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PROTECTIVE STOCKS AND BOTTLENECKS IN
PRODUCTION SYSTEMS

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ABSTRACT

In this paper we study the allocation of safety stocks and safety times in multi-echelon production systems. We concentrate on the impact of demand uncertainty on the amount of safety stocks in the presence of capacity constraints. We describe how this problem can be viewed as a Markovian Decision Process. The form of the optimal policies is discussed and illustrated with examples

I. INTRODUCTION

The purpose of this paper is to study the allocation of safety stocks and safety time in multi-echelon production systems with limited capacity. Uncertainty does exist in several forms - variability of demand from the forecast, uncertainty in the supply from stage to stage due to the variability in the yields from each production batch and uncertainty in the lead times for both production and procurement. Protection against these disruptions is required and this results in buffer stocks and/or safety times. Research on protective stocks in multi-stage systems has primarily been experimental. We refer the

reader to the work of BANERJEE [2], WHYBARK and WILLIAMS [20], BUECHEL [4], CARLSON and YANO [5], [6], COLLIER [9], DE BODT et al. [11], MEAL [17], NEW [19] and JOENSSON, LUNDELL and THORSTENSON [14].

The authors of this paper have shown in a recent paper [15] that the multi-echelon stochastic problem can be viewed as a Markovian Decision Problem. Because of the limited practical value of this approach heuristic solutions methods have been suggested based on the well known results of CLARK and SCARF [7], [8]. The authors also have demonstrated that generalized (s,S) policies appear to be optimal. The resulting (s,S) policies are next simulated in order to determine the amount of safety stock and safety time. The simulation is necessary because it is impossible to determine the exact amount of safety stock or time by only knowing the values of the policy variables. Safety stock is determined as the expected amount of on hand inventory at the time an order or a lot arrives. Safety time is defined as the expected time between the completion of a lot at a preceeding stage until any units from that lot are used in the production of a lot at the succeeding stage. The computational results indicate that both safety time and safety stock can exist at intermediate stages.

The above observations will be illustrated by means of an example in section II.

The purpose of this paper is to extend the previous analysis for situations where capacity constraints are limiting the production output at certain facilities.

In section II the multi-echelon system is introduced together with the underlying assumptions and a description is given of how this problem can be viewed as a Markovian Decision Process. In section III, we discuss the form of the optimal operating policies for the case in which there are capacity constraints at the end product level. In particular we

show that generalized (s,S) policies appear to be optimal. The discussion is illustrated with examples.

II. MODEL FORMULATION

A. METHODOLOGY

In this section, we introduce the multi-echelon production/inventory system and its underlying assumptions. We also describe how this system can be modelled as a Markovian Decision Process. Attention is restricted to the situation where the demand process is stationary and the planning horizon is infinite. The product structure we investigate, is depicted in figure 1. It is a two echelon serial structure, echelon 2 is the component level and echelon 1 is the final assembly level.

Figure 1 : A two echelon serial structure.

The model we present is based on the following assumptions :

1. A periodic review policy is followed;
2. The production and procurement lead times, which are measured in periods, are known constants;
3. The demand distributions are known; demand is independent from period to period for each final product;
4. All excess demand is backordered;
5. Production capacity is limited at the final assembly level.

We also assume that the relevant costs consist of production costs, at each stage, which are composed of a fixed set up cost per production run and a constant cost per unit produced, holding costs, which are charged proportional to the echelon stock at the end of a period and linear shortage costs, which are charged proportional to the number of final products backordered at the end of each period. Note that the constant cost per unit produced can be ignored in our analysis as we assume that costs remain stationary over time and we have an infinite planning horizon.

The multi-echelon stochastic production/inventory problem introduced above can be modelled as a Markov Decision Process. It is assumed that the system is observed at time $t = 0, 1, \dots, T$ and classified into one of a finite number of states i labeled $1, 2, \dots, N$. For every state i , a number of decision alternatives, k , are possible, labeled $1, 2, \dots, K$. (The number of possible policies might differ from state to state). Whenever the system is in state i and alternative k is chosen the system moves to state j with known transition probability p_{ij}^k for all $i, j = 1, 2, \dots, N$ and $k = 1, 2, \dots, K$. The cost structure is introduced through q_i^k the expected one-period cost incurred when the system is in state i and decision alternative k is chosen.

In terms of the multi-echelon stochastic production planning problem the states $i, i = 1, \dots, N$, represent the echelon inventory position (on hand and on order) at each stage. The total number of states equals the product of all possible echelon inventory positions at all stages. The decision alternatives $k, k = 1, \dots, K$ represent the production decisions at each stage. In case there are capacity constraints, the number of production decisions is restricted by the available capacity. The methodology is basically the same for both situations. The transition probabilities depend on the current state i , the decision made, k , and the demand distribution.

The proposed solution method is based on the value iteration or successive approximation method developed by HOWARD [13]. The recursive relation

is given by :

$$V_i(T) = \min_k \{q_i^k + \alpha \sum_j p_{ij}^k V_j(T-1)\}$$

$V_i(T)$ stands for the expected total (discounted) cost of a system starting in state i and evolving for T periods when an optimal policy is followed, α denotes the discount factor and q_i^k and p_{ij}^k are elements of the cost matrix Q and the transition matrix P^k . For large values of T , a stationary cost structure and stationary transition probabilities, the system will reach its steady state. This means that the decision to be taken in a certain state i is unaffected by the time period in which the decision has to be taken. After stabilisation, only a limited number of states will be possible; these are the recurrent states.

B. A NUMERICAL EXAMPLE

An example will illustrate the methodology introduced in section II.A. Consider a two-stage serial production system

Demand distribution

Demand per period	Probability
0	0.4
1	0
2	0.1
3	0.2
4	0.3
Mean = 2	
Variance = 3	

Demand is assumed to be independent from period-to-period, all excess demand is backordered. We further assume that the demand process is stationary, the costs are time independent and the planning horizon is infinite.

In a first step, we have to restrict the number of states and decision alternatives :

★ Minimum echelon 2 inventory	: for this problem = -2
★ Maximum echelon 2 inventory	: for this problem = 20
★ Maximum production at 2 = M2	: for this problem = 15
★ Minimum echelon 1 inventory	: for this problem = -6
★ Maximum echelon 1 inventory	: for this problem = 15
★ Maximum production at 1 = M1	: for this problem = 10

The decision alternatives, k , are formed by a combination of the production at 1, $Z1 \in \{0,1, \dots, M1\}$, and the production at 2, $Z2 \in \{0,1, \dots, M2\}$. The cost elements, q_i^k , represent the set-up cost and inventory related costs. Suppose that the echelon stock at facility 2 is 11 units and at facility 1 we have an echelon position of 3 (consequently the installation stock at 2 is 8 units). Furthermore, assume that we decide to produce 5 units at facility 1 and nothing at facility 2, then the expected costs are given by :

Setup cost at 1	= 19.00
Holding costs at 1 ($0.4 \times 3 + 1 \times 0.1$)(14)	= 18.20
Shortage costs (1×0.3)(150)	= 45.00
Holding costs at 2 ($8 - 5$)(5)	= 15.00
Work in Process (5)(5)	= 25.00
	<hr/>
	112.20

The solution for the stated problem is given in Table I.

State i		Steady State Probability	Decision k		Expected Costs q_1^k
Ech.2	Ech. 2		Z2	Z1	
6	4	.7808E-01	8	2	77
7	4	.4932E-01	7	3	82
7	5	.5205E-01	7	2	91
8	4	.2466E-01	6	4	87
8	5	.3288E-01	6	3	96
8	6	.2603E-01	6	2	105
9	5	.1644E-01	5	3	101
9	6	.1644E-01	5	2	110
10	2	.2342E-02	0	6	180.2
10	3	.3041E-01	0	5	107.2
10	4	.9411E-01	0	4	67
10	6	.8219E-01	0	2	85
10	8	.1041E+00	0	0	94
11	3	.1562E-01	0	5	112.2
11	4	.2027E-01	0	4	72
11	5	.6274E-01	0	3	81
11	8	.6575E-01	0	0	99
12	4	.7808E-02	0	4	77
12	5	.1014E-01	0	3	86
12	6	.3137E-01	0	2	95
12	8	.3288E-01	0	0	104
14	6	.3123E-01	0	2	105
14	7	.4055E-01	0	1	114
14	8	.1255E-01	0	0	114
(1)	(2)	(3)	(4)	(5)	(6)

Gain = 95.449493
 Service = .974479
 Fill Rate = .983726

TABLE I : The Recurrent States.

From Table I we learn that the production level is zero at the second echelon when the echelon inventory position at that stage is 10 or more. The production quantity is positive if the inventory position is less than 10. A closer look at the table shows that we always order up to a level of 14. Consequently, at echelon 2 we have a pure (s,S) or min-max policy with $s = 10$ and $S = 14$. At stage 1 we also have an (s,S) policy with $s = 8$ and $S = 8$. The first three states in Table I seem to

be in conflict with this policy. This, however, is not true, the (s,S) policy for the first state e.g. suggests we should produce 4 units but we only have 2 units available (the installation stock at stage 2 is 2 units), so production is two units. Sometimes it is impossible to reach the order-up-to level because of the unavailability of components at the preceding facility.

The optimal average cost per period is 95.45. The service level defined in terms of percentage of periods without stockouts is 97.4 %. The fill rate equals 98.3 %.

It is interesting to note that all policies are based on the echelon stock concept. The echelon stock for facility i equals the installation stock at that facility plus the on-hand and on-order stock position at all succeeding facilities. As a consequence, the echelon-holding costs reflect the value added cost structure.

The following statistics can be obtained from a simulation experiment :

Average order quantity at stage 1	:	2.97
Average order quantity at stage 2	:	6.79
Fill rate	:	98.3 %
Safety stock at stage 1	:	2.62
Safety stock at stage 2	:	0.099
Safety time at stage 2	:	0.24 periods

We can now test e.g. the impact of the cost structure on the amount of safety stock. In the present example the installation holding cost for stage 1 is 14 and for stage 2, the installation holding cost is 5. The value added ratio is $5/14 \times 100 = 35,71 \%$.

The optimal policy can be computed for the following value added ratios.

echelon holding costs					
Stage 2	1	3	5	7	9
Stage 1	9	9	9	9	9
Ratio	10%	25%	35,71%	43,75%	50%

The higher the ratio the more expensive the component. The results are given in the figure below :

Figure 2 : buffer stock as a function of the value added ratio.

The impact of the length of the lead time on safety stocks can also be examined. Clearly, the amount of safety stock is much higher when the lead times are longer.

We now extend the analysis to the case in which there is a capacity constraint at the end product level (stage 1). Assume that no more than 3 units can be produced during a specific time period.

The optimal policy is now given by

Echelon 1 $s = 8$ $S = 9$

Echelon 2 If the on-hand installation position at stage
 2 (OH2) is greater than or equal six units,
 there is no production

 If $OH2 < 6$ then we have that

$s = 11$ $S = \min \{ \text{Echelon stock at 1} + 9, 15 \}$

At first sight, this policy seems difficult to understand. Simulating the above policy (the simulation experiment always covers 7000 periods) indicates that the average order quantity at facility 1 is 2.85 units and for facility 2 the average order quantity equals 5.76 units. This shows that the production quantity is almost 3 for facility 1 and 6 for facility 2. This result can easily be explained by the fact that all facilities must be coordinated with the bottleneck, which is in our case facility 1. A more detailed analysis in section III will reveal that the safety stock at echelon 2 is increased. In a multi-echelon production system, the planning depends on the bottleneck processes.

III. SAFETY STOCKS AND BOTTLENECKS : COMPUTATIONAL RESULTS

A. DESCRIPTION OF THE EXPERIMENT

In this section we analyze the results obtained for eight test problems covering two lead time structures, two demand distributions and two cost combinations. In all cases, a two stage serial production system was examined.

The cost structures we used in the experiment are given in Table II.

FACILITY	Cost Structure 1		Cost Structure 2	
	Setup cost (K)	Echelon holding cost (H)	Setup cost (K)	Echelon holding cost (H)
1	30	5	9	9
2	9	9	30	5

TABLE II : Cost Data for the Experiment

In both cases the shortage cost equals 150.

Two demand distributions for the final product are used in the experiment. The expected demand is the same for both distributions; however, the variance is higher for the second distribution. The specifications are given in Table III.

DEMAND DISTRIBUTION A		DEMAND DISTRIBUTION B	
DEMAND PER PERIOD	PROBABILITY	DEMAND PER PERIOD	PROBABILITY
0	0	0	0.4
1	0.4	1	0
2	0.3	2	0.1
3	0.2	3	0.2
4	0.1	4	0.3
MEAN = 2		MEAN = 2	
VARIANCE = 1		VARIANCE = 3	

TABLE III : Demand Distributions for the Experiment.

The lead time combinations are given in Table IV.

LEAD TIME COMBINATION	LEAD TIME STAGE 1	LEAD TIME STAGE 2
1	1	1
2	2	1

TABLE IV : Lead Time Combinations used in the Experiment.

The resulting eight test problems are summarized in Table V.

PROBLEM NUMBER	COST STRUCTURE	DEMAND DISTRIBUTION	LEAD TIME COMBINATION
1	2	A	1
2	2	B	1
3	1	A	1
4	1	B	1
5	2	A	2
6	2	B	2
7	1	A	2
8	1	B	2

TABLE V : The Eight Test Problems.

For each problem, 3 experiments were conducted :

- The optimal policy is computed in case there are no capacity constraints;
- The optimal operating policies are computed for the same eight examples but with the restriction that production output at stage 1 (end product level) is limited to 3 units
- In a third experiment we examine what would happen if the optimal unconstrained policy was applied in a capacity constrained environment (simulation experiment).

For the experiments a and b the Markov Decision Model was used to find the optimal policies. Afterwards, a simulation experiment is run (covering each time 7000 time periods) to identify the amount of safety stock and safety time. The simulation was done for all test problems in the three experimental settings.

The results are summarized in Table VI.

B. ANALYSIS OF THE RESULTS

The examples introduced above can be split up in two groups depending on the cost structure. Cost structure 1 (problems 3, 4, 7 and 8) results in collapsed policies. A collapsed policy means that the available units from stage 2, are all pushed to stage 1, resulting in equal lot sizes at both stages. MAXWELL and MUCKSTADT [16] have shown that in this case the

PROBLEM	Experiment a (no cap. constraints)			Experiment b (cap. constraints)			Experiment c (un-constrained policy in a constr. environment)
	Stage 1	Stage 2	Exp.cost/period	Stage 1	Stage 2	Exp.cost/period	
1	(6,7)	(7,12)	79.1	(6,7)	If OH2 < 6, then s=6 S=min{ech.stock 1+9,12} otherwise no production	81.65	82.20
2	(8,8)	(10,14)	95.45	(8,9)	If OH2 < 6, then s=6 S=min{ech.stock 1+9,15} Otherwise no production	106.70	110.04
3	(6,9)	(7,10)	90.12	s=6 Production Quantity=3	s=8 Production quantity=3	95.26	110.89
4	(7,11)	(9,11)	111.09	(8,9)	s=11 S=Min{ech.Stock 1+6,12}	121.52	140.56
5	(8,9)except (10,14) if the ech.stock at 2 equals 10, then (9,10)		95.75	(8,10)	If OH2 < 6 s=10 S=Min{Ech Stock 1+9,14} Otherwise no production	99.25	100.40
6	(10,11)exc. (12,16) If the ech. stock at 2 eq. 12 then (10,12)		121.33	(11,12)exc. If Ech.stock at 2 is 14, then (11,11)	If OH2 < 6 s=14 S=Min{ech.1+9,14} Otherwise no production	130.58	138.03
7	(10,12)	(10,12)	114.80	(10,12)	s=10 Production Quantity=3	120.47	127.30
8	(10,14)	(11,14)except If Ech.Stock at 2 equals 10, then (11,15)	143.12	(11,13)	s=13 Production Quantity=3	153.32	180.79

TABLE VI : Operating Policies for each problem

lot sizes should be identical for all items (in a deterministic environment). The simulation experiment shows that this is indeed the case even in a stochastic environment. The average order quantities for both facilities are given in Table VII for the three experiments.

Problem		3	4	7	8
Experiment					
a. No capacity constraints	Fac.1	4.74	3.878	3.738	5.64
	2	4.74	3.878	3.738	5.64
b. With capacity constraints	Fac.1	3	2.86	3	3
	2	3	2.86	3	3
c. Unconstrained policy in a constrained environment	Fac.1	2.459	2.821	2.577	2.755
	2	4.73	3.878	3.738	5.646

TABLE VII : Average order quantities for problems 3, 4, 7 and 8.

It is interesting to observe what happens in experiment c. In this experiment we examine the behavior of an optimal unconstrained policy in an environment in which there actually is limited capacity. The result is that the collapsed policy can no longer be followed and this explains the important cost difference of on the average 13.91 % between experiment b and c.

Cost structure 2 results in totally different policies (examples 1, 2 5 and 6). It are (s,S) policies constrained by the fact that there is never a production order when the installation stock at facility 2 is 6 or more. Moreover, the production quantity is dominated by the bottleneck process. The production quantity at 2 is a multiple of the capacity constraint. The resulting average order quantities are summarized in Table VIII.

From Table VIII it can be seen that the impact of the capacity constraint in experiment c is not so great. The average cost difference between

Problem		1	2	5	6
Experiment					
a	Fac.1	2.624	2.977	3.016	3.869
	2	5.815	6.798	5.81	6.798
b	Fac.1	2.451	2.845	3	2.611
	2	5.816	5.761	6	6.96
c	Fac.1	2.451	2.55	2.599	2.736
	2	5.805	6.80	5.816	6.890

TABLE VIII : Average order quantities for problems 1, 2, 5 and 6.

experiment b and c is 2.66 %. The cost incurred by not taking into account the capacity constrained is not very high in this case.

Next, we examine the behavior of the safety stock and safety time. We recall that safety stock is defined as the expected amount of on hand inventory at the time an order or lot arrives. Safety time is defined as the expected time between the completion of a lot at the preceeding stage and the time at which units from that lot are used in the production of a lot at the succeeding stage.

The results for problems 1, 2, 5 and 6 are given in Table IX.

Problem	1		2		5		6	
Experiment	a	b	a	b	a	b	a	b
.Safety Stock at 1	1.995	1.831	2.621	2.8274	2.097	1.8615	3.056	3.42
.Safety Stock at 2	0.1389	0.2951	0.099	0.2383	0	0	0	1.066
.Safety Time at 2	0.1355	0.1082	0.2409	0.3940	0.196	0.1707	0.239	0.225
.Average numer of backorders	0.0509	0.0795	0.0346	0.0741	0.0795	0.101	0.083	0.117
.Fill rate	0.974	0.960	0.983	0.962	0.960	0.949	0.958	0.942

TABLE IX : Safety Stocks and time for problem 1, 2, 5 and 6

In all four cases the average number of backorders increases in a constrained environment. This explains for a great deal the increased costs per period in the b experiments. The increased backorder position

is associated with an increase in safety stocks and safety time at one or two facilities (problem 5 is an exception). The service level obtained (measured by the fill rate) in the capacitated case is slightly lower, that means that more safety stock would be required to obtain the same service level as in the uncapacitated case.

In Table X the results are given for problems 3, 4, 7 and 8.

Problem	3		4		7		8	
Experiment	a	b	a	b	a	b	a	b
.Safety Stock at 1	1.23	1.731	1.931	2.85	2.216	1.99	2.56	3.452
.Safety Stock at 2	0	0	0	0.139	0	0	0	0
.Safety time at 2	0	0.160	0.574	0.42	0	0	0.25	0.27
.Average number of backorders	0.101	0.075	0.0735	0.0735	0.068	0.129	0.1354	0.119
.Fill rate	0.949	0.962	0.963	0.963	0.956	0.935	0.932	0.940

TABLE X : Safety Stocks and time for problems 3, 4, 7 and 8.

Remember that problems 3, 4, 7 and 8 are characterized by a collapsed policy. The cost difference between the a and b experiments can no longer be explained by the increased shortage costs as was the case for the other problems. The cost difference here is caused by the lower order quantities (increased setup costs). As can be seen from Table X, the fill rate in a capacitated environment improves in a number of cases (because of more safety stock). It appears as if collapsed policies offer a better protection against uncertainty even in the case of capacity constraints. A collapsed policy means that the available units from stage 2, are all pushed to stage 1, consequently the protection against uncertainty is realized at stage 1. In case of capacity constraints this results in a faster response to changes in demand.

For the c experiments we can conclude that the cost difference with e.g. experiment b can be explained by the increased backorder position and consequently the increased shortage costs. The average number of

backorders (experiment c) is given in Table XI. For the problems 3, 4, 7 and 8 we have the additional problem that the collapsed policies are no longer followed.

Problem	1	2	3	4	5	6	7	8
Average number of backorders	0.0812	0.1568	0.1844	0.239	0.126	0.246	0.129	0.358

TABLE XI : Average number of backorders for experiment c.

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