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CHAINS**

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Summary

The present paper analyses the bullwhip problem generated by exponential smoothing algorithms in both ‘stand alone’ passing-on-orders mode, and within inventory controlled feedback systems. Results are predicted from transfer function analysis, and then confirmed by simulation via the Explorer supply chain software. A novel feature of the paper is the introduction of the ‘matched filter’ concept into the exponential smoothing algorithm. This adjusts the value of the smoothing constant depending on whether the Constant, Linear, or Quadratic forecasting model is used. It is shown that matching the filter via noise bandwidth equalises the output variance when the demand is a random signal. Hence some of the unwanted effects of using the Linear and Quadratic forecasting models are attenuated. We show that ‘matching the filter’ is a very valuable concept for controlling bullwhip both in stand alone mode as in inventory controlled feedback systems. However, there is little benefit obtained by using sophisticated forecasting methods within the inventory controlled feedback systems as their tracking ability is not substantially improved.

1. Introduction

Demand amplification is a problem met in real-world supply chains, as originally demonstrated by [1]. What happens as a consequence of this phenomenon is that an individual ‘player’ within the chain orders more products from his supplier than were demanded by his customer. Such behaviour is more likely to occur when orders throughout the chain are fluctuating, since the various ‘players’ start building-up (or running down) stocks in anticipation of further change. A major problem is that in many supply chains this effect multiplies up. The swing in marketplace behaviour then escalates many times before reaching the materials supplier. As the latter may be geographically as well as temporally separated from the marketplace, he might feel that such boom-and-bust

behaviour is a necessary part of earning a living whereas level scheduling might actually be more appropriate.

Research has shown that such demand amplification (now often termed ‘the bullwhip effect’, [2]) can be constrained by proper design and re-engineering of the supply chain [3]. In particular the importance of designing the chain according to established Principles of Material Flow Control has been recognised [4]. The basic philosophy is to root out and eliminate potential causes of bullwhip at source. A regular contributory factor to bullwhip is the forecasting algorithm used by various players within the chain [5], [6] and [7]. Such a bullwhip effect can be generated even if the basic principle used is ‘passing on orders received’, which is a policy already demonstrated as a good fall-back position when playing the Beer Game [8]. The reason for this counter-intuitive state of affairs is that the players forget that even using simple exponential smoothing algorithms can paradoxically lead to amplification over at least part of the frequency spectrum. As this paper will demonstrate, using either the Linear or Quadratic exponential smoothing algorithms, bullwhip is not only possible, but guaranteed! [9].

The theory developed enabled the noise transmission (i.e. the frequency band over which bullwhip is present) to be reduced via matching the bandwidth, according to the smoothing algorithm used. The paper presents the theory from a control engineering perspective, and supports the results via simulation experiments. These include the incorporation of forecasting algorithms within supply chain models. The conclusions are particularly important in terms of buffering value-added processes from excessive short term fluctuations.

2. Some Directly Relevant Results from Control Theory

In control and communications engineering we are dealing with transfer function models which can be written as the ratio of two polynomials in s , the Laplace Operator. The general form of the transfer function relating two signals within a system is then:

$$F(s) = K \left(\frac{1 + b_1 s + b_2 s^2 + \dots + (a_q - b_q) s^q}{1 + a_1 s + a_2 s^2 + \dots + a_q s^q} \right) \quad (1)$$

In this paper we may reasonably assume $K = 1$. We are also particularly interested in the Error Transfer Function $[1 - F(s)]$, which measures our failure or otherwise to follow particular signals. With forecasting applications this error transfer function is particularly important, since it demonstrates whether or not we can detect changes due to step, ramp or parabolic trends in market data. With $K = 1$ the error transfer function becomes:

$$E(s) = 1 - F(s) = \left(\frac{(a_1 - b_1)s + (a_2 - b_2)s^2 + \dots + (a_q - b_q)s^q}{1 + a_1 s + a_2 s^2 + \dots + a_q s^q} \right) \quad (2)$$

The transfer function approach is powerful for many reasons. These include the ability to predict system behaviour in the time and frequency domains, and the ready classification of real life systems into a number of standard models [10]. The classification method allows us to transfer good practice between many kinds of systems applications such as hydraulic power controls, radar trackers, aircraft landing systems, noise filters, machine tool control systems, and inventory systems [11]. Hence a database of ‘good’ designs evaluated in the format of equation (1) can be exploited in a guaranteed good standard of robust system design. It is also readily incorporated in CAD packages relevant to a wide range of industrial applications thus building onto known best practice.

The Final Value Theorem is a simple but powerful tool to evaluate steady-state errors when following deterministic inputs [12]. Some important results obtained via this Theorem include the following conditions for steady-state errors:

For a step input. : the system must be stable and $K = 1$

For a ramp input. : the system must be stable, $K = 1$, and $b_1 = a_1$

For a parabolic input : the system must be stable, $K = 1$, $b_1 = a_1$, and $b_2 = a_2$

These are exactly the same requirements as those needed to enable a Constant, Linear, and Quadratic Model based on exponential smoothing and subsequent forecasting.

3. Trend Detection: Achieving Zero Steady-State Error

In Appendix I the basic building block (and its discrete equivalent) as used in this paper is defined as a Type I system as follows:

$$F_I(s) = \left(\frac{1}{1 + \tau s} \right) \quad (3)$$

This building block can then be used to generate a Type II System ($K = 1$, $a_1 = b_1$), and a Type III System ($K = 1$, $a_1 = b_1$, $a_2 = b_2$)

FIGURE 1. JORDAN SIMULATION FORMAT OF THE THREE TYPES OF FORECASTORS

From Fig. 1, which is the Jordan simulation format [9], it is seen that the technique is to correctly manipulate the output signal from each stage of the simulation model so that the necessary and sufficient conditions for zero steady state error are met. It is readily shown by algebraic manipulation that the resulting transfer function for the Type II System is:

$$F_{II}(s) = \left(\frac{1 + 2\tau s}{1 + 2\tau s + \tau^2 s^2} \right), \text{ with } (a_1 = b_1). \quad (4)$$

And for the Type III System:

$$F_{III}(s) = \left(\frac{1 + 3\tau s + 3\tau^2 s^2}{1 + 3\tau s + 3\tau^2 s^2 + \tau^3 s^3} \right) \text{ with } (a_1 = b_1) \text{ and } (a_2 = b_2). \quad (5)$$

These fulfil the necessary and sufficient conditions for achieving zero steady state errors for the ramp and parabolic inputs respectively, a result that will be confirmed later via simulation. To obtain the exponential smoothing equivalents for $F_I(s)$, $F_{II}(s)$, and $F_{III}(s)$, all we need to do is replicate the block diagrams and signal manipulations and replace $F_I(s)$ with the basic exponential smoothing algorithm $ES(\alpha)$ as shown in Appendix I. Here the exponential smoother has been expressed in terms of the delay operator z^{-1} :

$$ES(\alpha) = \left(\frac{\hat{Y}}{Y_{obs}} \right) = \left(\frac{\alpha}{1 - (1 - \alpha)z^{-1}} \right) \quad (6)$$

Where the conversion factor between α and τ is given by:

$$\alpha = \left(\frac{1}{1 + \left(\frac{\tau}{\Delta t} \right)} \right) \quad (7)$$

4. Why Achieving Zero Error Has a Price to Pay

The key relationships between the forecasting and control/communications engineering approaches to trend detection may be summarised as shown in Table I.

EXPONENTIAL SMOOTHING TERMINOLOGY	CONTROL ENGINEERING TECHNOLOGY	STEADY STATE ERROR			SINGLE STAGE BULLWHIP FACTOR		
		TO A STEP INPUT	TO A RAMP INPUT	TO A PARABOLIC INPUT	STEP INPUT	FREQUENCY RESPONSE	NOISE VARIANCE
CONSTANT MODEL	TYPE I SYSTEM	ZERO	FINITE	INFINITE	1.00	1.00	1.00
LINEAR MODEL	TYPE II SYSTEM	ZERO	ZERO	FINITE	1.14	1.15	2.50
QUADRATIC MODEL	TYPE III SYSTEM	ZERO	ZERO	ZERO	1.21	1.29	4.125

TABLE I. SUMMARY OF BULLWHIP FACTORS FOR THE CONSTANT, LINEAR, AND QUADRATIC EXPONENTIAL SMOOTHING MODELS.

Henceforth for 'Constant Model' we may read 'Type I', for 'Linear Model' read 'Type II', and for the 'Quadratic Model' read 'Type III'. These analogies between control and

forecasting may then be exploited by transferring results obtained in one domain across to the other usually via generic models known as ‘Standard Forms’. Such a procedure allows us to establish the price to be paid by achieving zero steady-state error, which thereby improves one performance metric but may cause another to deteriorate.

For example, consider the Linear Model used to detect a ramp signal in the marketplace demand. This requires that ($a_1 = b_1$). But as shown in Appendix II from control theory, this means that in response to a step change, the forecast must overshoot (not *might* but *definitely will*). Herein lies the self-induced Bullwhip effect due to using this particular forecasting algorithm. Hence the penalty paid for trend detection is demand amplification. However, the situation is worsened on using the Quadratic Model requiring ($a_1 = b_1$) and ($a_2 = b_2$). This is because it may be shown that by reference to Appendix II, the Quadratic Model additionally has an overshoot on the ramp response, leading to an even greater overshoot to a step change in demand at the marketplace.

The step response ‘Bullwhip Factor’ shown in Table I reflect these important differences between the three forecasting models in their time response to a step input signal. Also shown in Table I is a measure of bullwhip that could also be used, which is defined in the frequency domain. Specifically, it is the peak amplitude ratio when the system is subjected to a sinusoidal input for all frequencies from zero to infinity. This is illustrated in Figure 2a where we have the frequency response curves for the three types of forecasters. The amplifications for the Linear and the Quadratic model are to be expected since since time and frequency domain are intimately related via the Fourier Transform. In particular, if the step response overshoots then the frequency response will do likewise. Remembering that in supply chains the nature of ‘bullwhip’ tends to be multiplicative rather than additive, these effects can rapidly build up to become a significant factor in demand amplification. It only requires a few internal or external ‘players’ to ostensibly ‘pass orders up the chain’ via trend detection algorithms to generate a substantial bullwhip effect. But there is worse news to come: the filtering (unwanted signal) capability also has been drastically impaired by moving from the Constant to the Linear Model and made even worse by selecting the Quadratic option (see the responses to high frequencies in figure 2a). However on the upside, control theory can help us achieve the best of both worlds in this respect via the matched filter concept [9]: by adjusting τ (and hence the smoothing parameter α), it will be possible to greatly reduce the noise transmission without sacrificing the trend detection capability.

5. Noise Bandwidth as a Measure of System Performance

Yet another bullwhip measure that we could use is the noise bandwidth as shown in Table I. This is a helpful measure of performance for comparing the filter characteristics of a system design. Such a metric is common in communications engineering [13]. It is defined as:

$$W_N = \int_0^\infty |F(j\omega)|^2 d\omega \quad (7)$$

where $F(j\omega)$ is the steady state response at excitation frequency ω . For reasonable values of transfer functions, W_N can be calculated directly from formulae based on Parseval’s Theorem. Alternatively the frequency response may be calculated at set points and W_N (being the surface under the squared frequency response curve) estimated via numerical integration. For our purpose the Parseval Theorem approach is to be preferred since it will lead to an analytic solution to our problem of finding a ‘fair’ comparison between the Type

I, Type II and Type III systems. This in turn will mean that the values of τ (and hence α) will vary according to the Type Number. In [14] it is shown that the noise bandwidths are:

$$\left. \begin{aligned} W_{NI} \text{ (Type I)} &= \frac{\pi}{2} \cdot \frac{1}{\tau_1} \\ W_{NII} \text{ (Type II)} &= \frac{\pi}{2} \cdot \frac{5}{2\tau_2} \\ W_{NIII} \text{ (Type III)} &= \frac{\pi}{2} \cdot \frac{33}{8\tau_3} \end{aligned} \right\} \quad (8)$$

For the special case where $\tau_1 = \tau_2 = \tau_3$ the noise bandwidths are in the ratio of 1 (Type I), 2.5 (Type II), 4.125 (Type III). ***This accounts for the difference in filter characteristics between the systems as the Type No. is increased.*** The high frequency response of the Type II and Type III models clearly lets more ‘noise’ through the system as confirmed by the larger noise bandwidths. In effect, for the Type II Model the bullwhip extends over a wide range of frequencies and which is nearly doubled again using the Type III Model, see Figure 2a. However, W_N has an important attribute which allows us to redesign our filters for better performance. ***This is that for an input of ‘white’ random noise (constant power density at all frequencies) and zero mean, W_N is a direct measure of the variance at the output from the filter.***

FIGURE 2A and 2B. FREQUENCY RESPONSE CURVES FOR THE THREE TYPES OF STAND-ALONE FILTERS IN THE CONSTANT AND MATCHED CASES

6. Matched Filters Equalising Noise Bandwidth by Design

We ***must*** accept that in order to detect ramp and parabolic trends, the consequence is to induce a bullwhip effect into our estimates. However we do have some scope in ensuring that the consequence is not the transmission of excessive ‘noise’ along with the message we are trying to track. The solution proposed here is to adjust the value of α downwards as we move from the Constant to the Linear Forecasting Model and downwards again as we move onto the Parabolic Model. According to our theory these ratios should be as follows: $\tau_{\text{(Linear Model)}} = 2.5 \tau_{\text{(Constant Model)}}$ and $\tau_{\text{(Parabolic Model)}} = 4.125 \tau_{\text{(Constant Model)}}$. Working back through the equation linking τ with α (equation (7)), we obtain the corresponding exponential smoothing parameters as $\alpha = 0.2$, $\alpha = 0.091$, and $\alpha = 0.057$ respectively. These are substantially different for the Constant, Linear, and Quadratic Models.

In figure 2b, it can be seen that, after adapting the smoothing parameters in the above described way, noise bandwidths are indeed equal for the three types of forecasting. To verify the approach, the three models so matched to have identical noise bandwidths, have been subjected to a random white noise signal. The output standard deviations have been estimated for a significant length of time series after discounting any initial transients. These new results are also shown in Table II, in which the noise rejection characteristics of all three models are compared for constant α values and for α varied according to the requirement for a Constant, a Linear, and a Parabolic Model. The output signal standard deviations estimated for the variable α model experiments are again within 2% of the predicted values and confirm that we have indeed matched the noise transmissions. The accuracy of prediction adds further confidence to the EXSMO transformation used for discrete simulation in support of an analogue based theory.

So in one sense only it *is* possible to simultaneously get the best of both worlds. We still have to tolerate the bullwhip levels shown in Table I. However, as the simulation results clearly demonstrate we can trend detect and avoid too much noise cluttering up our forecasts by adjusting the exponential smoothing constant downward according to our need to use the Constant, Linear, or Quadratic Models. To summarise, if we match the filter properties as suggested, we shall reduce the range of bullwhip frequencies generated by the Linear Model by 2.5:1, and similarly reduce the range of bullwhip frequencies generated by the Quadratic Model by 4.125:1. Both are highly significant reductions available to operations managers if the increased trend detection time is deemed acceptable.

SIMULATION CONDITIONS	STANDARD DEVIATION	MODEL USED		
		CONSTANT	LINEAR	QUADRATIC
CONSTANT α FOR ALL MODELS	MEASURED	0.2466	0.4031	0.5230
	PREDICTED	0.2466	0.3900	0.5008
α VARIED ACCORDING TO TYPE NO.	MEASURED	0.2466	0.2483	0.2411
	PREDICTED	0.2466	0.2466	0.2466

TABLE II. RESULTS OF SIMULATION EXPERIMENTS IN RESPONSE TO RANDOM NOISE INPUTS APPLIED TO BOTH MATCHED AND UNMATCHED FILTERS

7. Cost Analysis via the Bullwhip Explorer.

The Bullwhip Explorer package, [7] provides a useful tool for quantifying the performance of ordering decisions via analysis of the inventory and shortage costs coupled with production costs. This process yields a result that is easy to understand by practitioners unfamiliar with control theory concepts. For the present purpose the demand signal is represented by a set of discrete sine waves and costs calculated at each frequency and summed across the frequency range. Specifically within the software the inventory holding and shortage costs are defined as:

$$ICost = \frac{\sum_{w=0, step(\pi/200)}^{\pi} \left(\sum_{t=100}^{500} v * AINV_t \right)}{200} \quad (9)$$

where, $AINV_t$ = actual inventory level in time period t in response to excitation frequency w and v = cost weighting. In our simulations $v = 0.05$ per unit of $AINV$ per period (i.e. when $AINV_t > 0$) for inventory holding costs and 2 per unit of $AINV$ per period for shortage costs

(i.e. when $AINV_t < 0$), although this can be changed by the user of the system as desired. Thus the inventory and shortage costs are defined as the average costs of holding inventory (or the shortage costs) in response to 200 frequencies between 0 and π radians per time period, ignoring any start-up transients.

The production adaptation costs are defined as:

$$PCost = \frac{\sum_{w=0, \text{step } (\pi/200)}^{\pi} \left(\sum_{t=100}^{500} 1.5 * |ORATE_t - ORATE_{t-1}| \right)}{200} \quad (10)$$

where $ORATE_t$ is the production order rate in time period t to excitation frequency w . Thus the production adaptation cost is defined as the average of the sum of the incremental change in the production order rate in response to 200 excitation frequencies between 0 and π radians per time period, ignoring any start-up transients.

Table III, shows the results from the Bullwhip Explorer cost analysis for a constant, linear, and parabolic exponential smoothing for two cases. Firstly, the smoothing constants have been kept the same irrespective of Type No., as in the frequency analysis above. Secondly, we retain trend detection capability but adjust the smoothing constants as a function of Type No. The result is that failing to match the smoothing constant to Type No. will significantly increase the total costs as more trend detection is introduced. However, intelligent selection of the smoothing constants, as we move from Type I to Type III, can actually marginally reduce costs!

Case	Type I	Type II	Type III
Constant	239.79	243.33	283.47
Matched	239.79	229.58	226.84

TABLE III. TOTAL COSTS FOR THE DIFFERENT STAND-ALONE FORECASTORS.

8. Application to Supply Chain Dynamics

To fully understand the impact of forecasting methods on supply chain dynamics we use the well-established APIOBPCS model shown in causal-loop form in Figure 3, [15]. The APIOBPCS can be expressed in words as ‘Let the production targets be equal to the sum of some form of forecasted demand, smoothed over Ta time units, a fraction $(1/Ti)$ of the discrepancy in the finished goods inventory levels and a fraction $(1/Tw)$ of our WIP error’. The specific transfer function describing the APIOBPCS model used in this study is provided in Appendix IV.

It is readily shown that by adjusting model parameters and/or removing signal paths, APIOBPCS can represent a wide range of industrial ordering policies. This includes make-to-order, make-to-stock, level scheduling and pass-on-orders (also known as ‘order-up-to’). For certain conditions the APIOBPCS response is analytic for both deterministic inputs and for noise bandwidth. The response of ‘traditional’ supply chains is readily predicted in the frequency domain via coupling of appropriate transfer functions and sub-optimised using the filter concept [16]. APIOBPCS uses the constant exponential smoothing in the feed forward path that determines the extent to which present sales influence our ordering process. It is then a simple matter to substitute either the linear or parabolic exponential smoothing algorithm in the forward path.

FIGURE 3. CAUSAL LOOP DIAGRAM OF APIOBPCS.

The transfer function for Type II and Type III exponential smoothing within APIOBPCS are given in Appendix IV. Figure 4 shows the steady-state frequency response of a single stage APIOBPCS for constant, linear, and parabolic exponential smoothing for two distinctive cases. Firstly, the smoothing constants have been kept the same irrespective of Type No., for which it is self-evident that the range of bullwhip frequencies increases considerably as we introduce trend detection. Secondly, we retain trend detection capability but adjust the smoothing constants as a function of Type No. The result is clearly to significantly reduce the bullwhip frequency range. Hence the principle of matching filter capability applies equally well to closed loop ordering systems. Table IV illustrates how our bullwhip measures alter when using different forecasting systems in the matched and constant cases. The importance of matching the filters is clearly demonstrated in the three bullwhip measures.

FIGURE 4A and 4B. FREQUENCY RESPONSE CURVES FOR THE THREE TYPES OF APIOBPCS FILTERS IN THE CONSTANT AND MATCHED CASES ($T_i = T_w = 4$)

EXPONENTIAL SMOOTHING TERMINOLOGY	CONTROL ENGINEERING TECHNOLOGY	STEADY STATE ERROR			SINGLE STAGE BULLWHIP FACTORS			
		TO A STEP INPUT	TO A RAMP INPUT	TO A PARABOLIC INPUT		STEP INPUT	FREQUENCY RESPONSE	NOISE VARIANCE
CONST ANT MODEL	TYPE I SYSTEM	ZERO	ZERO	FINITE		1.24	1.42729	1.1781
LINEAR MODEL	TYPE II SYSTEM	ZERO	ZERO	FINITE	Matched	1.2592	1.46675	1.069901
					Unmatched	1.4110	1.80502	2.18166
QUADR ATIC MODEL	TYPE III SYSTEM	ZERO	ZERO	FINITE	Matched	1.2339	1.46951	1.0199
					Unmatched	1.4679	2.10247	3.32707

TABLE IV. SUMMARY OF BULLWHIP FACTORS FOR THE CONSTANT, LINEAR, AND QUADRATIC EXPONENTIAL SMOOTHING MODELS WITHIN INVENTORY CONTROLLED FEEDBACK SYSTEMS.

Using the Bullwhip explorer package as described earlier to analyse the cost performance of the three forecasters results in the costs shown in Table V. Here we can see a slight decrease in cost performance if a matched filter is used across the Type Nos. However, failing to match the filters produces increased costs as more sophisticated algorithms are used.

Case	Type I	Type II	Type III
Constant	359.54	408.21	451.77
Matched	359.54	329.70	319.71

TABLE V. TOTAL COSTS FOR THE APIOBPCS ORDERING SYSTEM.

Appendix III, however, shows how well the different forecasting methods perform in the APIOBPCS model in the time domain. Specifically the production completion rate against consumption is shown in Figure III.1. Significantly, all three types adequately track the step and the ramp input, however they all produce a constant error to the parabolic input. The integral of the error yields information on the drift in the actual inventory when responding to an input. Unsurprisingly all three types, when placed inside the APIOBPCS model, result in no inventory drift to a step input, a finite error in inventory to a ramp and infinite error in inventory to a parabolic input. Thus, although we have shown that when using type 2 and type 3 forecasters in feedback systems, the matched filter concept should be applied in order to reduce bullwhip, a more fundamental question first has to be asked ‘Why bother using sophisticated forecasting algorithms within feedback ordering policies?’

9. Conclusions

The idea that every system (including every ordering algorithm) may be treated as a filter is not new. However the concept has only been applied in the supply chain context in recent years. As shown in the paper, some surprisingly simple conclusions may be drawn, including the prediction of system induced bullwhip emanating from trend detection

exponential smoothing algorithms in the stand-alone case or passing on orders mode. However, even if trend detection is obligatory, the range of bullwhip frequencies still lies within the discretion of the system designer. Here we have shown that reducing the exponential smoothing constant as the Type No. increases makes a significant contribution to reducing the bullwhip frequency range, which has been verified via the Bullwhip explorer software via a cost analysis. There is also a consequential attenuation at high frequencies.

The advantages of using more sophisticated forecasting methods within inventory controlled ordering systems are, however, less obvious as their inclusion does not guarantee trend detection via zero steady state error in the inventory response, so the question has to be asked, 'Why bother using sophisticated forecasting algorithms?' They do, however, reduce the production adaptation and inventory costs as demonstrated by the bullwhip explorer if the matched filter concept is used. If the filters are not matched however, costs will significantly increase!

Acknowledgements

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Appendix I

Developing Elemental Dynamic Building Blocks for Analogue and Discrete Simulation

Recognising the problems associated with exponential smoothing and identifying possible solutions is made easier by viewing trend detection as needing particular combinations of elemental building blocks. These need to be recognisable in both analogue (continuous, s Transform) and discrete (sampled-data, z Transform) format. Transferring knowledge from one domain (where the problem may have already been met and solved) is thereby made much more transparent. In the analogue domain the elemental block we use has the transfer function:

$$F_I(s) = \left(\frac{1}{1 + \tau s} \right) \quad (I.1)$$

which is in fact the Type I first order system where τ is termed the time constant. To relate this filter (termed Type I because it has zero error to a step input) to exponential smoothing of the usual form:

$$\hat{Y}_{(K+1)} = \hat{Y}_{(K)} + \alpha (Y_{obs(K+1)} - \hat{Y}_{(K)}) \quad (I.2)$$

using the EXSMO relationship established by [17] to link α and τ . This is based on the Matsuburu Time Delay Theorem [18] which matches the area under the step response error curve. The result is the fundamental relationship:

$$\alpha = \left(\frac{1}{1 + \left(\frac{\tau}{\Delta t} \right)} \right) \quad (I.3)$$

In equation I-3 Δt is the sampling interval for the data available at the K th and $(K+1)$ th instants. Experience has shown that this equation is surprisingly accurate for practical applications, except where Δt approaches τ in which case the Tustin approximation [19] should be used. For the purpose of demonstrating the equivalence principles in this paper, equation I-3 proves to be both simple and accurate. As [20] has pragmatically put it when transforming from continuous to discrete time, ‘The proof of the pudding is in the eating!’. Fig. I.1 shows the analogue and discrete transfer functions obtained via the EXSMO transformation. Hereinafter in this paper we shall regard these two formats as being interchangeable when moving from one domain to the other.

FIGURE I.1. COMPARISON OF FIRST ORDER LAG AND EXPONENTIAL SMOOTHING

Appendix II

Graphical Proof of the Time Domain Bullwhip Properties of the Linear and Parabolic Exponential Smoothing Models in Stand Alone mode

(a) Linear (Type II) Model

Let $E_u(t)$, $E_r(t)$, and $E_f(t)$ be the error responses as a function of time when the model is subjected to a step, ramp, and parabolic input respectively. From the integral properties of these input signals it is clear that:

$$E_r(t) = \int_0^t E_u(t) dt \quad (\text{II.1})$$

If we wish the trend (i.e. response to a ramp input) to be detected with zero offset it follows that $E_r(\infty) \equiv 0$. Hence

$$\int_0^\infty E_u(t) dt \equiv 0 \quad (\text{II.2})$$

But when $t = 0$, there will be a finite positive error. Hence there must be a contribution to the right hand side of equation (II. 1) from this source. Thus the only way in which there will be zero steady state offset for a ramp input is for this positive error to be balanced out by a later contribution to this integral caused by an excursion into a region of negative error. Fig. II.1, which is due to Porter [21], shows this effect in detail in graphical form for the Linear (Type II) Model. It is a physical explanation as to why this model must contribute a bullwhip effect due to essential overshoot to a step change if a ramp trend is to be followed with zero steady-state offset.

(b) Parabolic (Type III) Model

The relationship between the ramp and parabolic errors is given by:

$$E_f(t) = \int_0^t E_r(t) dt \quad (\text{II.3})$$

If we wish the parabolic input to be detected with zero offset then clearly $E_f(\infty) \equiv 0$.

$$\text{Hence } \int_0^\infty E_r(t) dt \equiv 0 \quad (\text{II.4})$$

But when the model is responding just after $t = 0$ there must be a positive error contribution to the right hand side of Equation II.2. This must be cancelled out by a later negative error contribution to the integral, and this can only be caused by an overshoot in the ramp response. Fig. II.1 again illustrates the point.

(c) Relationship Between the Constant, Linear and Parabolic Models

Note that the Parabolic Model must have $E_u(\infty) = 0$, $E_r(\infty) = 0$, and $E_f(\infty) = 0$.

In equation (1) this is guaranteed if the system is stable **and** $K = 1$, $(b_1 = a_1)$, $(b_2 = a_2)$. In other words as the Type numbers are increased then the previous necessary conditions are carried forward and supplemented by additional criteria. As summarised in Fig II.1 the practical effect is that the Constant Model has no overshoot to the step response, the Linear Model has at least one overshoot to the step response, and the Parabolic Model has one overshoot and one undershoot to the step response. Note that the issue of more than one overshoot or undershoot does not arise with these exponential smoothing models. This is because with the particular structure used, it is not possible for the system characteristic equation to have complex roots.

FIGURE II.1. GRAPHICAL PROOF OF THE BULLWHIP PROPERTIES IN THE TIME DOMAIN OF THE THREE TYPES OF FORECASTORS IN STAND ALONE MODE (Key: Ess –Error in steady state, Ei – Integral of the error).

Appendix III

Graphical Proof of the Time Domain Bullwhip Properties of the Linear and Parabolic Exponential Smoothing Models in APIOBPCS feedback mode

FIGURE III.1. GRAPHICAL PROOF OF THE BULLWHIP PROPERTIES IN THE TIME DOMAIN OF THE THREE TYPES OF FORECASTORS IN APIOBPCS MODE (Key: Ess –Error in steady state, Ei – Integral of the error).

Appendix IV. Transfer Functions of APIOBPCS Required.

The three APIOBPCS models can be described via the following transfer functions where,

- equation III.1 represents the *ORATE/CONS* for the APIOBPCS model with Type I forecasting,
- equation III.2 represents the *ORATE/CONS* for the APIOBPCS model with Type II forecasting and
- equation III.3 represents the *ORATE/CONS* for the APIOBPCS model with Type III forecasting.

and where *ORATE* stands for order rate and *CONS* stands for customer consumption or thus demand,

$$\frac{E^{4s} (4sTi + Tw + s(Ta + Ti)Tw)}{(1 + sTa)(Tw + Ti(-1 + E^{4s}(1 + sTw)))} \quad (III.1)$$

$$\frac{E^{4s} (4s(1 + 2sTa)Ti + Tw + s(Ti + Ta(2 + s(Ta + 2Ti)))Tw)}{(1 + sTa)^2 (Tw + Ti(-1 + E^{4s}(1 + sTw)))} \quad (III.2)$$

$$\frac{E^{4s} (4s(1 + 3sTa(1 + sTa))Ti + ((1 + sTa)^3 + s(1 + 3sTa(1 + sTa))Ti)Tw)}{(1 + sTa)^3 (Tw + Ti(-1 + E^{4s}(1 + sTw)))} \quad (III.3)$$

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