBRECHT, VANDER EECKEN and VANDERVEKEN [7] [2]. Configurations which do not satisfy one of the above definitions are called general multistage systems (see figure 1d).

(a) Assembly structure

(b) Arborescence structure

(c) Serial structure

(d) General structure

Figure 1 : Examples of Multistage Inventory Systems.

As mentioned before we will concentrate on the assembly structure. The model holds under the following assumptions : (a) the time varying demand pattern over a finite horizon is assumed to be known (b) no backlogging is allowed (c) at each stage we have two types of cost functions, which are assumed to be constant over time : the production cost function is of the fixed charge type (a setup cost and a variable production cost) and the inventory holding cost function, penalizing the end of period inventory, which is a linear function of the amount kept in inventory.

Because of the computational difficulties encountered by searching for an optimal solution we concentrated on heuristic solution procedures. We selected 22 heuristic methods and evaluated their performance with respect to the optimal solution method.

The paper is organized as follows. In section II, the optimal solution method developed by CROWSTON and WAGNER [3] is presented. Section III focusses on the selected heuristic approaches and section IV describes the simulation experiment. Based on a preliminary simulation a number of heuristics will be eliminated because of their bad performance. This elimination procedure is explained in section V. In section VI the remaining sub-optimal methods are tested more extensively using a larger set of experiments. The conclusions of the study are given in section VII.

II. THE OPTIMAL SOLUTION METHOD

Assume an assembly structure with N facilities. The stages are numbered in the following way : facility 1 produces the final product, the other facilities are numbered in such a way that each stage i has a smaller number than all its predecessors. Let $s(i)$ be the immediate successor of stage i and $p(i)$ the set of immediate predecessors. By convention $s(1)$ refers to the customer of the finished product. Let $S(i)$ denote the set of all successors of stage i , and similarly $P(i)$ is the set of all predecessor stages of i . No time lags are involved in transferring units between facilities, and production is assumed to be instantaneous.

The following symbols are used : r_t stands for the finished product requirement in period t , $t=1, \dots, T$, F_i denotes the fixed set-up cost and h_i stands for the inventory holding cost per unit, facility i , $i=1, \dots, N$. Without loss of generality we assume that one unit of output of each facility requires one unit of input from the immediate predecessor stages. As a con-

sequence all order quantities are determined in units of finished product.

The optimal solution method of CROWSTON and WAGNER [3] holds under the following cost assumptions :

- production costs are nonincreasing over time
- inventory holdings costs are decreasing over stages.

This last assumption implies that

$$(1) \quad h_i \geq \sum_{j \in p(i)} h_j \quad i=1, \dots, N,$$

or, the incremental holding cost H_i has to be nonnegative :

$$(2) \quad H_i = h_i - \sum_{j \in p(i)} h_j \geq 0, \quad i=1, \dots, N$$

Note that the above conditions are automatically satisfied for our analysis because we assume constant costs over time.

Under these assumptions, there exists an optimal solution satisfying the following properties [9] :

$$(3) \quad I_{i,t-1} \cdot x_{i,t} = 0, \quad \forall(i,t) \quad \text{and,}$$

$$(4) \quad \text{if } x_{i,t} > 0 \text{ then } x_{s(i),t} > 0, \quad \forall(i,t)$$

with $I_{i,t}$: inventory at the end of period t , facility i

$x_{i,t}$: production quantity period t , facility i .

Note that condition (4) implies that in an optimal solution, production plans must be "nested", LOVE [10] .

Let $\pi = \{\pi_1, \pi_2, \pi_3, \dots, \pi_t, \dots, \pi_T\}$ be a zero-one string

$$\begin{aligned} \pi_t &= 1 & \text{if } x_{it} > 0 \\ \pi_t &= 0 & \text{if } x_{it} = 0 \end{aligned}$$

As a result of property (3) we have that :

$$(5) \quad x_{it} = \sum_{j=t}^{t'} r_j \quad \text{if } \pi_j = 0, j = t+1, \dots, t' \text{ and} \\ \pi_t = \pi_{t'+1} = 1$$

Suppose a zero-one string π , and the corresponding x_{it} , $t=1, \dots, T$, have been selected for stage i . Then the corresponding production and inventory holding costs are determined in the following way :

- (a) The echelon stock or total inventory in the system at the end of period t which has undergone processing at stage i equals :

$$(6) \quad Y_{it} = \sum_{j=1}^t (x_{ij} - r_j) \quad t=1, \dots, T$$

Total echelon inventory holding cost, $h^i(\pi)$, for stage i according to schedule π is given by :

$$(7) \quad h^i(\pi) = \sum_{t=1}^T H_i \cdot Y_{it}$$

- (b) Total setup costs, $F^i(\pi)$, for stage i are obtained by :

$$(8) \quad F^i(\pi) = F_i \cdot \sum_{t=1}^T \pi_t$$

For a given π , the acceptable production schedules for the predecessor stages of facility i are given by the set $\sigma(\pi)$:

$$(9) \quad \sigma(\pi) = \{\pi' \mid \pi_t - \pi'_t \geq 0, \forall t\}$$

The set $\sigma(\pi)$ is constructed in such a way that (4) holds

The optimal production schedules for each stage are determined by recursion formula (10), starting with facility N and ending with stage 1 (final stage).

Let $f^i(\pi)$ be the minimum total cost for stage i given π is chosen for stage i and given optimal production plans for all predecessor stages :

$$(10) \quad f^i(\pi) = h^i(\pi) + F^i(\pi) + \sum_{j \in p(i)} \min_{\pi' \in \sigma(\pi)} f^j(\pi')$$

CROWSTON and WAGNER showed that the number of comparisons can be decreased using a more efficiently structured recursion relation.

Define $\sigma^1(\pi)$ as follows :

$$(11) \quad \sigma^1(\pi) = \{ \pi' \in \sigma(\pi) \mid \sum_{t=2}^T (\pi_t - \pi'_t) = 1 \}$$

$$\text{Hence } \sigma(\pi) = \pi \cup \bigcup_{\pi' \in \sigma^1(\pi)} \sigma(\pi')$$

$$\text{Let } f^j(\pi^{\#}(\pi')) = \min_{\pi'' \in \sigma(\pi')} [f^j(\pi'')]$$

And thus

$$\min_{\pi' \in \sigma(\pi)} f^j(\pi') = \min [f^j(\pi), \min_{\pi' \in \sigma^1(\pi)} f^j(\pi^{\#}(\pi'))]$$

The revised recursion formula becomes :

$$(12) \quad f^i(\pi) = h^i(\pi) + F^i(\pi) + \sum_{j \in p(i)} \min [f^j(\pi), \min_{\pi' \in \sigma^1(\pi)} f^j(\pi^{\#}(\pi'))]$$

As one should expect, the computer time of the dynamic programming algorithm increases exponentially with the number of periods but only linearly with the number of stages regardless the characteristics of the assembly structure (see section VI).

Note that CROWSTON and WAGNER also developed a branch-and-bound algorithm which is intended for cases where the number of time periods is large but where the structure is close to serial [3].

III. HEURISTIC PROCEDURES

Many heuristic methods have been proposed to solve the lot sizing problem for an assembly structure. By examining these heuristics we can distinguish two types of solution procedures; the Level-by-Level and the multi-level or period-by-period heuristics. The level-by-level procedures assign production quantities to a specific facility over all periods and use the resulting production program as requirements for all immediate predecessor stages. These heuristics start with the finishing facility (the facility producing the final product, i.e., stage 1).

The multi-level procedures on the contrary, assign production quantities to all stages on a period by period basis. Such an approach has a number of interesting advantages. First, a period-by-period approach enhances the practical value of the method mainly because the current decisions can be based on near future and thus more accurate data. Moreover, most decisions are made on a rolling schedule basis where one is interested in the first "best" move. Second, period-by-period approaches are promising with respect to cost performance because this approach considers the cost structure and the product structure of the total system simultaneously, this is a highly desirable characteristic in a practical environment.

Three, the computational effort to arrive at a good solution is reduced compared with multi-pass level-by-level procedures (see section VI).

Besides that, our myopic approach is justified because of the existence of planning horizons for all dynamic lot sizing problems.

A. Level-by-Level procedures

a. Single-pass heuristics

The most common form of heuristic is to consider the stages sequentially, starting with the lowest echelon stage and scheduling each stage with a

single-stage lot-sizing procedure, such as period balancing, least unit cost, Silver and Meal, etc. For each single-stage method several sets of cost parameters can be used. In our analysis we incorporated 4 sets of cost parameters. Let SCP_i and ICP_i denote respectively the set-up and inventory holding cost parameters for stage i .

1. Set (SS,h) : the cost parameters equal the costs of the considered facility:

$$\begin{aligned} SCP_i &= F_i \\ ICP_i &= h_i \end{aligned} \quad i=1, \dots, N$$

2. Set (CS,h) : here we use as set-up cost parameter the sum of all set-up costs of all preceeding facilities increased by the set-up cost of stage i itself :

$$\begin{aligned} SCP_i &= F_i + \sum_{j \in P(i)} F_j \\ ICP_i &= h_i \end{aligned} \quad i=1, \dots, N$$

3. Set (SS,H) : as inventory cost parameter we use the incremental inventory holding cost of the considered stage, while the single set-up cost F_i is used :

$$\begin{aligned} SCP_i &= F_i \\ ICP_i &= h_i - \sum_{j \in p(i)} h_j = H_i \end{aligned} \quad i=1, \dots, N$$

4. Set (WS,h) : the inventory holding cost is h_i , while we use as set-up cost parameter, the sum of the single stage set-up cost and a weighted sum of the set-up costs of all immediate predecessor stages. This parameter which is based on the average time between orders (TBO) has been suggested by McLaren and Whybark [10] .

$$SCP_i = F_i + \sum_{j \in p(i)} \frac{TBO_i}{TBO_j} \cdot F_j$$

$$\text{with } TBO_i = \left[\frac{2 \cdot F_i \cdot T}{T} \right]^{1/2} \quad , i=1, \dots, N$$

$$\left[\begin{array}{c} h_i \sum_{t=1}^T r_t \end{array} \right]$$

$$ICP_i = h_i$$

There exists in literature a wide range of possible cost parameters which are basically variants of the four sets presented. We refer the reader to [1] and [2] where other sets of cost parameters are used.

The procedure followed by using a level-by-level approach can be briefly described as follows. Let r_t^i denote the requirement at stage i period t . For stage 1 we have that $r_t^1 = r_t$, $t=1, \dots, T$.

If $i \neq 1$, the production vector of the immediate successor stage $s(i)$ has to be taken as demand vector for stage i , or, $r_t^i = x_{s(i),t}$, $t=1, \dots, T$.

The following single-stage lot sizing rules are included in the experiment :

- Part Period Balancing (PPB)
- Least Unit Cost (LUC)
- Silver-Meal (SM)
- Order Moment (OM)
- The optimal Wagner-Whitin algorithm (WW).

We briefly describe the Order Moment heuristic [12], and we refer the reader to the literature for a detailed description of the other approaches.

The Order Moment heuristic states that an order is placed covering periods ℓ through $t-1$, the first time the following inequality holds :

$$(13) \sum_{j=\ell}^t r_j^i \cdot (j-\ell) \geq \sum_{j=1}^{ITBO_i} E(r) \cdot (j-1) + ITBO_i \cdot E(r) \cdot (TBO_i - ITBO_i)$$

$$t=\ell, \ell+1, \dots$$

$$\text{with } TBO_i = \left[\frac{2 \cdot SCP_i}{ICP_i \cdot E(r)} \right]^{1/2}$$

$ITBO_i$ = the greatest integer less than or equal to TBO_i

$$E(r) = \frac{1}{T} \cdot \sum_{j=1}^T r_j$$

b. A Multipass heuristic

S.C. Graves [5] developed this very promising multipass level-by-level heuristic. A single-pass heuristic sequentially schedules each stage and then stops, whereas the multipass heuristic of S. Graves continues to revise the current production plan in an iterative fashion until no further improvements are possible. The basic idea is to revise the cost structure at each iteration based on a marginal cost concept. The procedure consists of solving iteratively a number of Wagner-Whitin problem. The author shows that his procedure is monotonic and convergent. His procedure is further improved by a stage collapsing routine in which various stages are "collapsed" into single-stage problems. It is interesting to note that the proposed heuristic can easily handle general production structures. Our computational experience indicates that this multipass procedure is by far the best level-by-level heuristic available up till now. The interested reader is referred to [5] for more details concerning the logic of the suggested approach.

c. Other Approaches

Crowston, Wagner and Henshaw [4] developed three heuristics for the constant demand infinite horizon case. These three procedures, the single pass, the multiple pass and the modified multiple pass heuristic can be adopted to handle the variable demand patterns. Our computational experience with these methods shows that these adapted procedures deviate on the average by 25 % from optimal. For this reason we exclude these methods from further consideration.

Finally, we have to mention the work by McLaren [12] who developed the Multi-Level Wagner and Whitin algorithm using dynamic programming. This method comes up with near optimal production programs but, as mentioned by the author, the method requires a lot of computer time especially when the number of levels in the assembly system increases. For this reason we excluded this method from our comparative analysis.

B. A Period-by-Period procedure : The Coefficient Approach

The Coefficient Approach is a multi-level procedure which means that order quantities are assigned to all facilities on a period by period basis. Assuming no initial inventories, the requirements of period one have to be produced in the first period so that we are first interested in the requirements of period 2. The first question is whether it is advantageous to include r_2 in a lot of period 1 and the second question is then "where" should it be added, in other words should r_2 be included in none, some or all of the facilities. To answer both questions simultaneously, a coefficient is computed indicating by its sign and magnitude whether cost reductions may be expected by producing r_2 in period 1 at some facility. The procedure starts with facility N and stops when facility 1 is reached. Next the procedure is repeated for periods 3 upto T. The cost coefficient satisfies the Part Period Balancing criterion.

This section is organized as follows : first a description is given of the procedure and next an example is introduced to illustrate the method.

START : $t=1$

STEP 1 : if $t \neq T$: $OP_i = t$, $\forall i$
 $\alpha_i = 1$, $\forall i$
 $Y_i = 0$, $\forall i$
 $x_{i,t} = r_t$, $\forall i$

STEP 2 : if $t=T$ go to step 5

else, . $W_i : V_i = 0$

. $t = t+1$

. Compute coefficients U_i , $i=N, N-1, \dots, 1$

if $\alpha_i = 1$

$$(14) U_i = F_i + \sum_{j \in P(i)} F_j - [Y_i + r_t \cdot (t - OP_i)] \cdot h_i$$

if $\alpha_i = 2$

$$(15) U_i = F_i + \sum_{j \in P(i)} U_j - (h_i - \sum_{j \in P(i)} h_j) \cdot [r_t \cdot (t - OP_i) + Y_i]$$

$i = N+1$

. $i = N+1$

The value of α_i is determined in step 4.

The above expressions correspond to the well known Part Period Balancing criterion adjusted of course for the multi-level environment. We donnot only consider the setup costs and the (accumulated) inventory holding costs for facility i itself but we also include the cost consequences for all preceeding facilities of i .

STEP 3 : . $i = i-1$

$$. V_i = \sum_{j \in P(i)} V_j : W_i = 1$$

. If $V_i \leq U_i$ then $V_i = U_i$
and $W_j = 1$ for $\forall j \in P(i)$

If $V_i > U_i$ then $W_i = 0$

. If $i=1$ go to the next step, else, repeat step 3

In this step we compare $V_i (= \sum_{j \in P(i)} V_j)$ with U_i , if U_i is the maximum, we prefer to carry r_t as inventory at facility i instead of carrying inventory

at some preceeding facility (ies) $j \in P(i)$. In this case, W_i is set equal to 1, (as well as all W_j , $j \in P_i$) indicating that r_t is produced in period OP_i at facility i . In the other case, if $V_i > U_i$ we set $W_i = 0$, indicating that a new order will be placed.

STEP 4 : . If $V_i = 0$ go to step 1

. For $\forall i$, with $W_i = 1$ we have that

$$x_{i,OP_i} = x_{i,OP_i} + r_t$$

$$x_{i,t} = 0$$

$$Y_i = Y_i + r_t(t - OP_i)$$

In this case, r_t is carried in inventory at facility i , and the parameter α_i remains unchanged.

. For $\forall i$, with $W_i = 0$ we have that

$$x_{i,t} = r_t$$

$$Y_i = 0$$

$$\alpha_i = 2$$

if $t \neq T$, set $OP_i = t$

In this case a new setup is incurred at stage i , and α_i is set equal to 2, indicating that formula (15) will be used in step 2.

. go to step 2

STEP 5 : If $x_{1,T} = 0$ stop.

else . form $A = \{i \mid x_{i,T} > 0\}$

. $i = N+1$

STEP 6 : . $i = i-1$

. $V_i = 0$

. If $i \notin A$: $U_i = -\infty$ and go to step 7
else

$$U_i = \sum_{j \in p(i) \cap A} U_j + F_i - (h_i - \sum_{j \in p(i)} h_j) \cdot r_T(T - OP_i)$$

. go to next step

STEP 7 : If $i > 1$ return to step 6, else, $i = N+1$ and repeat steps 3 (for all i) and 4 (for $i \in A$) but instead of returning to step 2, stop.

Steps 5, 6 and 7 constitute a feedback mechanism that is used when production is positive at some of the facilities in period T . In this case the question is asked whether it is not advantageous to include r_T in a previously determined lot. Such a feedback mechanism is introduced because of the period-by-period approach.

EXAMPLE : Consider the following 5 period, 3 facility problem.

Step 1 : $OP_i=1$, $\alpha_i=1$ and $Y_i=0, \forall i$, $x_{i,1}=r_i=40$, $i = 1,2,3$

Step 2 : $V_i=0$, $\forall i$ and $t=2$ and using formula (14) :

$$U_3 = 90 - (80) \cdot 1 = 10$$

$$U_2 = 30 - (80) \cdot 1 = -50$$

$$U_1 = 100 + 30 + 90 - (80) \cdot 3 = -20$$

Step 3 : $V_3=0 < U_3=10$, hence $V_3=10$ and $W_3=1$

$V_2=0 > U_2=-50$, hence $V_2=0$ and $W_2=0$

$$V_1 = \sum_{j \in p(1)} V_j = 10 + 0 = 10 > U_1 = -20 \text{ hence } V_1=10 \text{ and } W_1=0$$

Step 4 : $W_3=1$, $x_{3,1} = 40 + 80 = 120$

$$Y_3 = 80(2-1) = 80$$

$$\alpha_3 = 1 \text{ (unchanged)}$$

$$W_2=W_1=0 \text{ , } x_{2,2} = x_{1,2} = 80, OP_2 = OP_1 = 2$$

$$\alpha_2 = \alpha_1 = 2$$

We now turn to period 3

Step 2 : $V_i=0$, $\forall i$, $t=3$

$$\alpha_3=1 \quad U_3 = 90 - [80 + 10(3-1)] \cdot 1 = -10$$

$$\alpha_2=2 \quad U_2 = 30 - [10(3-2)] \cdot 1 = 20$$

$$\alpha_1=2 \quad U_1 = 100 - 10 + 20 - (3-2) [10(3-2)] = 100$$

As mentioned before, the coefficients correspond to the Part Period Balancing criterion. Consider e.g. U_1 . By not producing r_3 in period 3 but instead satisfying v_3 according to the previously arrived at production plan we obtain total savings in setup costs of 120 ($F_1+F_2+F_3$) and incur cumulative holding cost of 120 ($Y_3+h_1r_3+h_3r_3$ which is equivalent to $(r_2+r_3)h_1+r_3h_3$) resulting in net savings of 100 (U_3).

Step 3 : $V_3=0 > U_3=-10$, hence $V_3=0$ and $W_3=0$
 $V_2=0 < U_2=20$, hence $U_2=20$ and $W_2=1$
 $V_1=20+0 < U_1=100$, hence $V_1=100$ and $W_1=1$ and consequently $W_2=W_3=1$

Step 4 : $W_1=W_2=W_3=1$, $x_{3,1}=130$, $Y_3=100$, $\alpha_3=1$
 $x_{2,2}=90$, $Y_2=10$, $\alpha_2=2$
 $x_{1,2}=90$, $Y_1=10$, $\alpha_1=2$

We now examine the next period, $t=4$

Step 2 : $V_i=0$, $\forall i$, $t=4$

$$\begin{aligned}\alpha_3=1 \quad U_3 &= 90 - [100 + 60(4-1)] \cdot 1 = -190 \\ \alpha_2=2 \quad U_2 &= 30 - [10 + 60(4-2)] \cdot 1 = -100 \\ \alpha_1=2 \quad U_1 &= 100 + (-190-100) - (3-2) [60(4-2) + 10] = -320\end{aligned}$$

Step 3 : $V_3=0 > U_3$, hence $V_3=0$ and $W_3=0$
 $V_2=0 > U_2$, hence $V_2=0$ and $W_3=0$
 $V_1=0 > U_1$, hence $V_1=0$ and $W_3=0$

Step 4 : Since $V_1=0$ we go to step 1. Note that V_1 zero for a specific period defines a reorder period. This means that a new setup is incurred at all stages in this period.

Step 1 : $OP_i=4$, $\forall i$

$$\begin{aligned}\alpha_i &= 1 \\ Y_i &= 0 \\ x_{1,4} &= x_{2,4} = x_{3,4} = 60\end{aligned}$$

Step 2 : $V_i = 0$, $\forall i$, $t=5$, $\alpha_i = 1$
 $U_3 = 90 - [80(5-4)] \cdot 1 = 10$
 $U_2 = 30 - [80(5-4)] \cdot 1 = -50$
 $U_1 = 100 + 90 + 30 - [80(5-4)] \cdot 3 = -20$

Step 3 : $V_3=0 < U_3=10$, hence $V_3=10$ and $W_3=1$
 $V_2=0 > U_2=-50$, hence $V_2=0$ and $W_2=0$
 $V_1=0+10 > U_1=-20$, hence $V_1=10$ and $W_1=0$

Step 4 : $W_3=1$, $x_{3,4}=140$, $Y_3=80(5-4) = 80$
 $W_2=W_1=0$, $x_{2,5} = x_{1,5} = 80$
 $Y_2 = Y_1 = 0$
 $\alpha_2 = \alpha_1 = 2$

Next we return to step 2, but since $t=T$ we go to step 5

Step 5 : $A = \{2,1\}$, $i=4$

Step 6 : $i=3$, $i=3 \notin A$: $U_3 = -\infty$
 $i=2$, $i=2 \in A$: $U_2 = 30 - (1) - 80(5-4) = -50$
 $i=1$, $i=1 \in A$: $U_1 = -50 + 100 - (3-2) \cdot 80(5-4) = -30$

Repeating steps 3 and 4 indicates that the current solution remains unchanged.

Solution

$x_{i,t}$	$t =$	1	2	3	4	5
i						
1		40	90	0	60	80
2		40	90	0	60	80
3		130	0	0	140	0

Note that this solution is optimal.

IV. DESCRIPTION OF THE SIMULATION EXPERIMENT

In order to examine the effectiveness of the different heuristics, a two-step simulation experiment was conducted. Consider the four assembly structures shown in figure 2. The time horizon is 8 periods.

The setup cost F_i is generated randomly from a uniform distribution in the range $[500, 10,000]$. The inventory holding cost per unit per period, for the finishing stage (h_1) is set equal to 100. The holding costs for the other facilities are determined in a random way but with the additional restriction that the sum of the inventory holding costs h_j , $j \in p(i)$ is less than or equal to h_1 . The procedure goes as follows, after h_1 is set equal to 100, the inventory holding cost for a preceding facility, say h_2 , is selected using a uniform distribution defined over the range $[0, 100]$. For the next facility, the holding cost is determined by selecting a number from a uniform distribution defined over the range $[0, h_2]$. The procedure is repeated for all remaining facilities. As a result of this procedure the inventory holding cost tends to decrease rapidly to more we reach the last facility N . Another consequence of this generation procedure is that the ratio S_i/h_1 tends to decrease as one proceeds up the product hierarchy (to the final product or stage 1). The majority of our test problems had a decreasing setup-to-carrying cost ratio, which seems to be in agreement with most practical inventory systems [2].

In a first step of the simulation experiment all heuristics (22) described in this paper were tested on 6 problems per assembly structure (for each assembly structure we selected 3 demand patterns and for each demand pattern two different cost structures). Based on the results of this preliminary experiment, only those heuristics which deviate by no more than 10 % from the optimal solution were retained for further investigation.

In a second simulation experiment the remaining heuristics were analyzed using 96 problems. For each assembly structure (4) we selected 12 demand patterns and for each demand pattern two different cost structures.

Section V reports on the results of the first experiment and section VI focuses on the experimental analysis of the second step of the simulation experiment. The demand patterns of the second simulation run are given in Appendix I. At the same time a number of characteristics are given of the optimal solutions for the four assembly structures. The following two measures of complexity are given : the percentage of setup periods in an optimal solution and the average number of periods with zero production for all facilities.

a) Assembly Structure I

b) Assembly Structure II

Figure 2 : The four assembly structures

c) Assembly Structure III

d) Assembly Structure IV

Figure 2 : The four Assembly Structures

V. SIMULATION EXPERIMENT - PHASE ONE

Table I shows the cost performance of the 22 heuristic procedures according to the 4 assembly structures selected (the cost of the optimal solution is set equal to 100). The last column gives the overall average per heuristic method. The following methods were selected for further investigation : OM(WS,h), OM(CS,h), WW(SS,h), WW(WS,h), PPB(SS,h), PPB(WS,h), SM(WS,h), SM(SS,h), the multipass heuristic of S. Graves and the Coefficient Approach.

VI. SIMULATION EXPERIMENT - PHASE TWO

The results of the 96 problems are given in Table II. Table III shows the computation times as a function of the length of the planning horizon T, of several methods with respect to assembly structure I. Since the single-pass heuristics all require about the same computer time only, the times for the WW(WS,h) method are given.

VII. CONCLUSIONS

Due to the serious disadvantages of the optimal solution methods (computer time and storage requirements) we were interested in the performance of several heuristic procedures. The best method is the Multipass heuristic of S. Graves followed by the Single Pass WW(WS,h). Next we have our period-by-period Coefficient Approach. All three methods are within 1,5 % from optimal.

The computer times of the single-pass methods and the Coefficient Approach are comparable whereas the Multipass methods is characterized by much higher computer times.

As explained in the introduction of section III, we believe that a period-by-period approach is of great practical significance and hence the Coefficient Approach is worth considering. Moreover, the period-by-period approach based on cost coefficients per requirement has been proven to be a very promising technique for solving capacitated lot sizing problems [9].

Structure	I	II	III	IV	Average Co Deviation
Method					
A. LEVEL-BY-LEVEL					
A.a. <u>Single Pass Heuristics</u>					
Order Moment : OM(cost set)					
OM(SS,h)	115,26	116,22	113,98	117,65	115,78
OM(WS,h)	108,02	105,76	109,12	109,71	108,15
OM(CS,h)	106,31	104,02	107,06	102,64	105,01
OM(SS,H)	182	149,79	122,37	134,64	147,20
Wagner-Whitin : WW(cost set)					
WW(SS,h)	104,30	102,3	104,9	105,47	104,24
WW(WS,h)	101,50	100,2	101,76	101,91	101,34
WW(CS,h)	117,44	115,97	112,76	114,23	115,10
WW(SS,H)	199,36	149,50	134,84	168,62	162,96
Part Period Balancing :					
PPB(cost set)					
PPB(SS,h)	109,00	103,52	106,52	106,22	106,32
PPB(WS,h)	105,20	101,71	102,46	102,21	102,90
PPB(CS,h)	121,76	113,44	118,28	116,54	117,51
PPB(SS,H)	208,08	158,36	128,83	140,09	158,84
Silver-Meal : SM(cost set)					
SM(SS,h)	112,67	106,65	111,71	108,52	109,89
SM(WS,h)	109,46	103,20	107,6	104,79	106,26
SM(CS,h)	119,71	115,00	117,71	115,29	116,93
SM(SS,H)	207,07	126,66	126,65	132,69	148,27
Least Unit Cost : LUC(cost set)					
LUC(SS,h)	122,75	122,74	119,52	110,05	118,69
LUC(WS,h)	120,59	122,58	118,21	106,92	117,08
LUC(CS,h)	137,24	129,52	140,81	145,16	138,18
LUC(SS,H)	227,30	180,15	149,13	131,17	171,94
A.b. <u>Multipass Heuristic</u>					
S. Graves	100,29	100,26	100,01	100,21	100,19
B. PERIOD-BY-PERIOD					
Coefficient Approach (CA)	102,66	101,13	100,91	100,73	101,36

TABLE I : Cost performance Simulation Experiment-Phase One

Structure Method	I	II	III	IV	Average cost Deviation
A. LEVEL-BY-LEVEL					
A.a. Single Pass					
OM(WS,h)	107,12	106,82	107,15	108,44	107,38
OM(CS,h)	105,76	104,65	108,52	102,07	105,25
WW(SS,h)	104,18	104,04	104,54	105,74	104,63
WW(WS,h)	101,16	100,76	101,43	101,69	101,26
PPB(SS,h)	106,24	104,66	105,55	105,93	105,59
PPB(WS,h)	102,83	101,70	102,08	101,74	102,09
SM(SS,h)	109,94	107,13	109,61	108,56	108,81
SM(WS,h)	105,81	104,07	105,65	104,10	104,91
A.b. Multipass					
S. Graves	100,23	100,33	100,48	100,24	100,32
B. PERIOD-BY-PERIOD					
Coefficient Approach	101,96	101,69	101,22	100,84	101,43

TABLE II : Results of the second simulation run.

T Method	4	6	8	10
Optimal Method	35	176	1150	10327
Coefficient Approach	18	29	38	49
S. Graves Multipass	380	675	1015	1295
WW(WS,h)	16	24	31	37

TABLE III : Execution times in m.sec (x) without input and output statements

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(x) IBM 3033

APPENDIX I : Demand Patterns used for the simulation experiment and problem characteristics.

Period	1	2	3	4	5	6	7	8
Pattern								
1	498	620	438	455	458	346	540	645
2	558	835	140	506	149	269	745	798
3	101	414	1336	101	935	889	101	118
4	526	446	525	511	463	469	456	579
5	522	328	705	764	330	140	772	434
6	338	1154	1174	539	368	370	17	39
7	494	464	473	553	533	383	552	548
8	434	410	562	644	860	443	443	202
9	393	833	740	18	44	301	412	1257
10	559	609	487	477	380	429	519	533
11	254	187	860	666	685	500	647	196
12	451	18	862	1150	294	338	18	865

Assembly Structure	I	II	III	IV
Average number of set-up periods in the optimal solutions	62,75	60,27	59,5	37,77
Total number of set-up periods using lot-for-lot	144	128	144	64
%	43,6%	47,09%	41,31%	59,02%
Average number of periods with zero production for all facilities in a given period	1,04	1,22	1,11	1

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